FACULTY
OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

DEPARTMENT OF
COMPUTER SCIENCE
AND ENGINEERING

CENTRE
OF COMPUTER GRAPHICS
AND VISUALIZATION

PLZEŇ
CZECH REPUBLIC

http://graphics.zcu.cz

COMPUTER GENERATED DISPLAY HOLOGRAPHY

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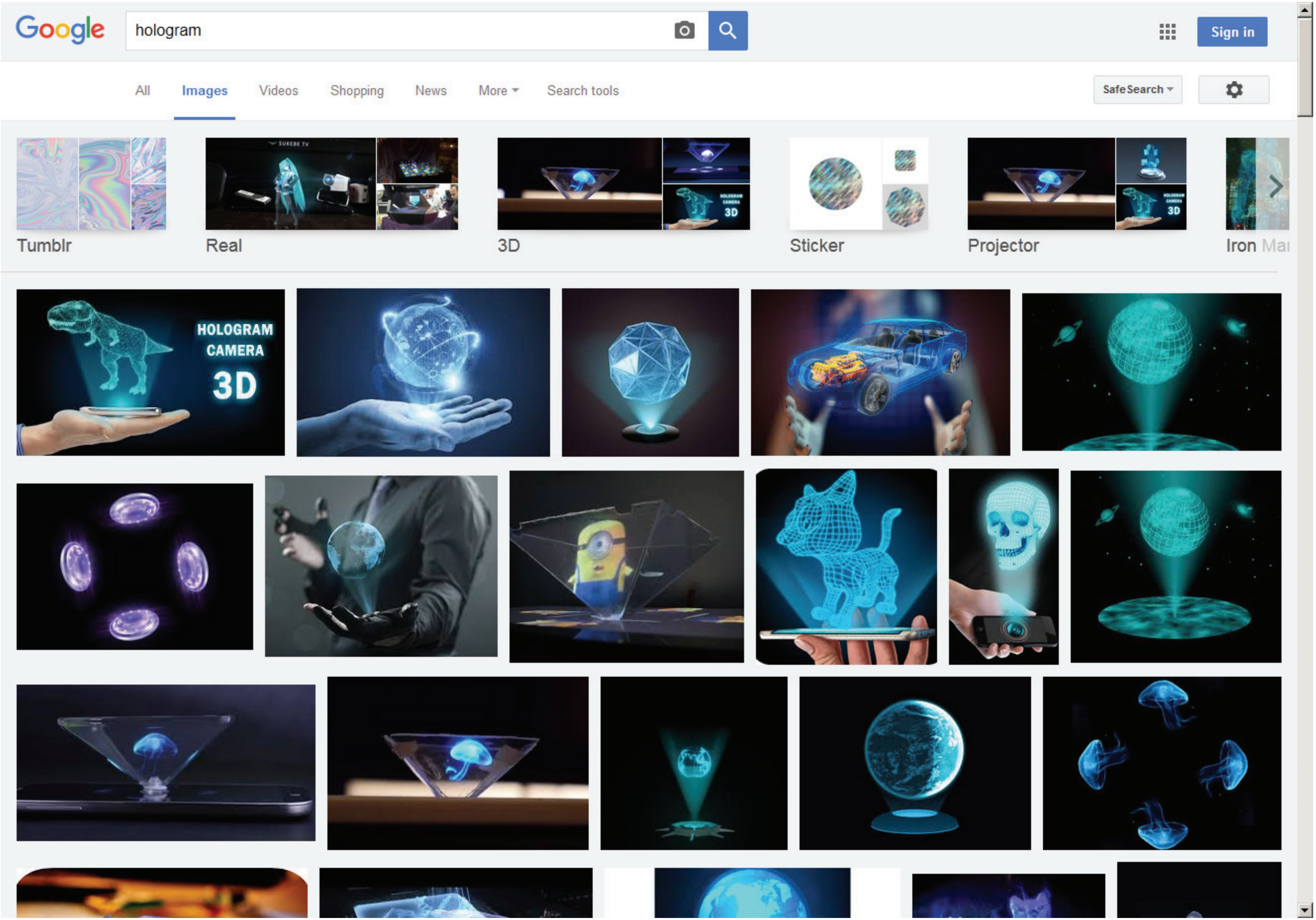
A tutorial at Eurographics 2017
Lyon, France, 24 April 2017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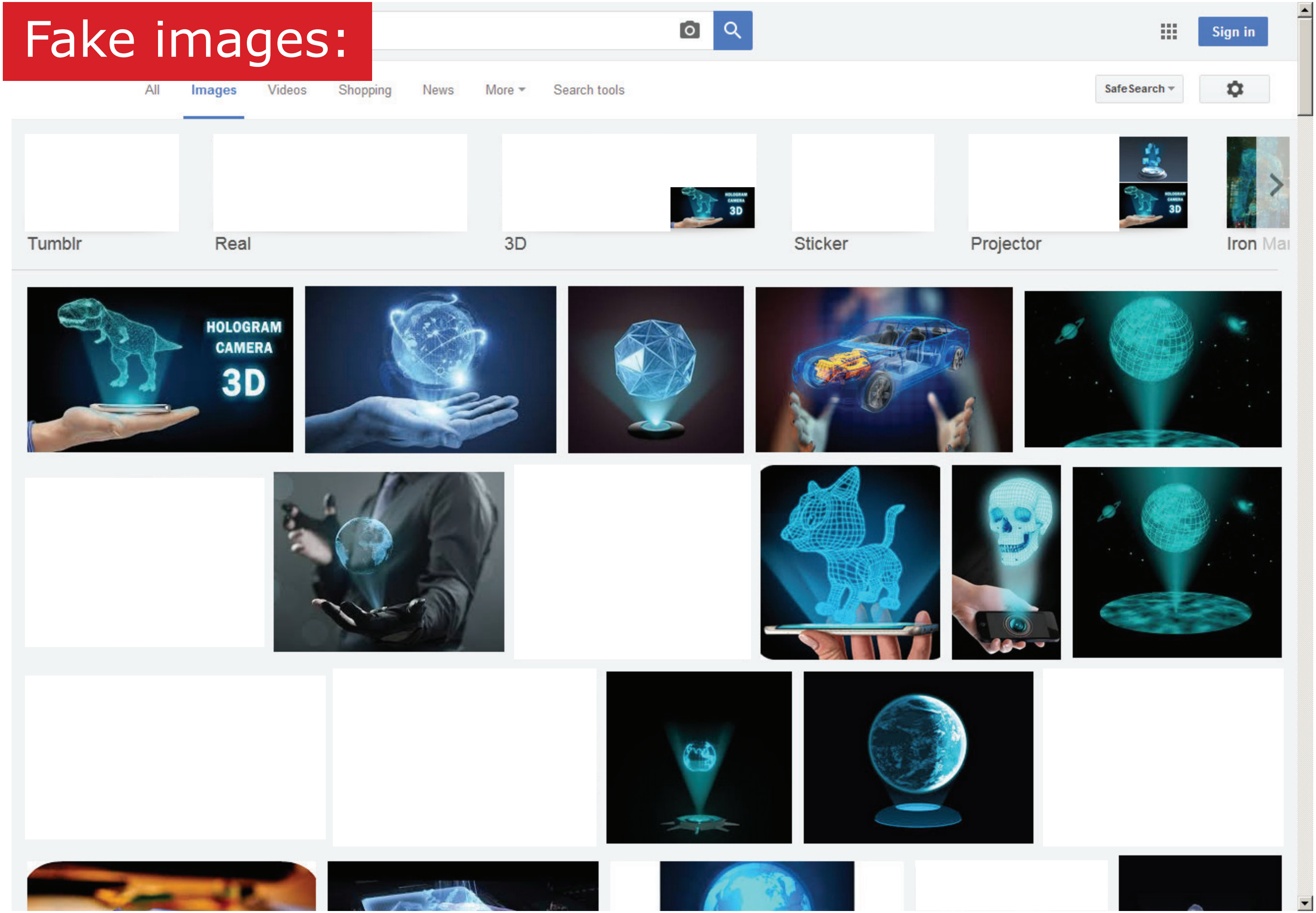
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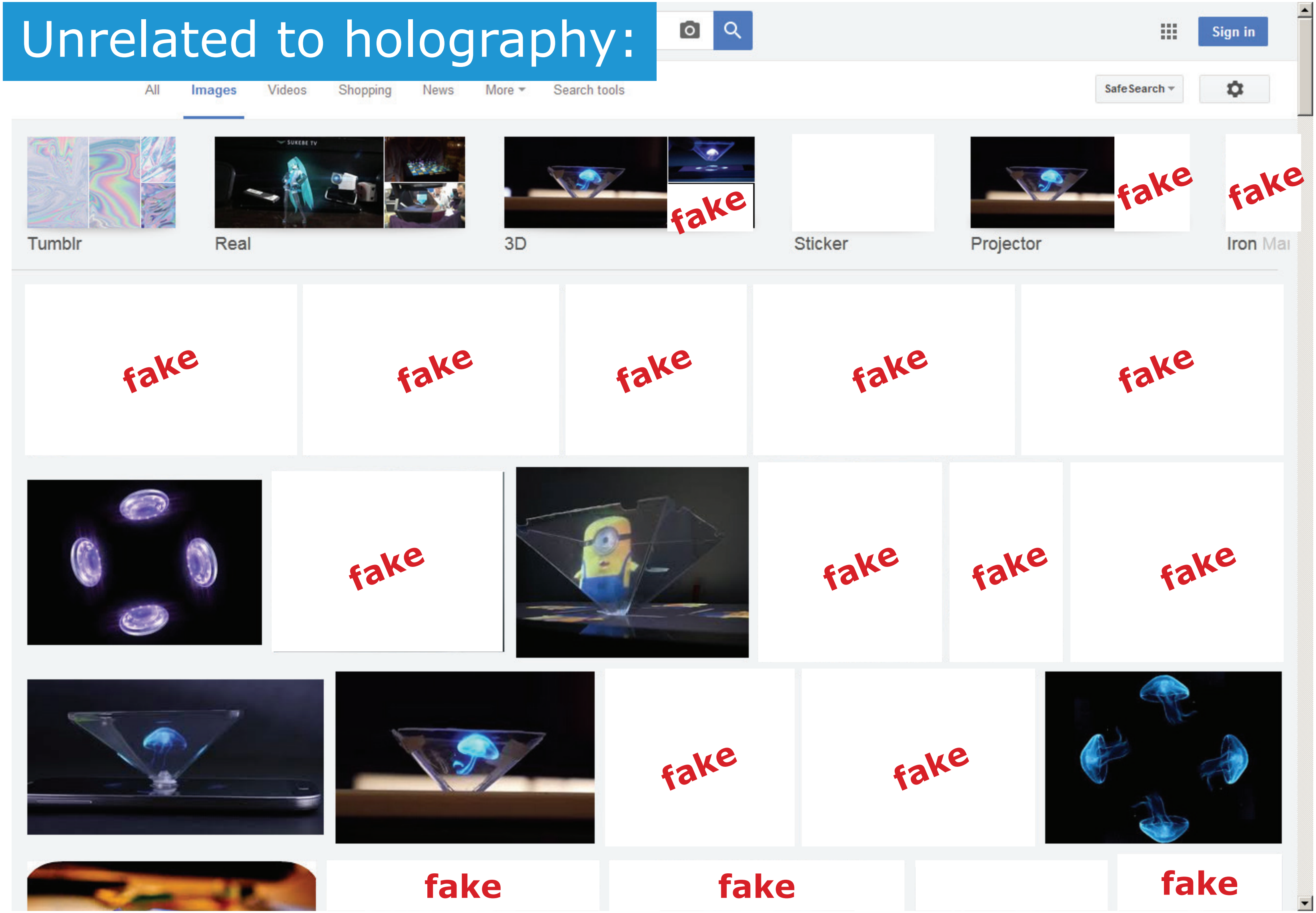
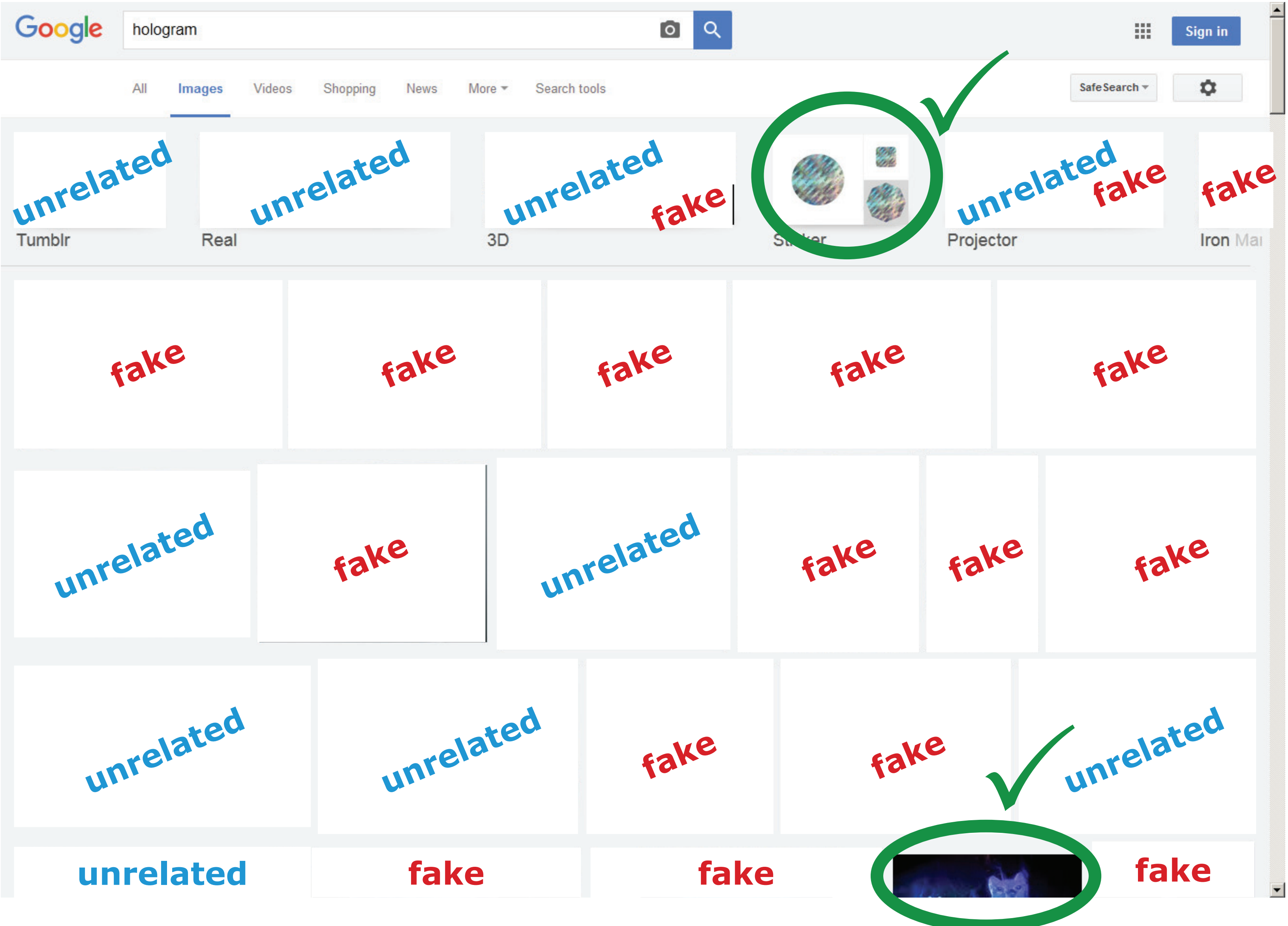
Unrelated to holography:

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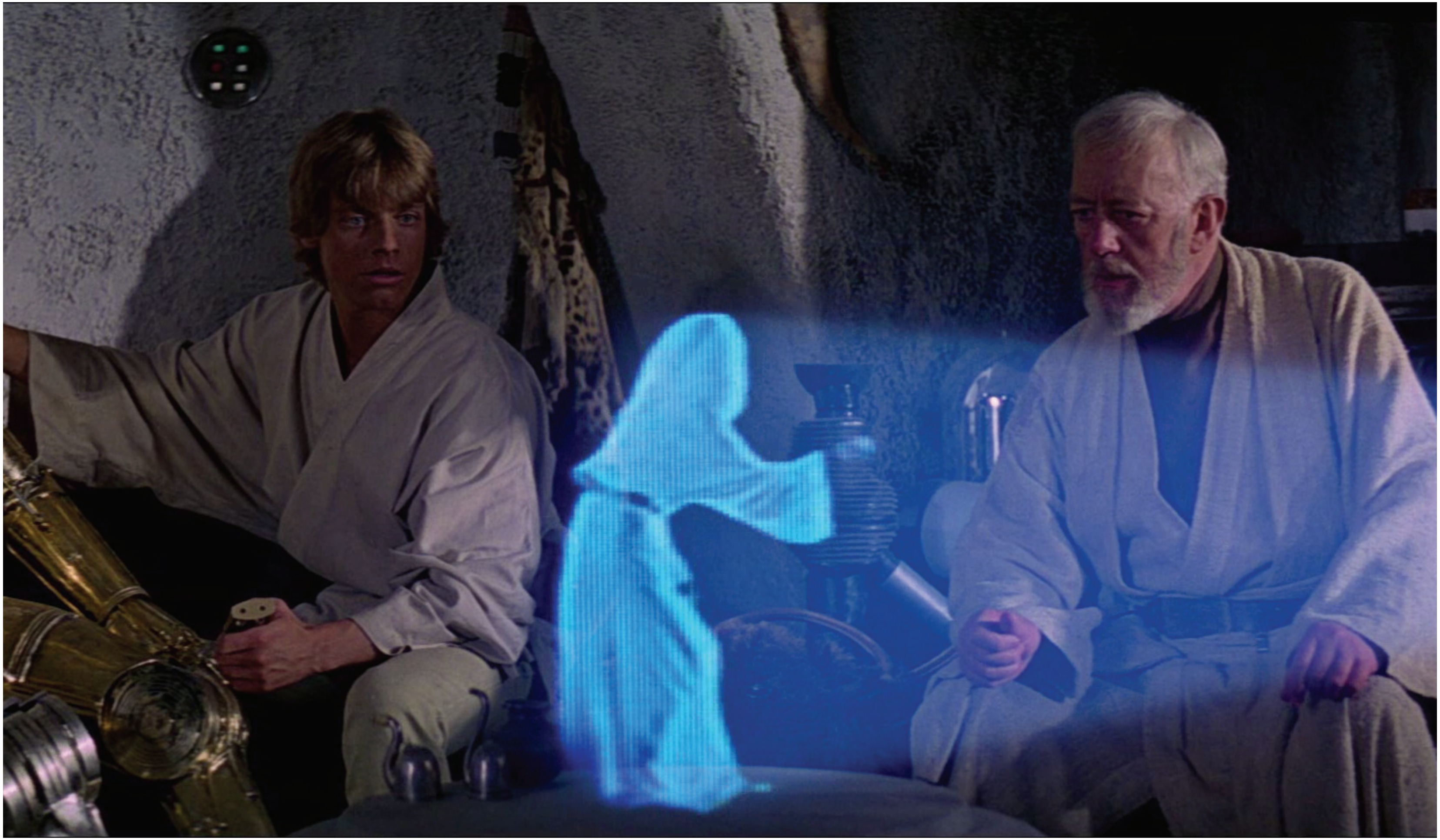
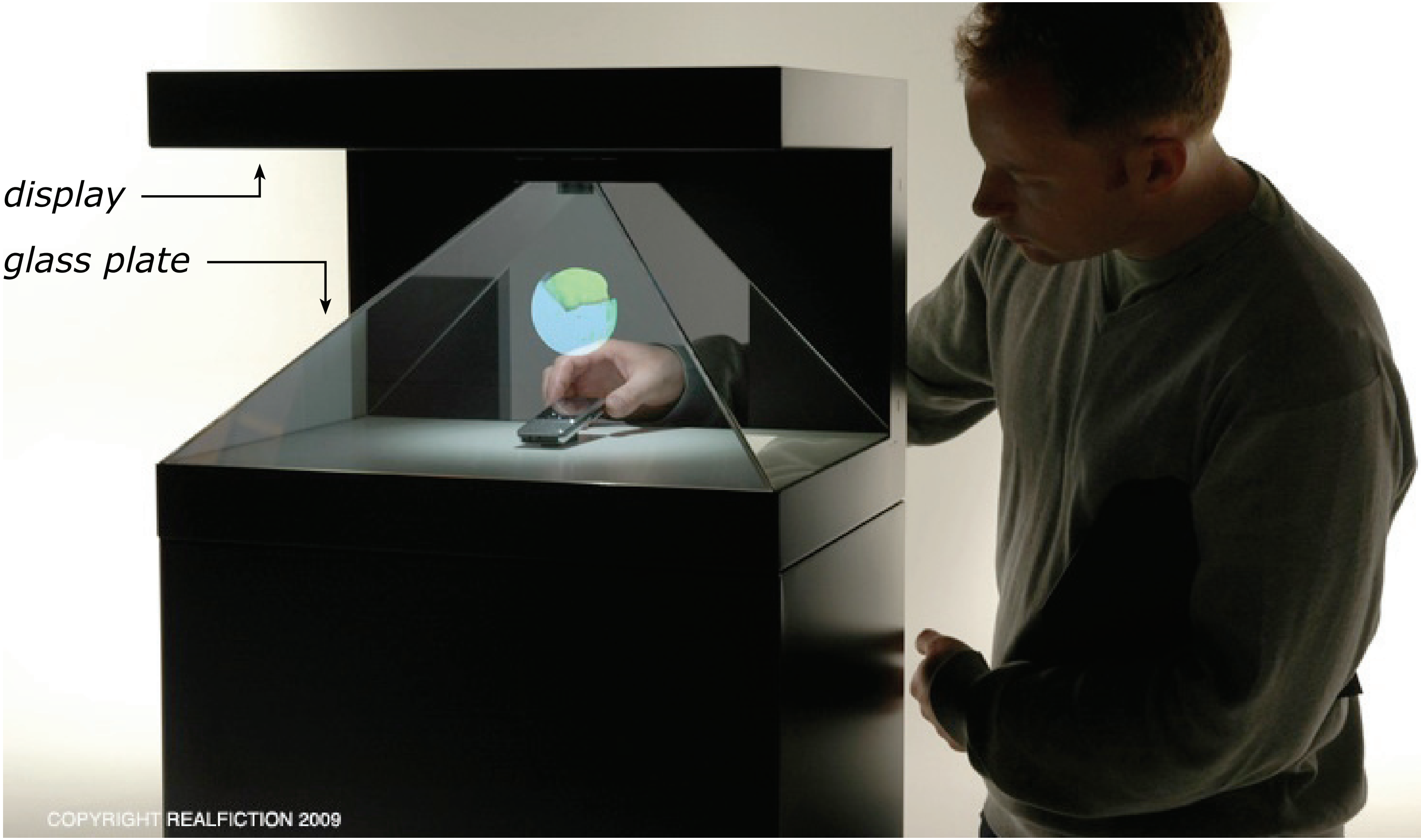
Tutorial contents



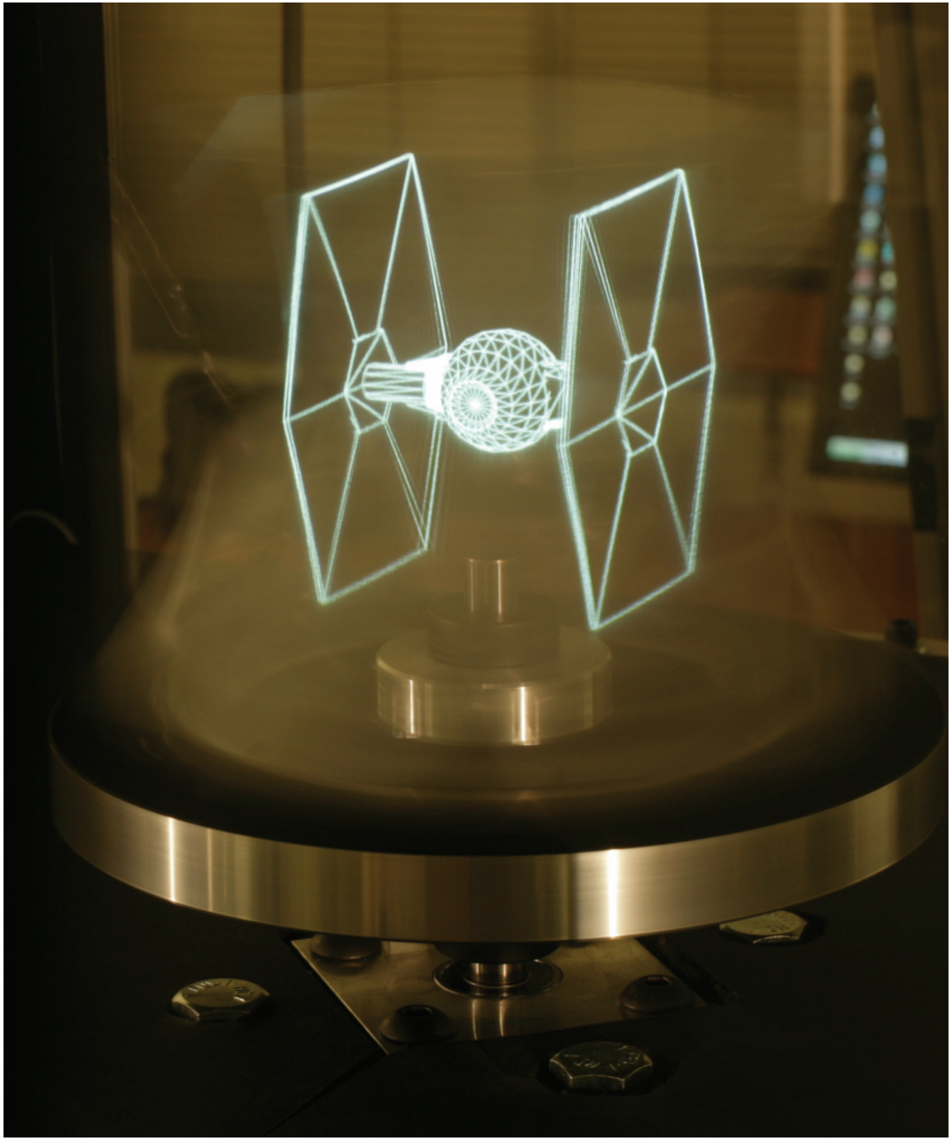
Star Wars: A New Hope (directed by G. Lucas, 1977)

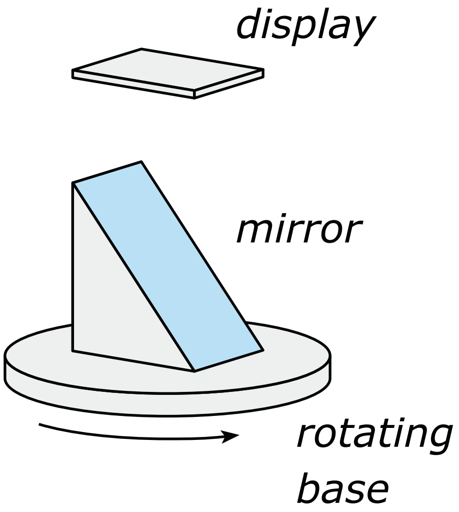
Microsoft HoloLens: visualization of augmented reality

Kagamine Rin & Len at a Hatsune Miku concert

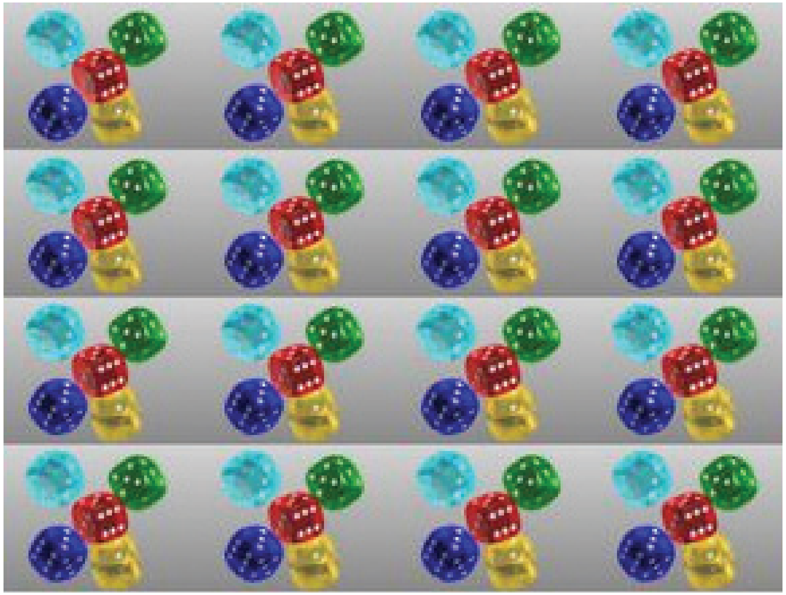


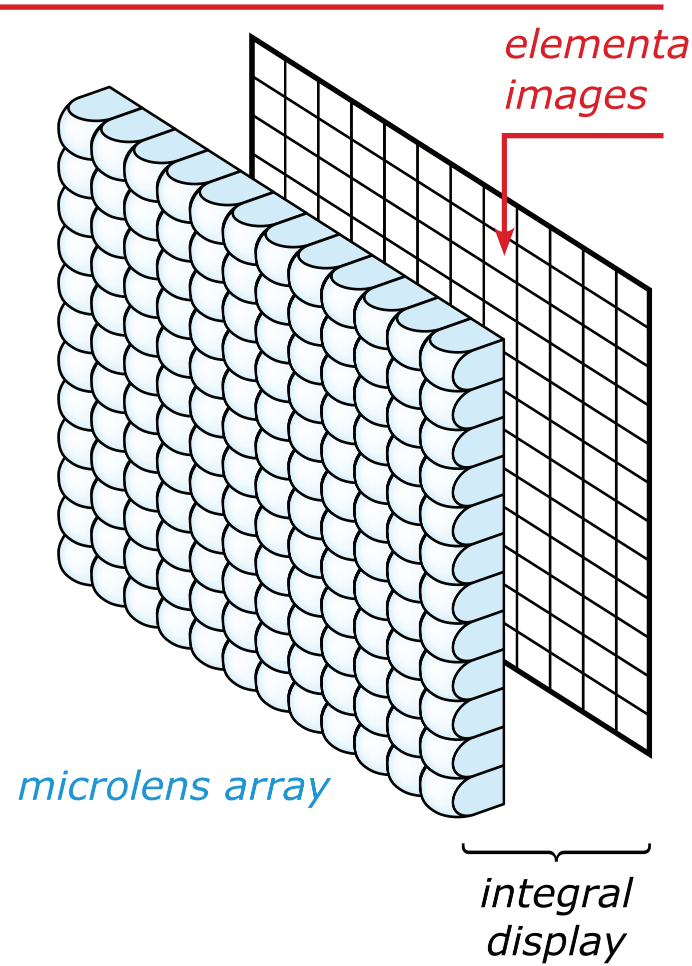
Cheoptics 360™ by viZoo



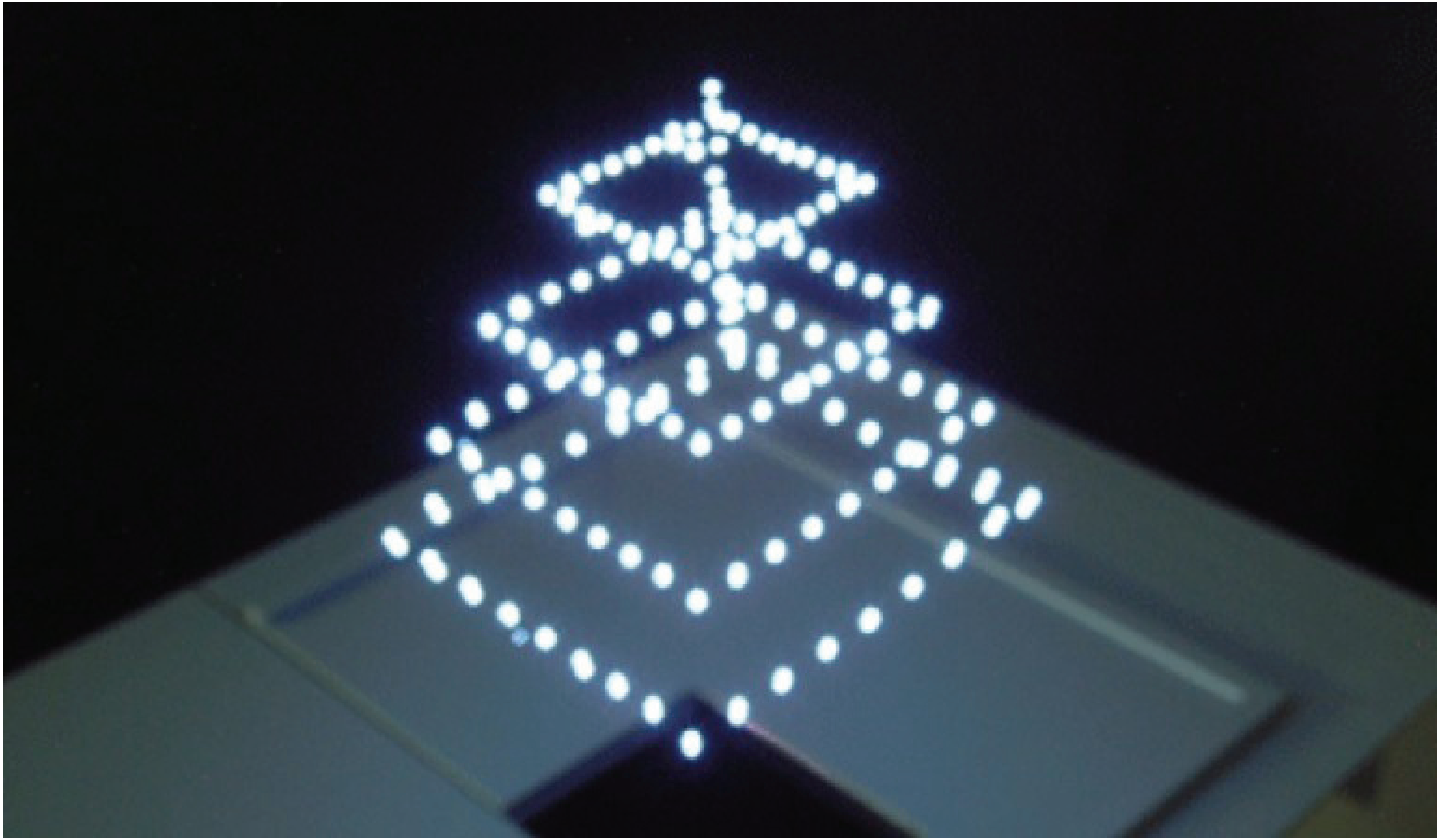


360° Light Field Display
University of Southern California

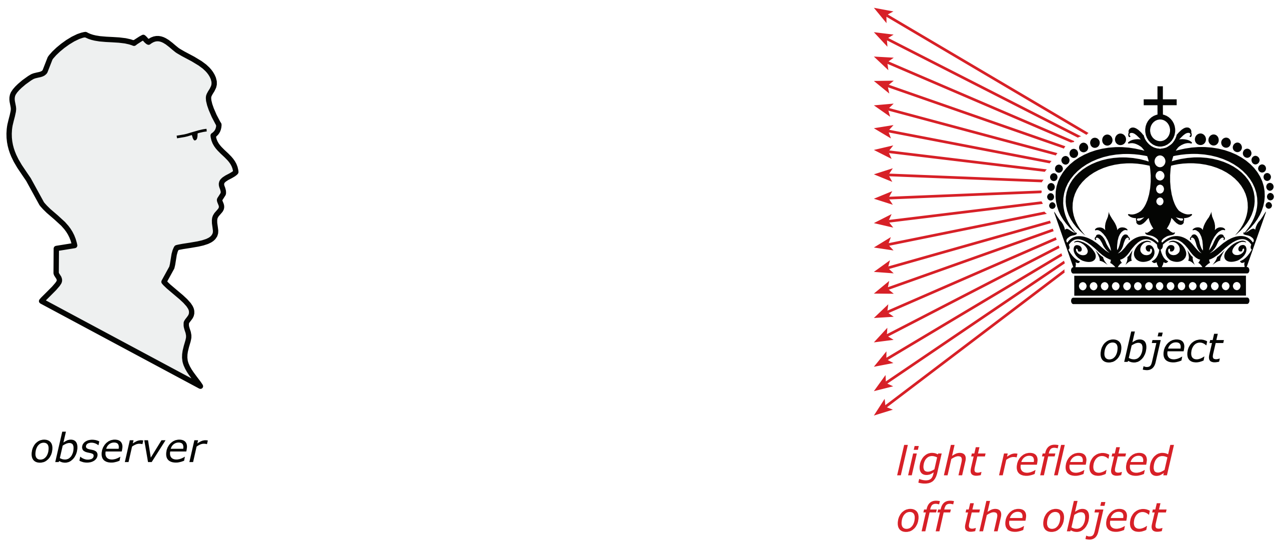


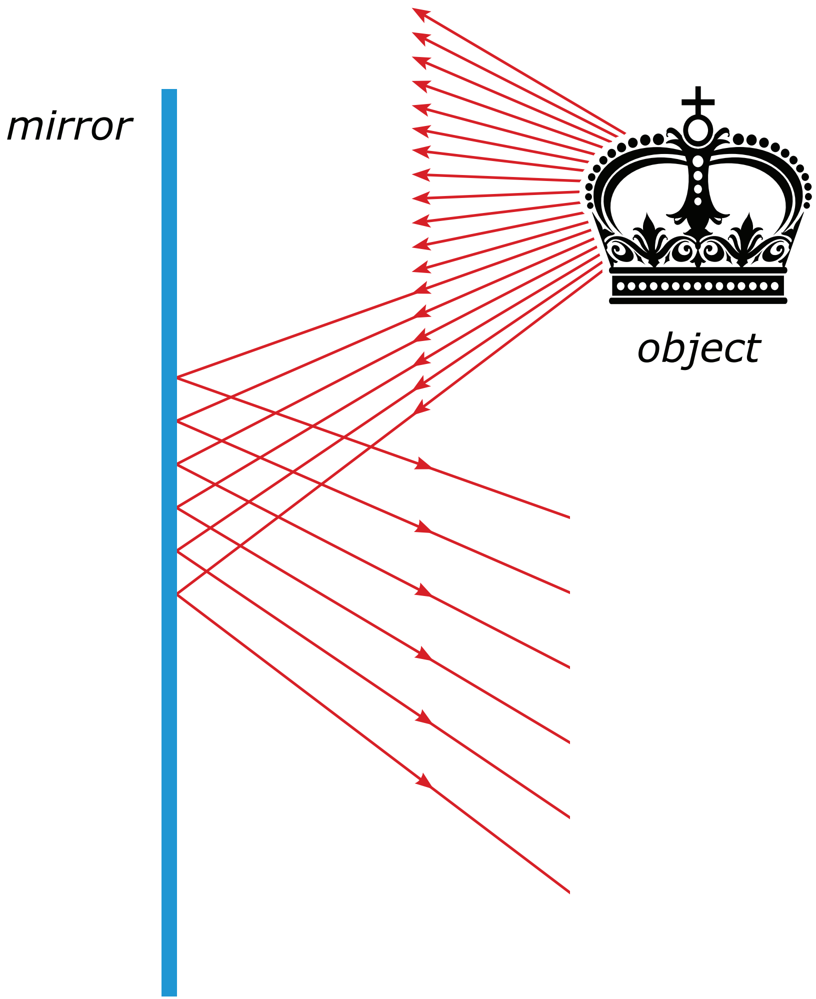


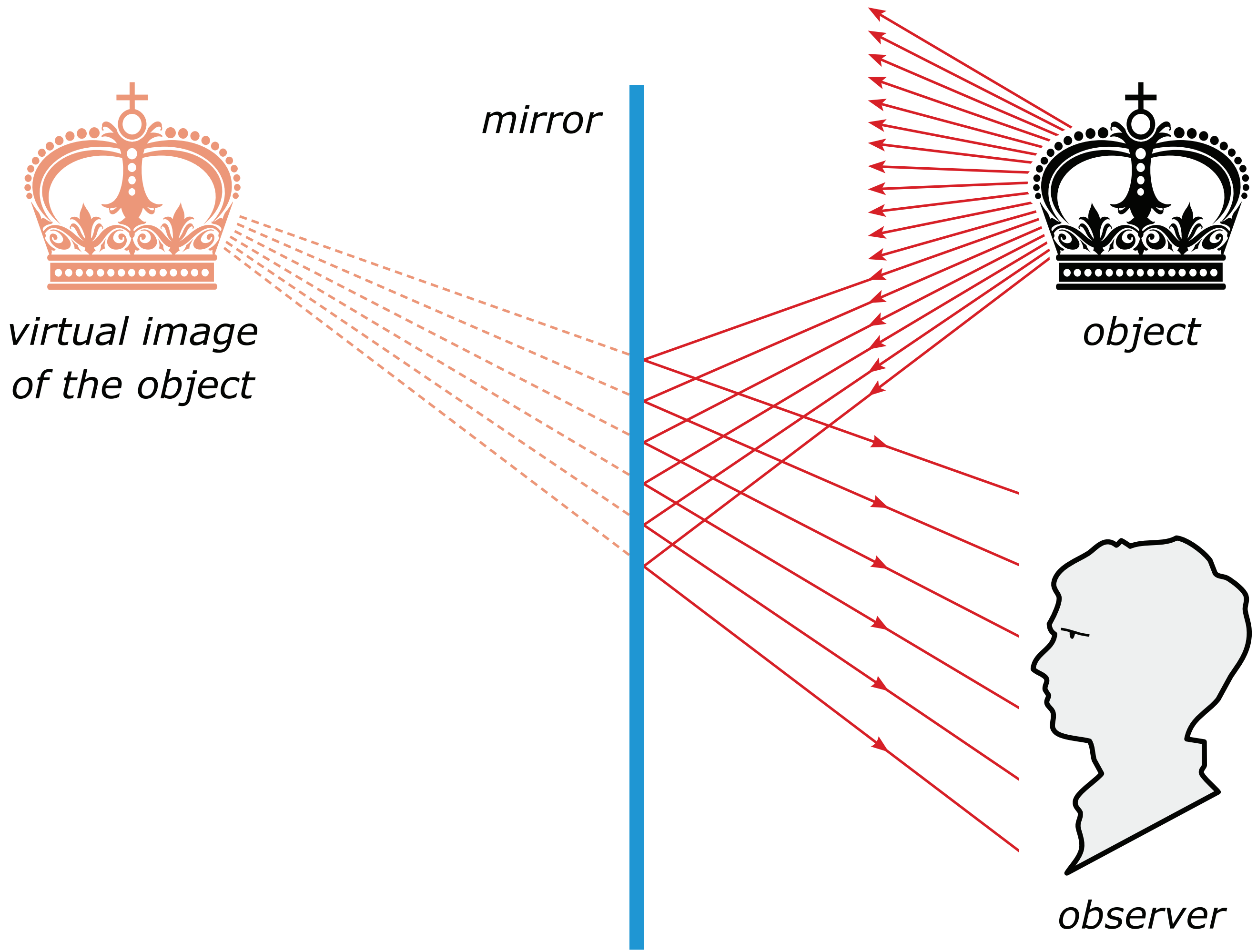
Integral (light field) display
(nVidia near-to-eye prototype)

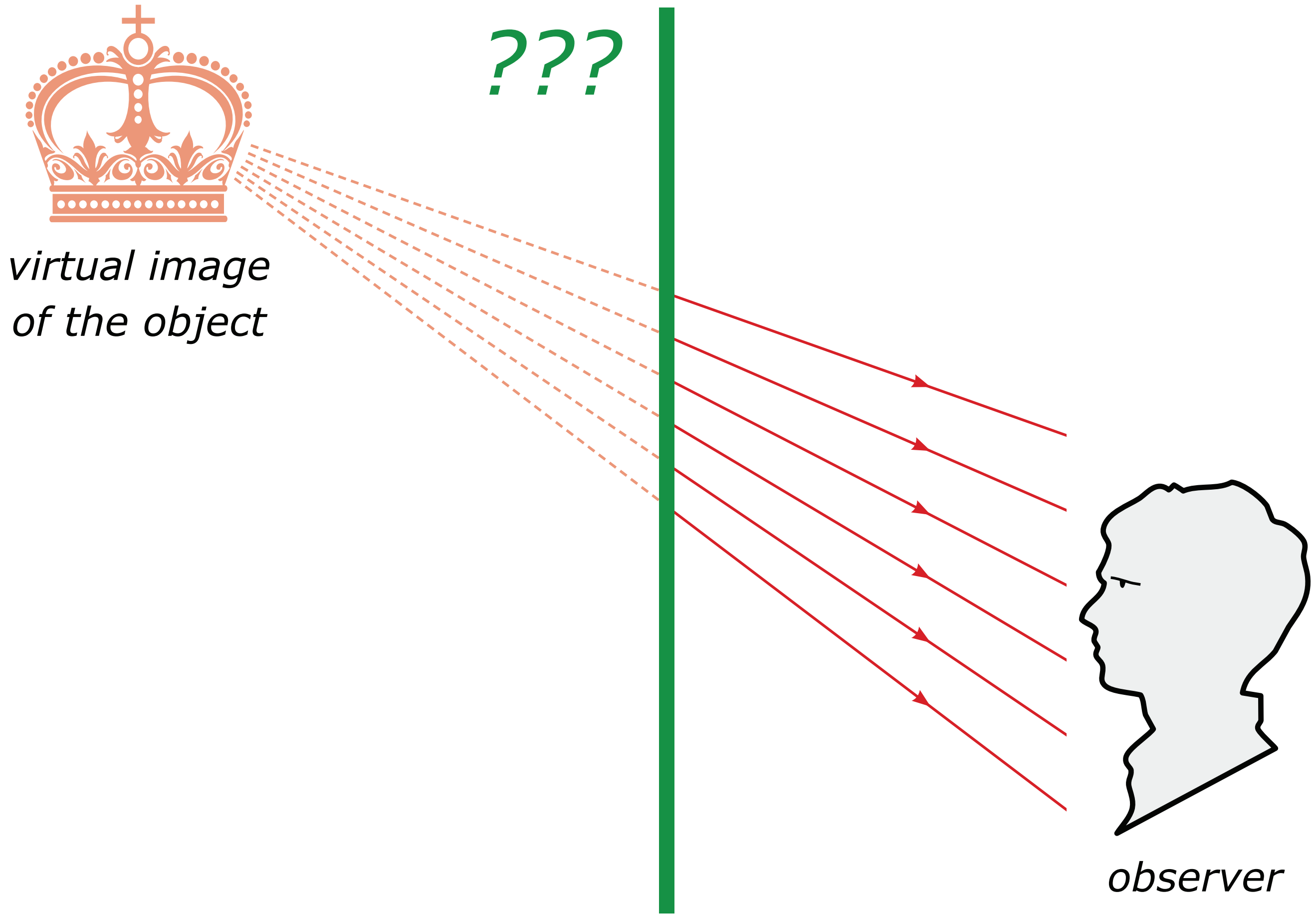


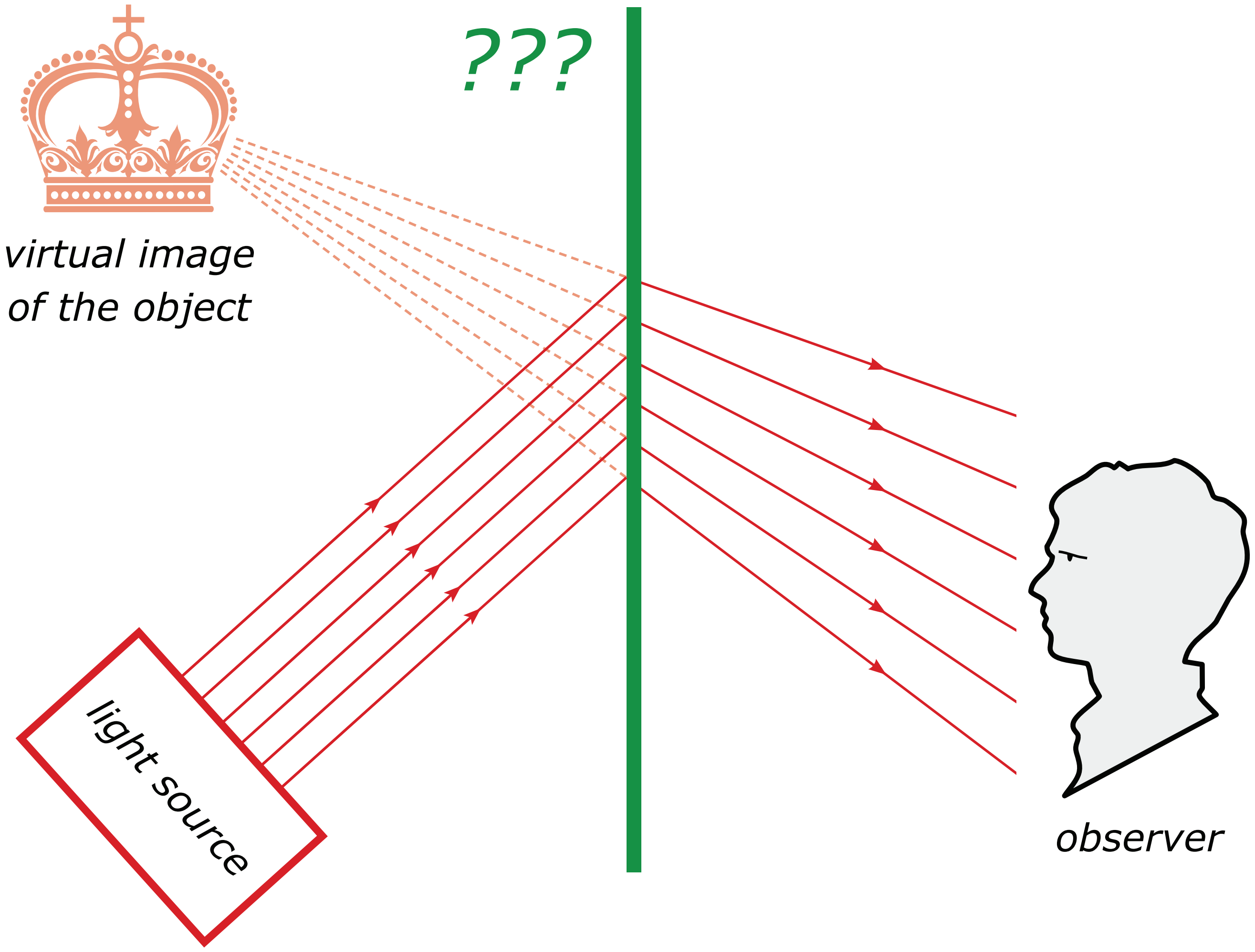
Plasma volumetric display by Burton Inc.


observer
object
light reflected
off the object


mirror
object


mirror
virtual image
of the object
object
observer


???
virtual image
of the object
observer


???
virtual image
of the object
light source
observer

Light diffraction

- depends on frequency $f = 1 / d$ of the pattern
  output angle of the rays: grating equation
  $\sin\theta_{out} = m\lambda / \mathrm{d} + \sin\theta_{in}$

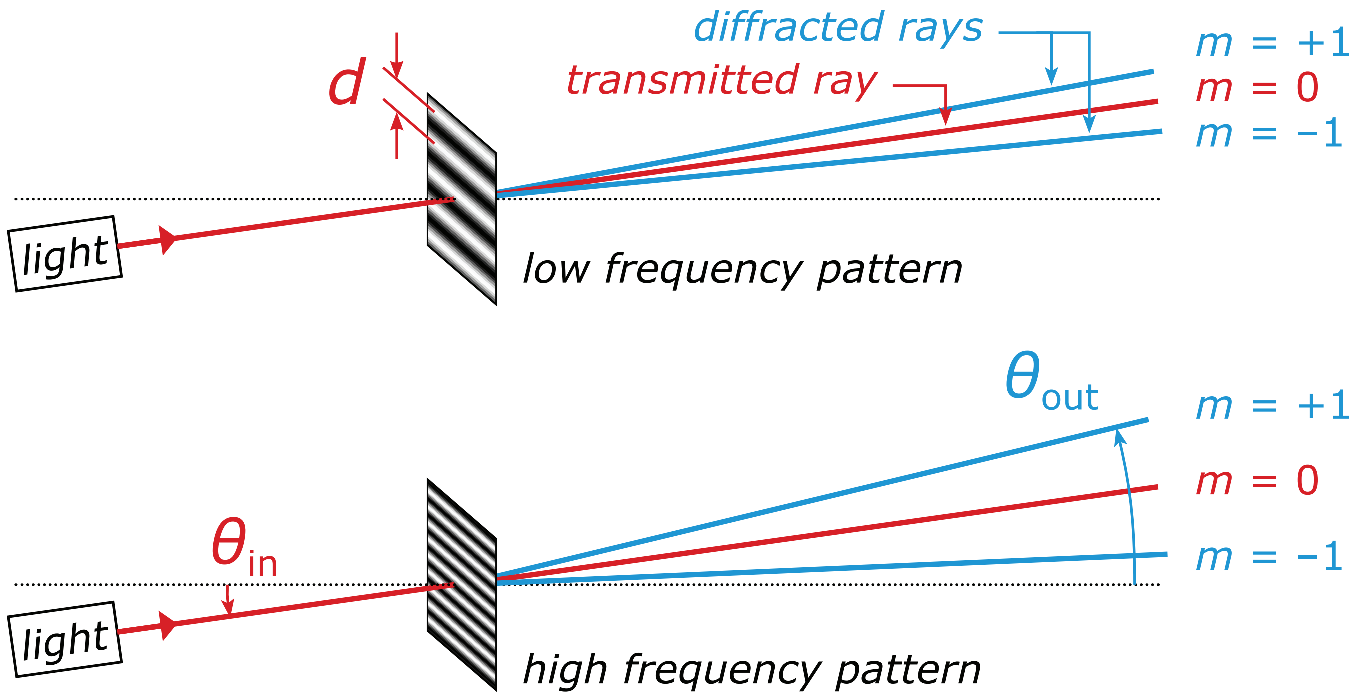
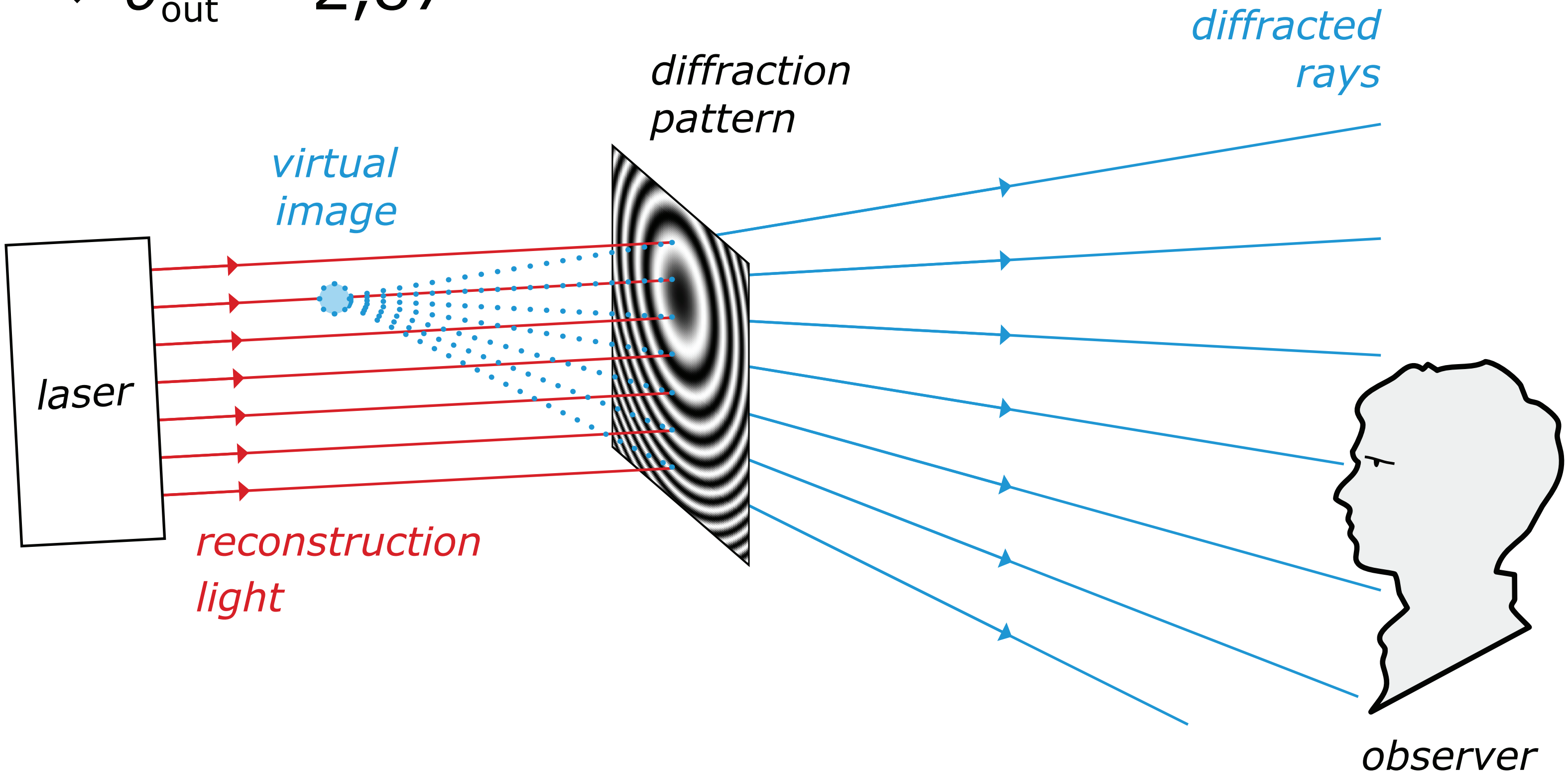


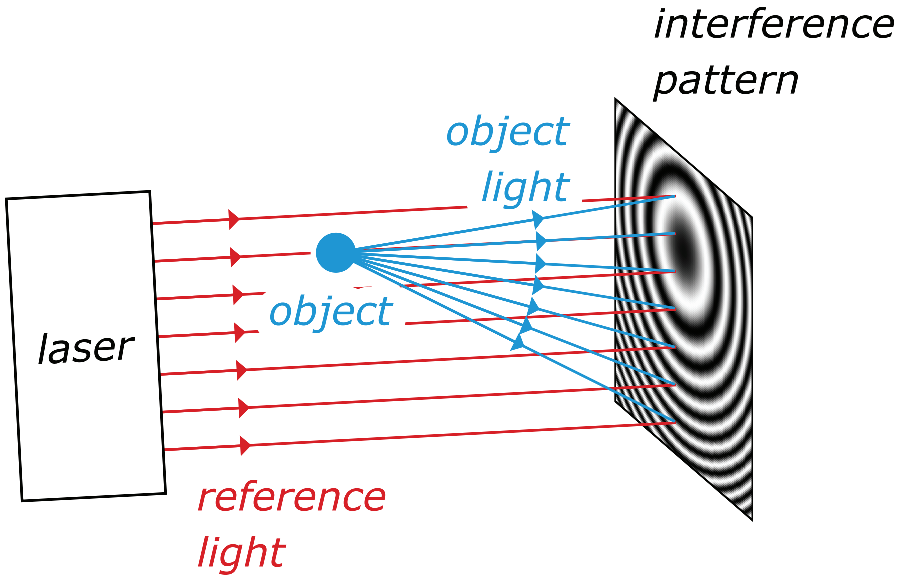
Image formation by means of diffraction

- grating equation: $\sin\theta_{out} = m\lambda / d + \sin\theta_{in}$
- example: λ = 0,5 µm d = 10 µm θ_{in} = 0 m = 1
  $\Rightarrow$ θ_{out} = 2,87°



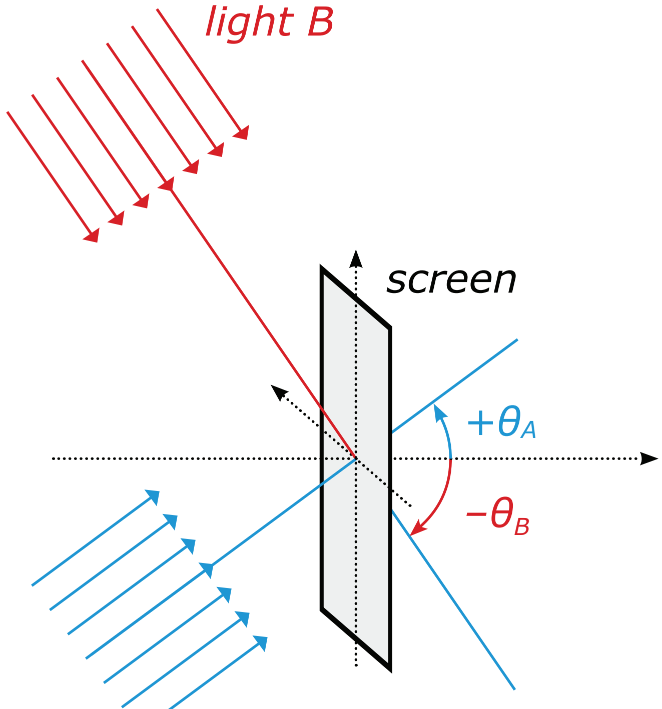
Diffraction pattern formation using interference

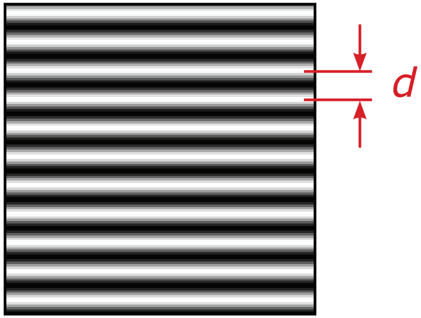
- an interference pattern can be recorded and subsequently used as a diffraction pattern



Light interference

- two "coherent" light beams "interfere": create a pattern of light and dark stripes





light intensity on the screen: the interference pattern

$$d = \frac{\lambda}{\sin\theta_\mathrm{A} - \sin\theta_\mathrm{B}}$$

example: $\lambda = 0{,}5\ \mu\mathrm{m}$

$\theta_\mathrm{A} = 45°$

$\theta_\mathrm{B} = -45°$

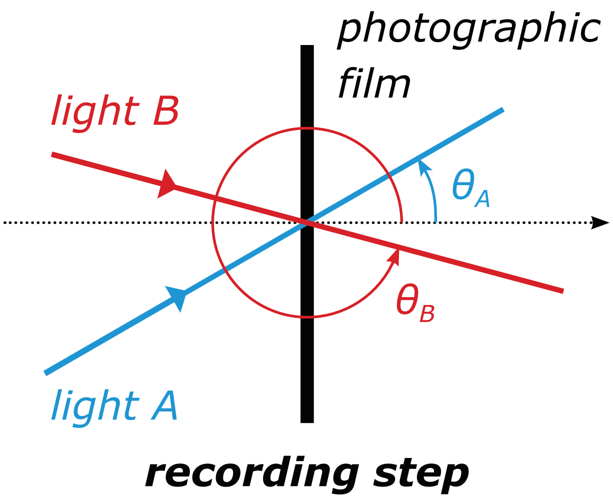
$\Rightarrow\ d = 0{,}35\ \mu\mathrm{m}$

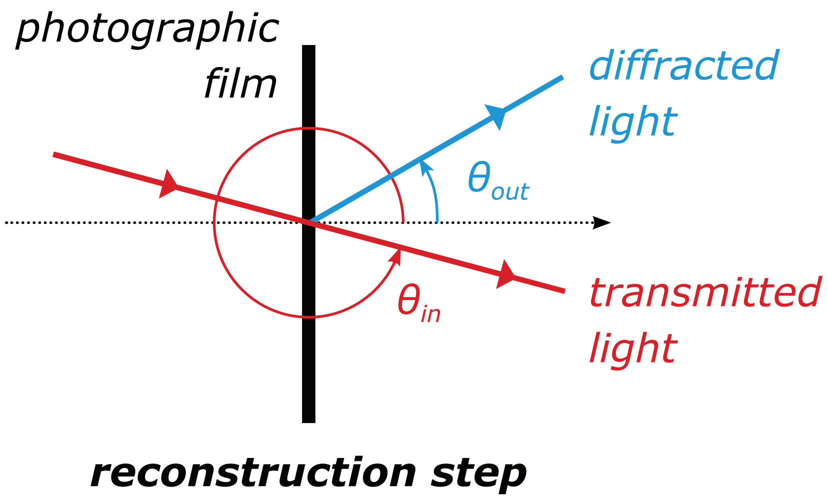
Principle of holography

- interference equation: $d = \lambda_1 / (\sin\theta_A - \sin\theta_B)$
  grating equation: $\sin\theta_{out} = m\lambda_2 / d + \sin\theta_{in}$
- after substitution of d: the $\sin\theta$ equation

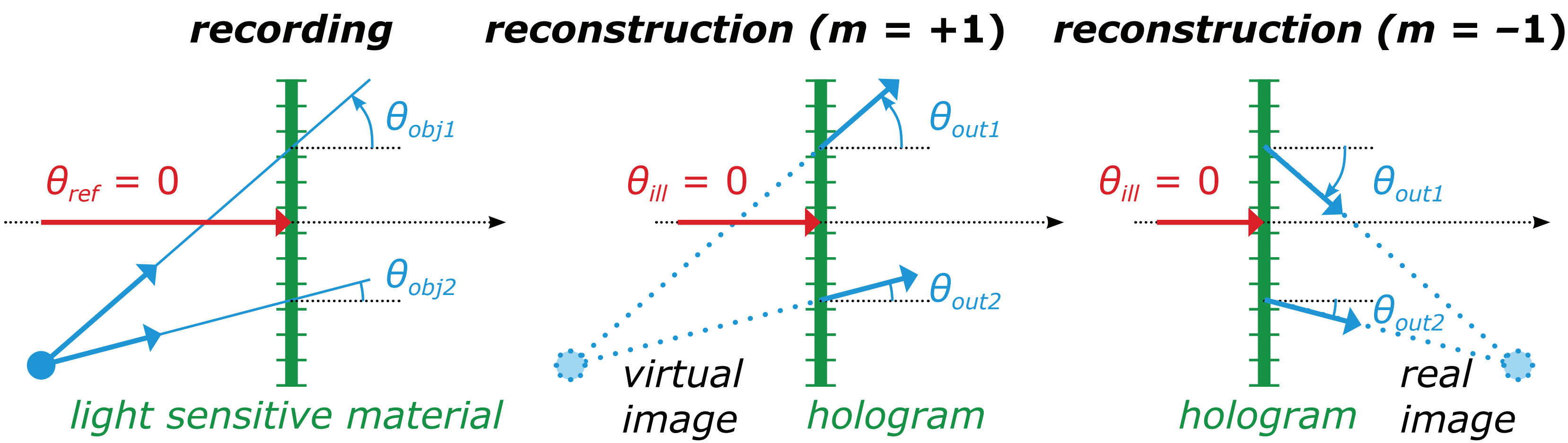
$$\sin\theta_{out} = m \frac{\lambda_2}{\lambda_1} (\sin\theta_A - \sin\theta_B) + \sin\theta_{in}$$

- for m = 1, $\lambda_1 = \lambda_2$, $\sin\theta_B = \sin\theta_{in} \Rightarrow \sin\theta_{out} = \sin\theta_A$



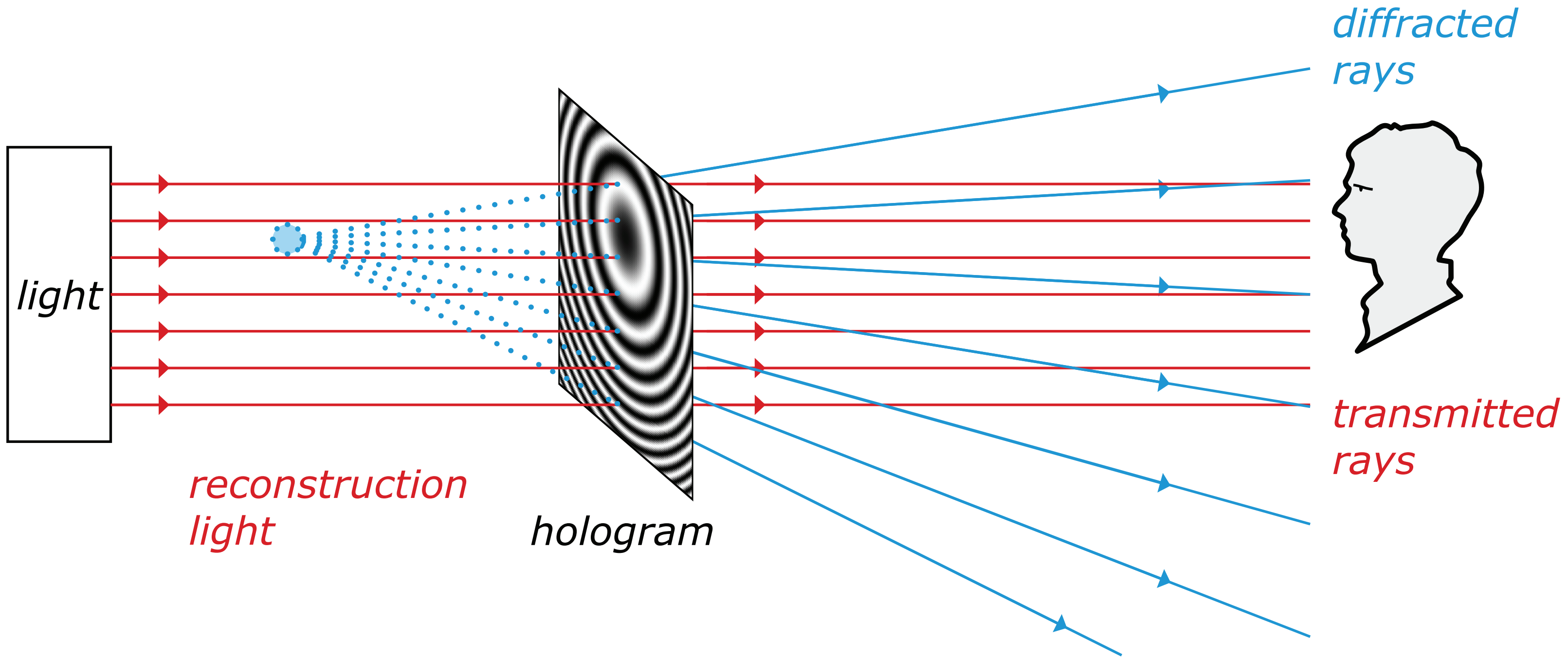


- hologram: the interference pattern of
  - an object wave: $\theta_{obj}\ (= \theta_A),\ \lambda_1 = \lambda_{ref}$
  - a reference wave: $\theta_{ref}\ (= \theta_B),\ \lambda_1 = \lambda_{ref}$
- hologram observation: illuminate it by
  - an illumination wave: $\theta_{ill}\ (= \theta_{in}),\ \lambda_2 = \lambda_{ill}$
- $\sin\theta_{out} = m\dfrac{\lambda_{ill}}{\lambda_{ref}}(\sin\theta_{obj} - \sin\theta_{ref}) + \sin\theta_{ill}$
- example: $\lambda_{ill} = \lambda_{ref}$, $\theta_{ill} = \theta_{ref} = 0$



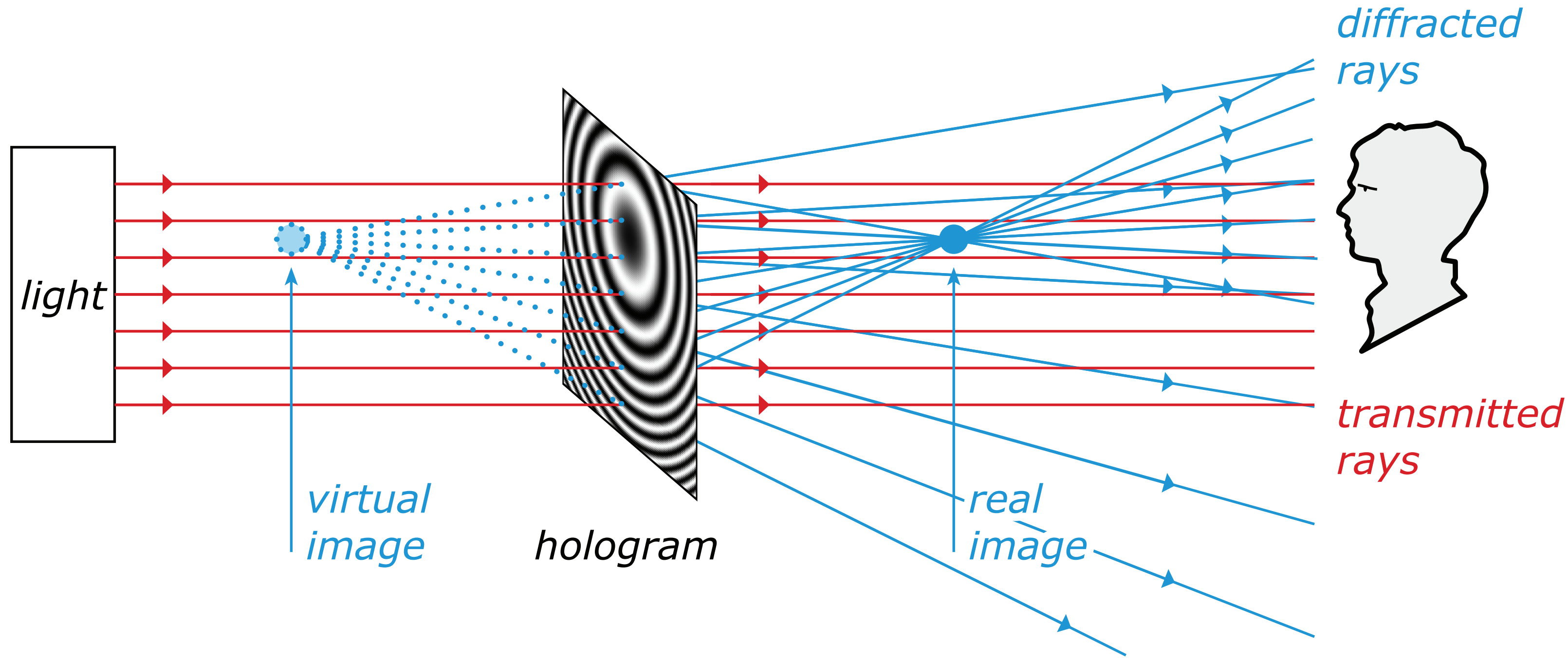
Virtual image formation

- illuminate hologram with a light source
- light beams diffract on the interference pattern
- diffracted rays are the same as the rays from the original object



Real image formation

- output angle of the rays: $\sin\theta_{\text{out}} = m\lambda \,/\, d + \sin\theta_{\text{in}}$
- for $m = -1$, rays can create real image of the scene
- both rays for $m = +1$ and -1 appear at once
  $\Rightarrow$ no need to distinguish between them



Classical holography

- capturing the interference pattern of laser lights using a photosensitive material
  - requires high quality lasers
  - requires high resolution recording materials (currently up to 10 000 lines/mm)
  - requires vibration-free environment
  - usually requires chemical processing
- reconstructing the hologram using light source
  - custom lighting setup required
- properly recorded and illuminated holograms provide ultra realistic image

Digital holography (DH)

- light sensitive sensor (e.g. CCD or CMOS) instead of photochemical light sensitive material
  - very fast
  - cannot capture high spatial frequencies (currently about 250 lines/mm)
- numerical simulation of the hologram reconstruction
- digital processing of the captured hologram instead of its visual inspection
  - automatic evaluation
  - allows processing hard to achieve in classical holography

Computer generated holography (CGH)

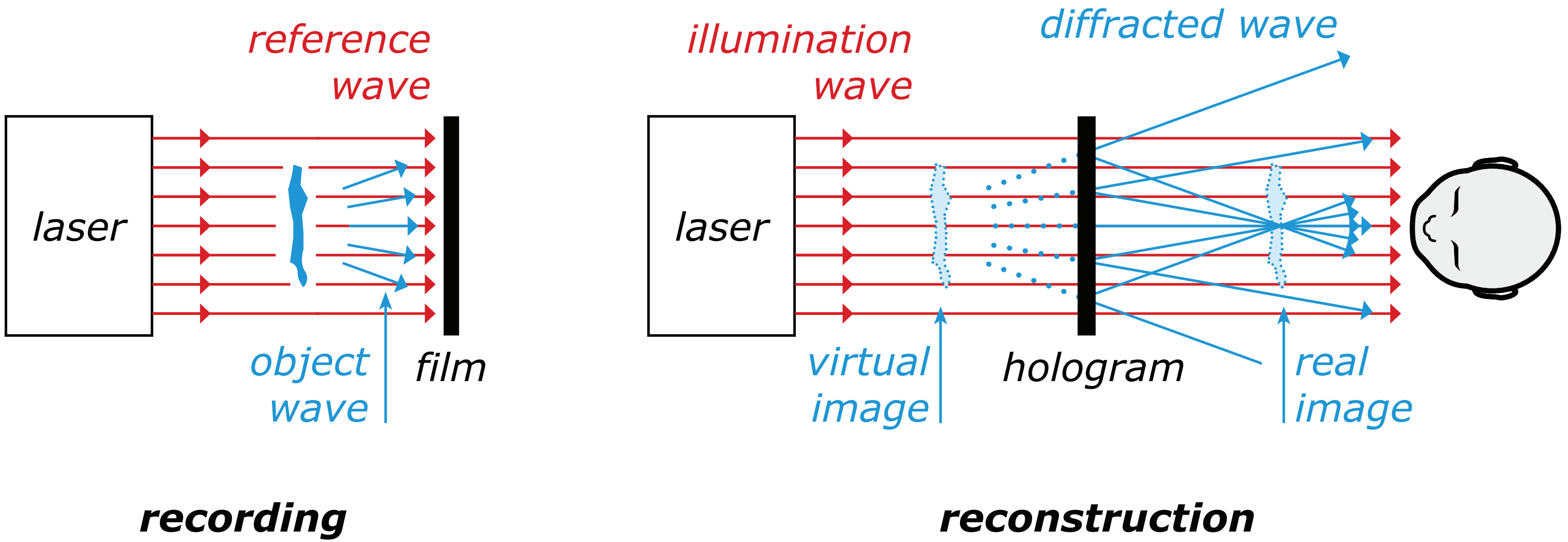
- numerical simulation of the **hologram recording process** (“sort of”)
- electronic display of a hologram
  - e.g. microdisplays with very fine pixels (spatial light modulators), currently up to 130 lines/mm
- “printing a hardcopy”
  - laser lithography
    expensive, up to 600 lines/mm
  - electron beam lithography
    very expensive, up to 10 000 lines/mm
- other technologies – let us talk about them later

Computer generated display holography (CGDH)

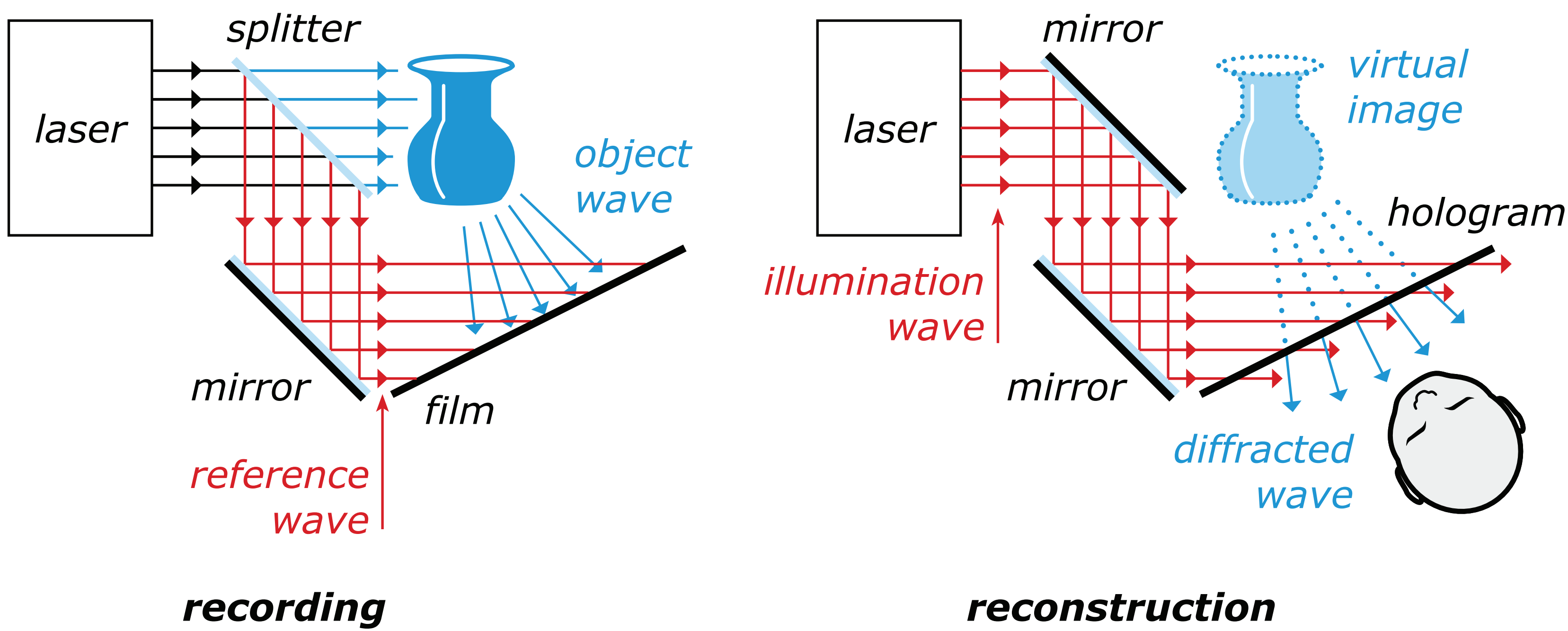
- computer generated hologram of a 3-D scene for **display purposes**
- computer graphics
  - makes a digital image to be displayed on a common electronic display
- computer generated display holography
  - makes a pattern to be displayed on a holographic display
- combination approaches are common, e.g., computer graphics for image rendering, subsequent classical holography for making an interference pattern

Basic hologram recording setups

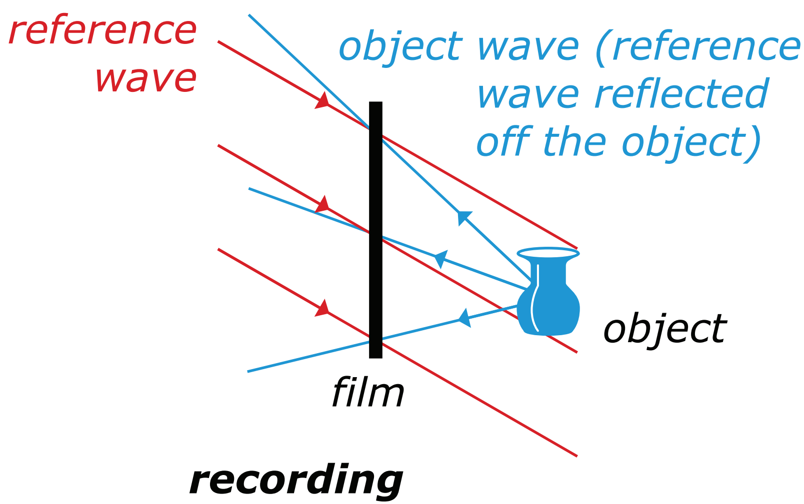
- on-axis (Gabor) hologram
  - mostly for transparent objects (restrictive)
  - image damaged by the 0th order, ±1st orders overlap (bad)
  - low spatial frequencies (100 lines/mm – good)

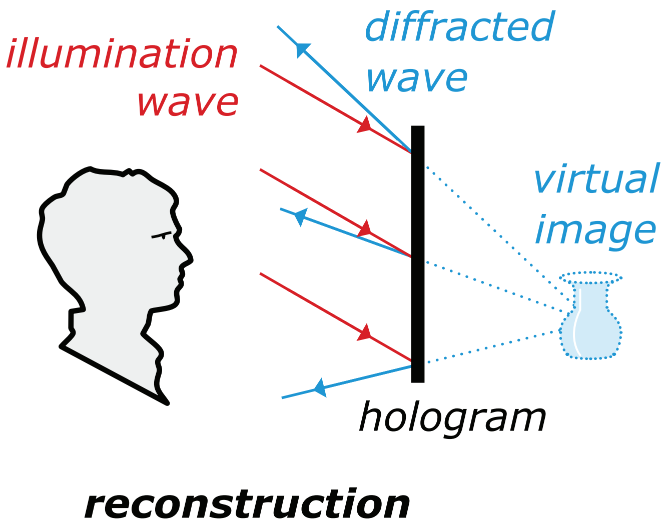


- off-axis transmission (Leith-Upatnieks) hologram
  - for both opaque and transparent objects
  - clear image (good)
  - high spatial frequencies (1000 lines/mm – bad)
  - visible in laser light only (uncomfortable)



- reflection (Denisyuk) hologram
  - the simplest setup (good)
  - visible in white light (good)
  - simply allows colour imaging (very good)
  - high spatial frequencies (4000 lines/mm – bad)
  - the diffraction pattern is volumetric, i.e., 3-D, not planar, i.e., 2-D (very bad)





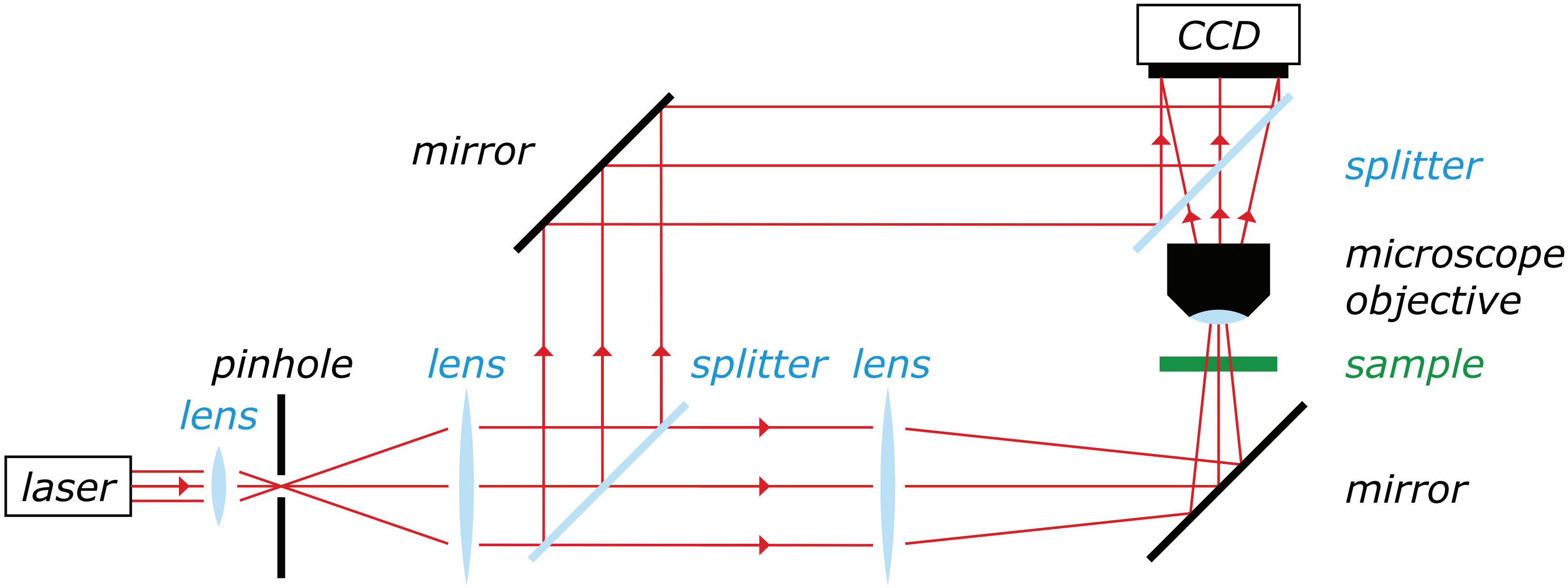
Applications of holography

- cultural heritage conservation
  - holograms instead of real exhibits
  - the exhibit too valuable or fragile, multiple exhibitions at once, multiple views of the same exhibit at once
  - almost perfect image of the exhibit, scale 1 : 1

A full colour Denisyuk hologram of the "15th anniversary Fabergé Easter egg", A. Sarakinos, HIH, 2015.

- microscopy, visual inspection
  1. perfect recording of light
     (from a biological sample, a bubble chamber, …)
  2. hologram examination
     (unlimited time of observation,
     examination in safe environment,
     holograms can be archived, …)

- digital holographic microscopy
  - acquisition of a digital hologram
  - numerical reconstruction
    ⇒ signal filtering, unwanted diffraction removal, numerical analysis, …

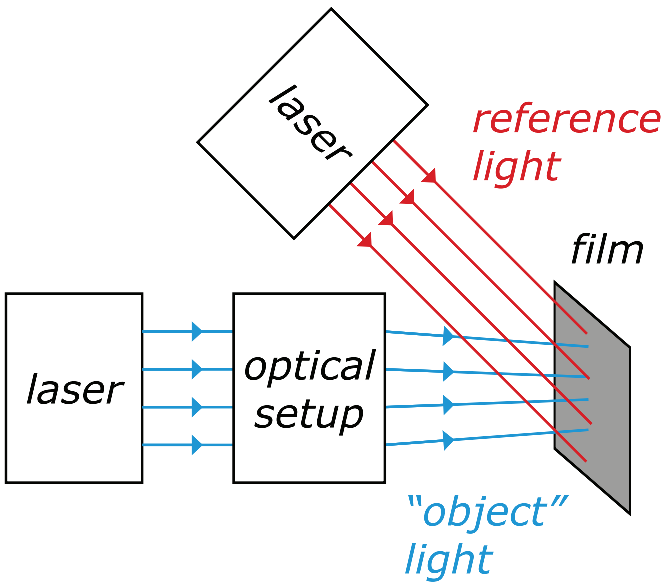


- enhancing electron microscopy
  - original D. Gabor idea behind holography (although in fact, it never worked)
  - hologram recording with electron beam (λ is 100 000× smaller than for visible light)
  - hologram enlargement, visible light illumination ⇒ image 100 000× bigger

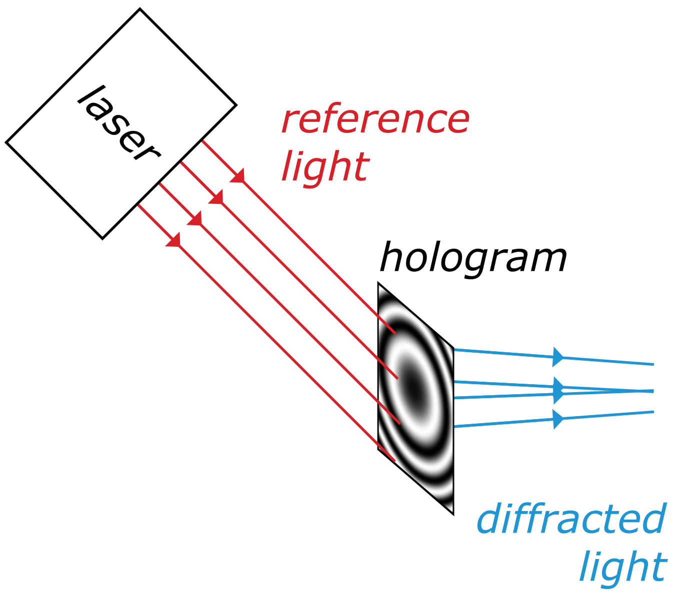
in the $\sin\theta$ equation: $\lambda_{\mathrm{ill}} / \lambda_{\mathrm{ref}} = 100\,000$

$$\sin\theta_{\mathrm{out}} = m\,\frac{\lambda_{\mathrm{ill}}}{\lambda_{\mathrm{ref}}}\left(\sin\theta_{\mathrm{obj}} - \sin\theta_{\mathrm{ref}}\right) + \sin\theta_{\mathrm{ill}}$$

- holographic optical elements (HOE)
  - mimicking any optical element
  - cheaper, easier aberration correction, …
  - also called diffractive optical elements (DOE) (the difference between HOE and DOE is subtle)

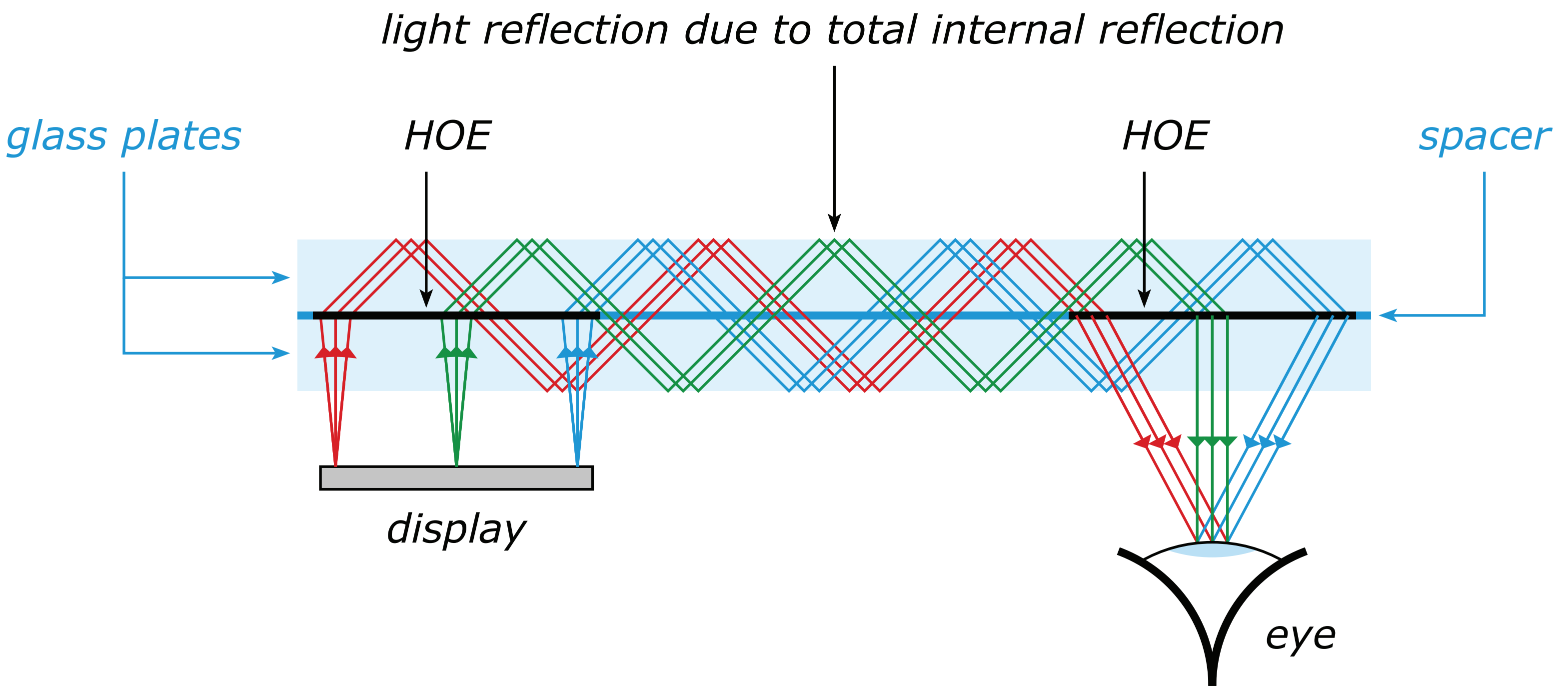


holographic optical element recording



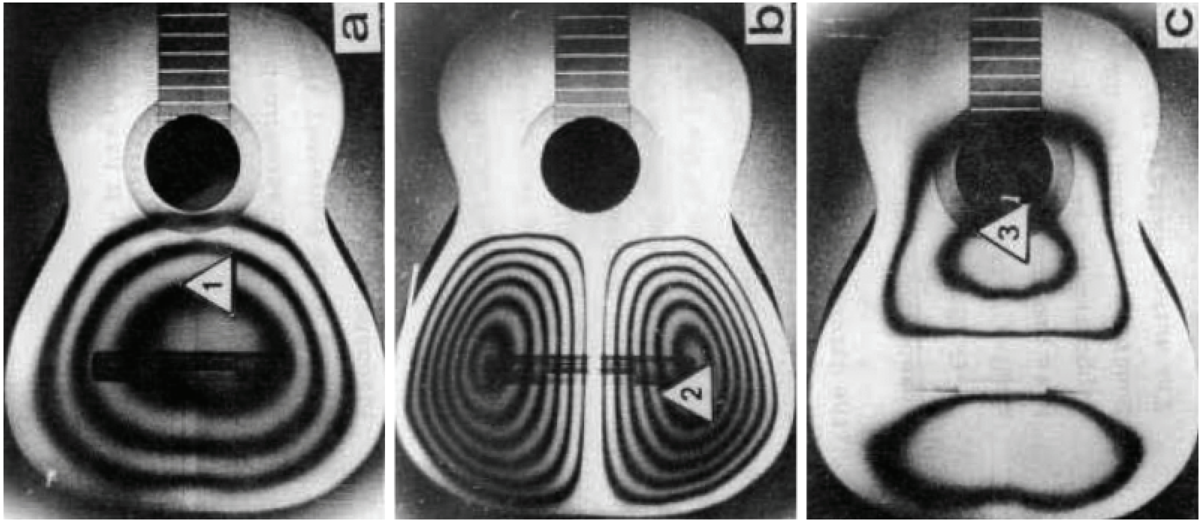
holographic optical element usage

– example: holographic optical element (waveguide coupler) for augmented reality head-up displays



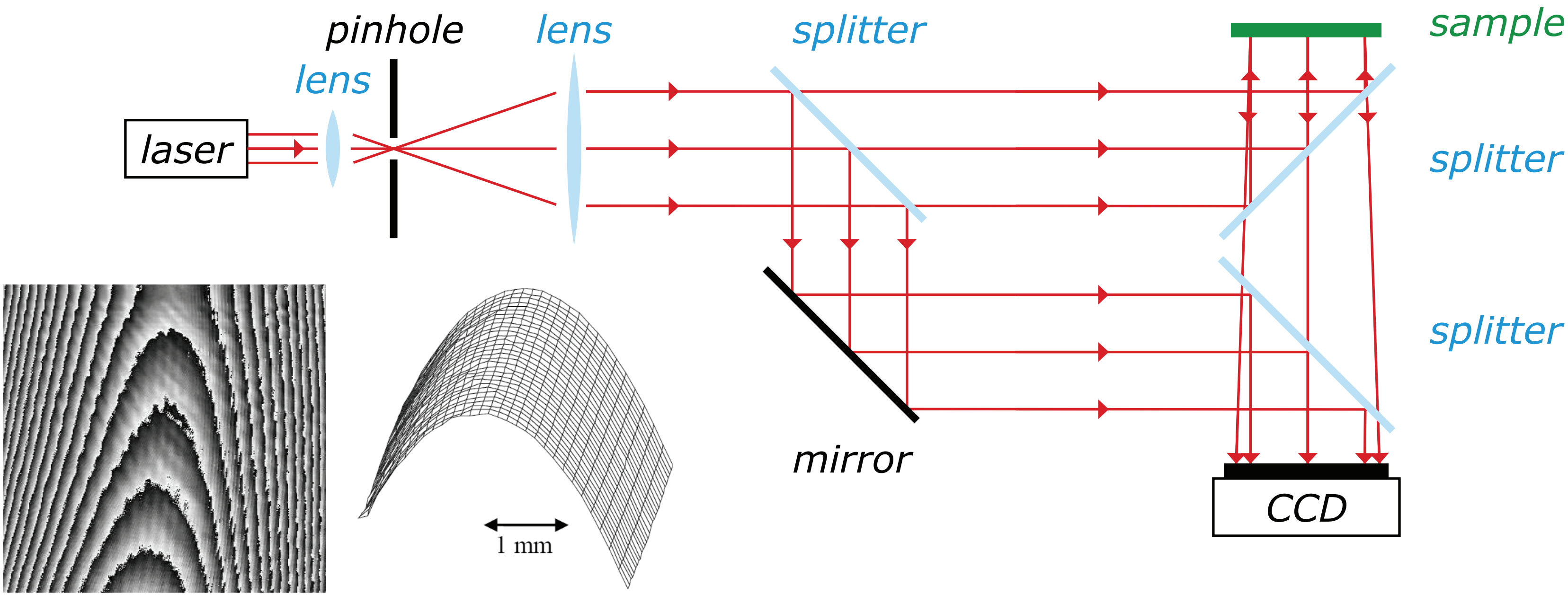
- non-destructive testing
  - double object recording on one hologram: shifts between recordings smear hologram fringes
  - taking a hologram of a vibrating object: vibration causes loss of hologram fringes

  ⇒ no fringes = no image = black strips on the object



K. Molin, N. Stetson, Institute of Optical Research, Stockholm (1971)

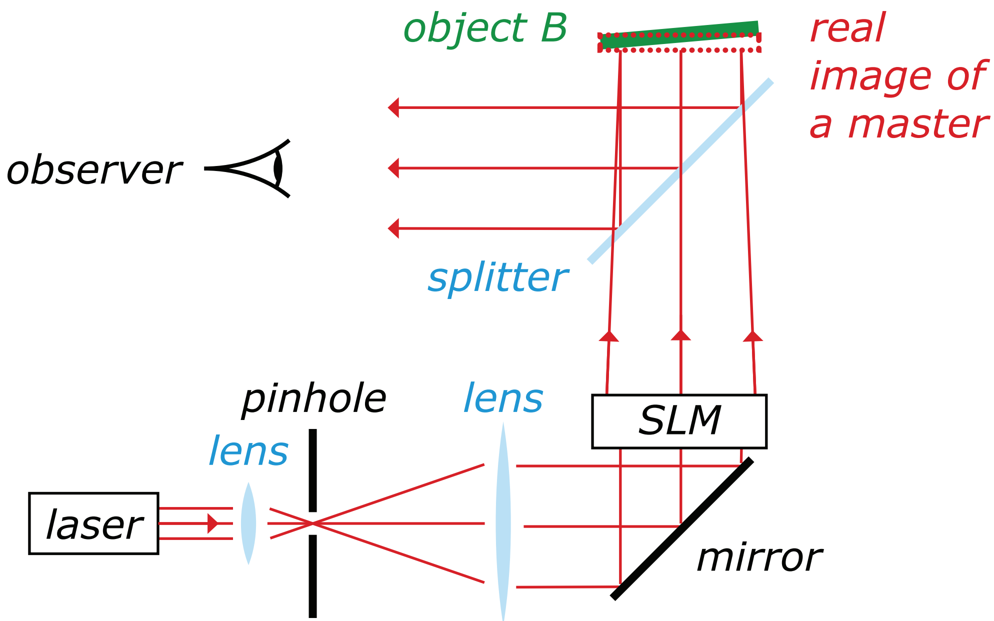
- surface metrology
  - digital hologram of a real object
  - numerical reconstruction of a hologram
  - reconstructed phase ~ surface bumpiness



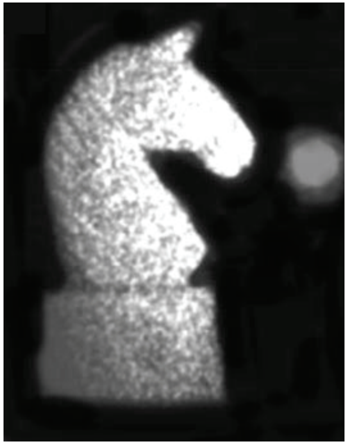
captured phase *unwrapped phase*

(Schnars et al.: Digital Holography and Wavefront Sensing)

- remote digital holographic interferometry
  - hologram of a master sample (A)
  - reconstruction of a real image of a master over a tested object B
  - contours ~ objects differences



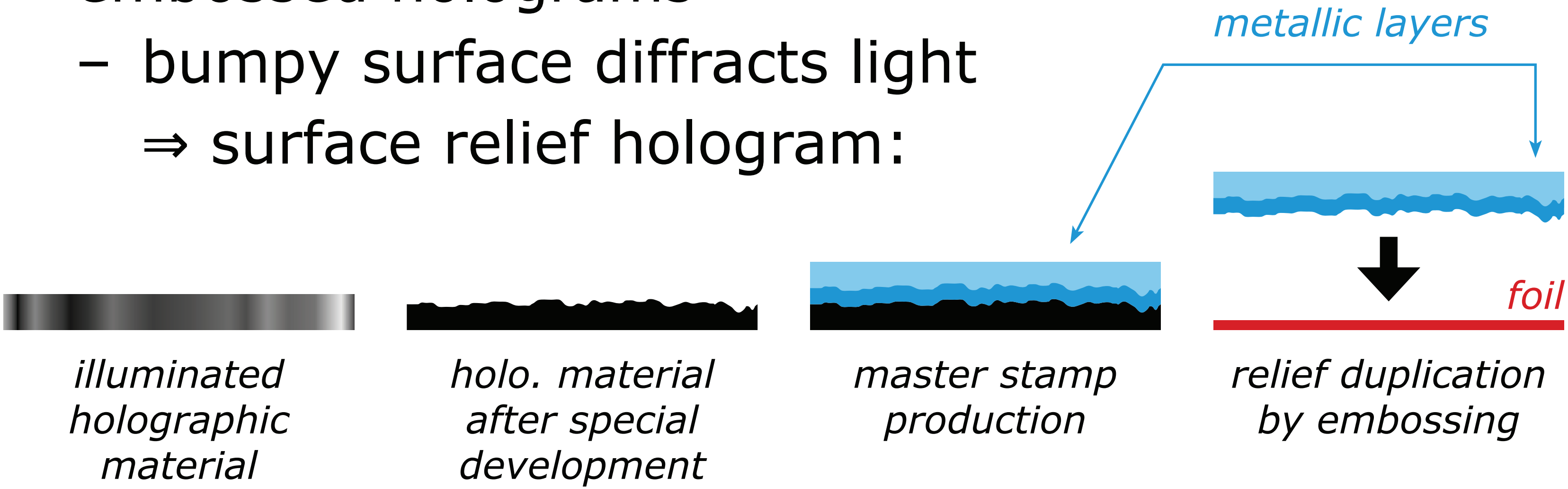
(Schnars et al.: Digital Holography and Wavefront Sensing)



master

contours

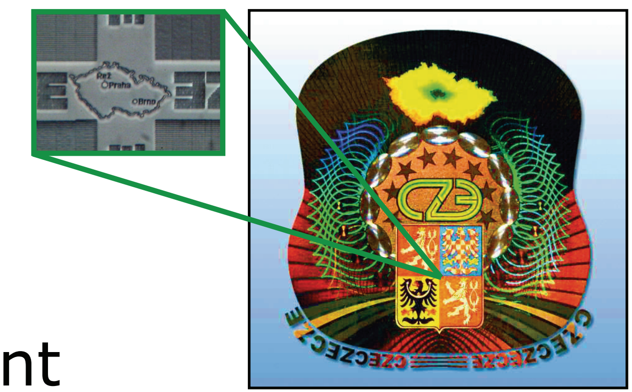
- embossed holograms
  - bumpy surface diffracts light
    ⇒ surface relief hologram:



  - making a master stamp expensive
  - making embossed copies cheap
  - can contain hidden features

  ⇒ hard to counterfeit

  ⇒ suitable as a security element



hologram by Optaglio

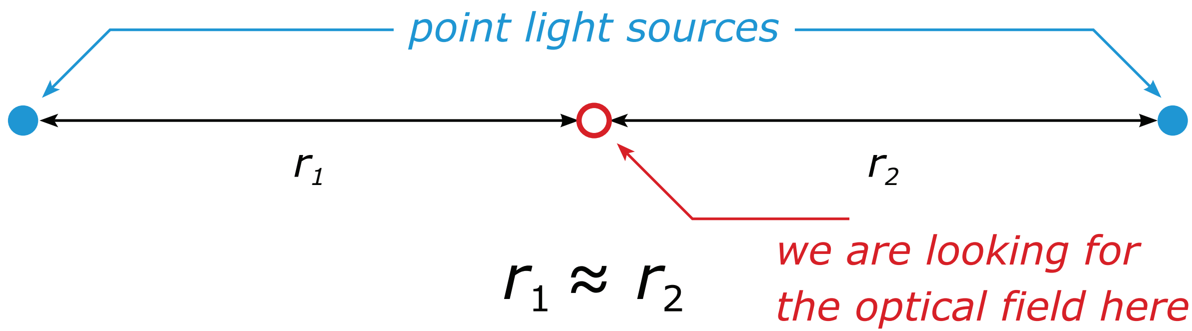
BASIC TOOLS OF COMPUTER GENERATED DISPLAY HOLOGRAPHY

Nature of light

- force interaction between (oscillating) point charges
- a point source of light: movement up and down $\sim \underset{\text{amplitude}}{A} \cos(\underset{\text{phase}}{\varphi} - \omega t)$
- optical field ($\sim$ electromag. force) at a distance r:

$$u(r, t) = \frac{A}{r} \cos\left[\varphi - \omega\left(t - \frac{r}{c}\right)\right] = \underbrace{A'(r)}_{\text{amplitude at } r} \cos(\underbrace{\varphi'(r)}_{\text{phase at } r} - \omega t)$$

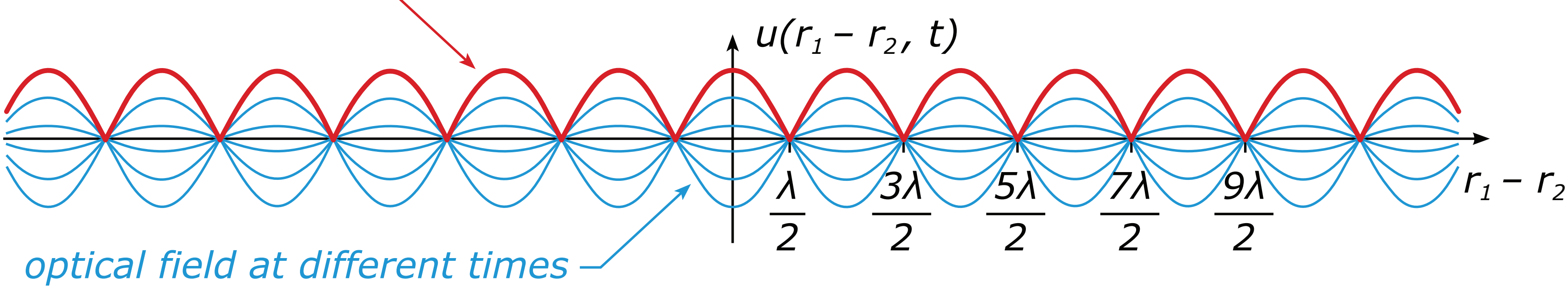
T	period of oscillation	1.7×10^{-15} s
$f = 1/T$	(time) frequency	600 THz
$\omega = 2\pi/T$	angular frequency	
c	speed of light	
$\lambda = cT$	wave length	0.5 µm
$k = 2\pi/\lambda$	wave number	1.2×10^{7} m^{-1}



$$r_1 \approx r_2$$

$$\frac{A}{r_1}\cos(kr_1 - \omega t) \quad + \quad \frac{A}{r_2}\cos(kr_2 - \omega t)$$

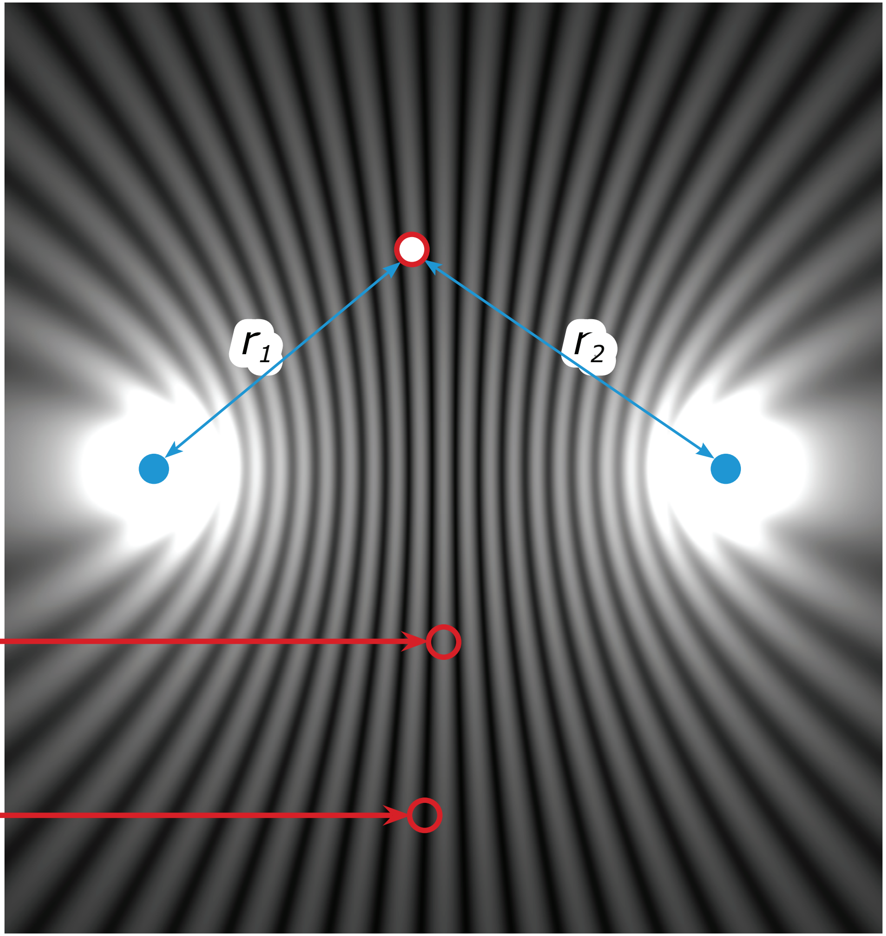
$$\approx \underbrace{2\,\frac{A}{r_1}\cos\left(\frac{k(r_1 - r_2)}{2}\right)}_{A'} \cos\left(\underbrace{\frac{k(r_1 + r_2)}{2}}_{\varphi'(r)} - \omega t\right)$$



- photographic film reacts on time average of light intensity ∝ $(A')^2$

⇒ cannot distinguish close "dimmer" light from distant "brighter" light

- constructive × destructive interference



intensity of light

Phasor arithmetic

- $j^2 = -1$
- $e^{jx} = \cos x + j \sin x$

- $u(r, t) = A(r) \cos[\varphi(r) - \omega t] = \mathrm{Re}\{A(r)\, e^{j[\varphi(r) - \omega t]}\}$
  $= \mathrm{Re}\{\underbrace{A(r)\, e^{j\varphi(r)}}_{\textit{phasor}}\, e^{-j\omega t}\}$

- phasor (complex amplitude):
  $U(r) = A(r)\, e^{j\varphi(r)}$

- light amplitude: $A = |U|$
  light phase: $\varphi = \arg(U)$
- light intensity:
  $I = |U|^2 = U U^* = A\, e^{j\varphi} A\, e^{-j\varphi} = A^2$

Advantage of phasor arithmetic

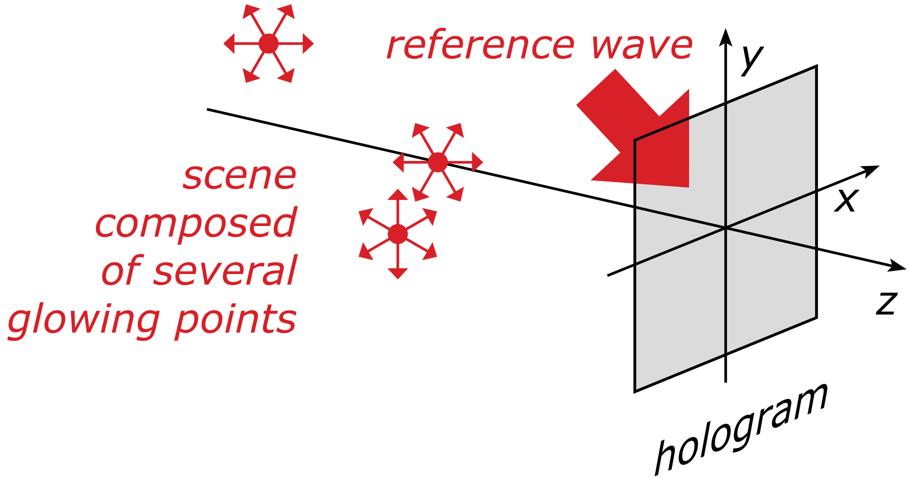
- optical field – time dependent function:
  $u(r, t) = A(r) \cos(\varphi(r) - \omega t)$
- its phasor (complex amplitude):
  $U(r) = A(r) \exp[\, \mathrm{j}\varphi(r) \,]$
- sum of optical fields:
  $A_1(r) \cos(\varphi_1(r) - \omega t) + A_2(r) \cos(\varphi_2(r) - \omega t) + \cdots$
  $= ?$
- in phasor arithmetic:
  $\mathrm{A}_1(r) \exp[\, \mathrm{j}\varphi_1(r) \,] + \mathrm{A}_2(r) \exp[\, \mathrm{j}\varphi_2(r) \,] + \cdots$
  $= U_{\mathrm{total}}(r)$
- optical field (if needed):
  $u_{\mathrm{total}}(r, t) = \mathrm{Re}\{U_{\mathrm{total}}(r)\, e^{-\mathrm{j}\omega t}\}$

Hologram recording simulation

- assume hologram in the plane $z = 0$
- calculation of a hologram of a synthetic scene:
  for every point $(x, y, 0)$ of the hologram:
  - get the complex amplitude U_{obj} of the object wave at $(x, y, 0)$
  - get the complex amplitude U_{ref} of the reference wave at $(x, y, 0)$
  - calculate captured intensity at $(x, y, 0)$
    $I(x, y, 0) = |U_{\mathrm{obj}} + U_{\mathrm{ref}}|^2$

Computer generated hologram of a point cloud

- the simplest algorithm in CGH
- basic building block of advanced algorithms of computer generated display holography



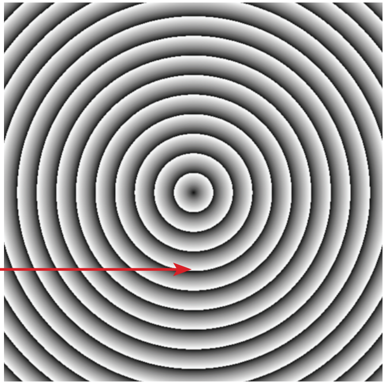
- spherical wave
  - light emitted by a point light source
  - r: distance from the light source
  - complex amplitude:

    $$U(r) = \frac{A}{r} \exp(\mathrm{j}[kr + \varphi])$$

  - locally resembles a plane in a big distance
  - rays: “directions perpendicular to wavefronts”

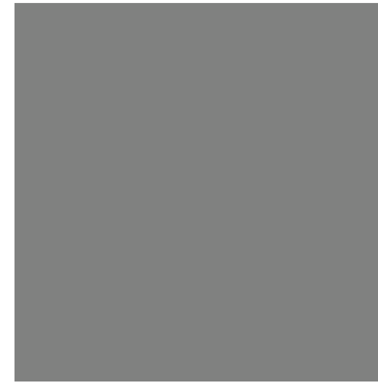
amplitude

wavefronts:
surfaces of constant phase

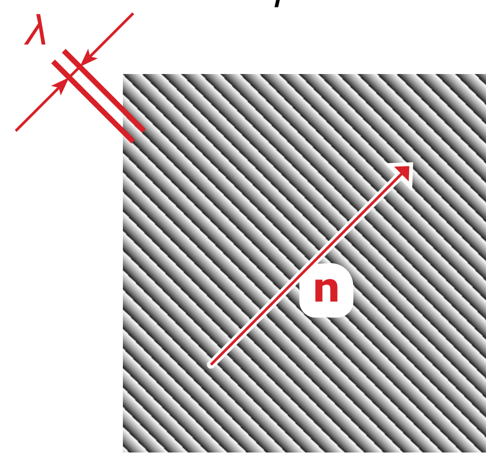


phase

- plane wave
  - light emitted by a point light source located in a direction $-\mathbf{n}$ far away, $|\mathbf{n}| = 1$
  - $\mathbf{n}$ is a direction of light propagation and the normal vector of the wavefronts
  - point in space $\mathbf{x} = (x, y, z)$
  - wavefront plane equation $\mathbf{n} \cdot \mathbf{x} = const.$
  - wavefronts separation λ
  - complex amplitude: $U(\mathbf{x}) = A \exp(\mathrm{j}[k\mathbf{n} \cdot \mathbf{x} + \varphi])$



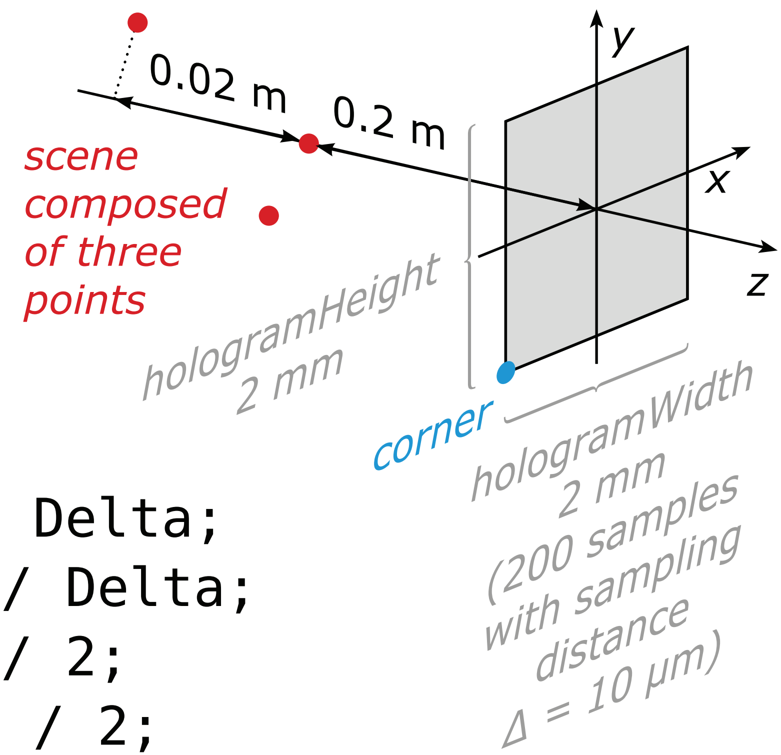
amplitude



phase

Really unoptimized Matlab (Octave) code

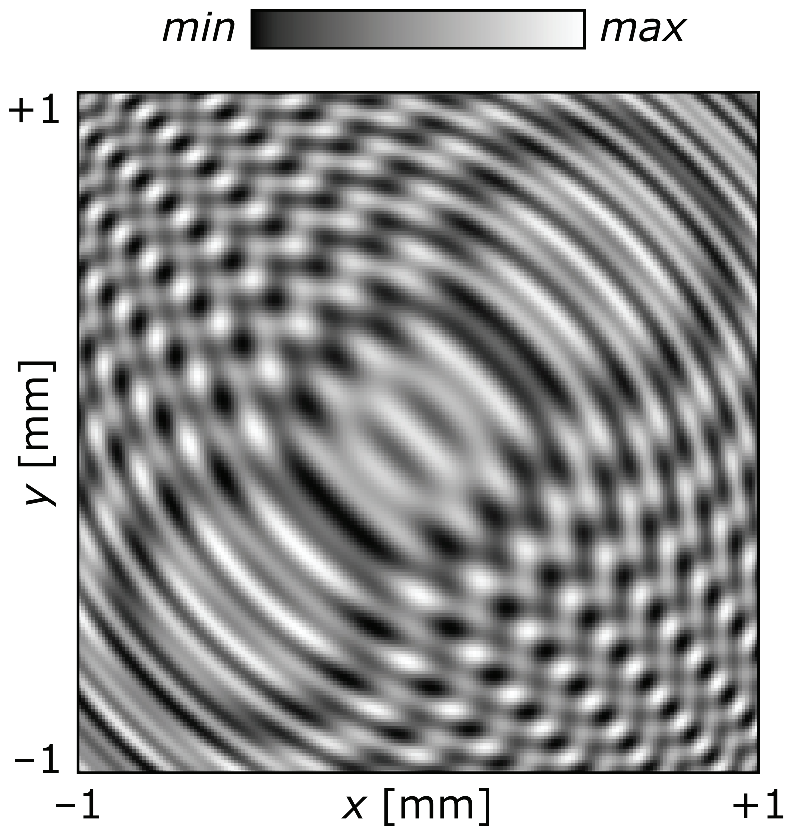
Initialization



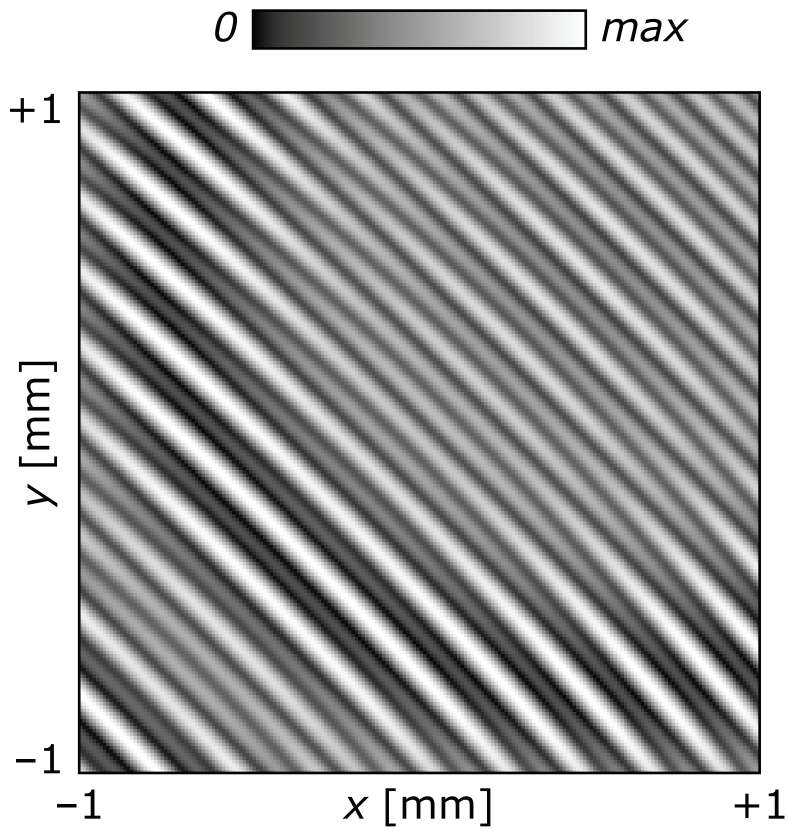

```
lambda       = 532e-9;
hologramHeight = 2e-3;
hologramWidth  = 2e-3;
hologramZ = 0;
Delta        = 10e-6;
samplesX  = hologramWidth / Delta;
samplesY  = hologramHeight / Delta;
cornerX   = -hologramWidth / 2;
cornerY   = -hologramHeight / 2;
points    = [        0,                 0, -0.2;
      -hologramWidth/4,-hologramHeight/4, -0.2;
       hologramWidth/4, hologramHeight/4, -0.22];
```

Object wave calculation

```
k = 2*pi/lambda;
objectWave = zeros(samplesY, samplesX);
for s = 1:rows(points)
  for column = 1:samplesX
    for row = 1:samplesY
      x = (column-1) * Delta + cornerX;
      y = (row-1)    * Delta + cornerY;
      r = sqrt((x         - points(s, 1))^2 ...
             + (y         - points(s, 2))^2 ...
             + (hologramZ - points(s, 3))^2);
      objectWave(row,column) += exp(1i*k*r) / r;
    end
  end
end
```

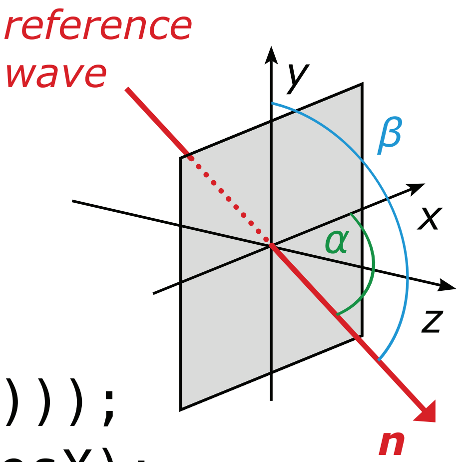


Real part of the object wave
(Just for information;
it has no physical meaning!)

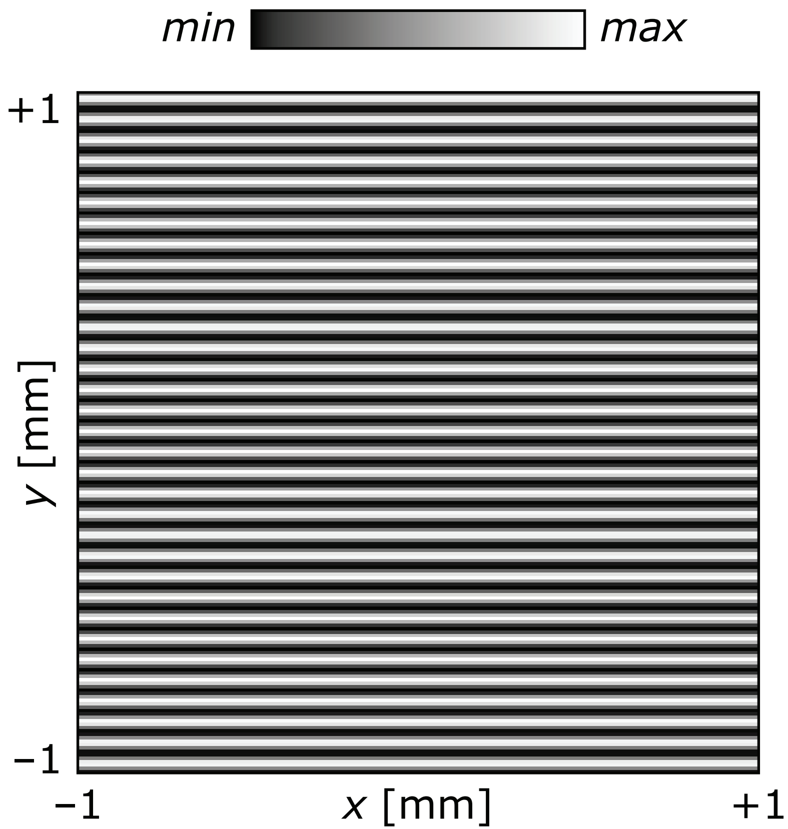


Intensity of the object wave
(Just for information;
it has no practical use!)

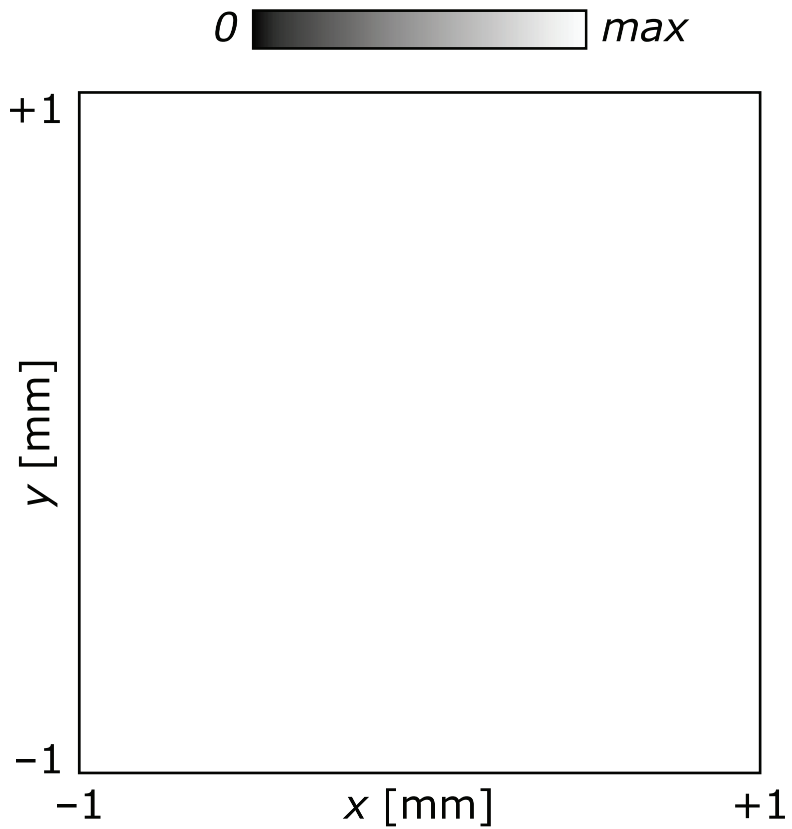
Reference wave calculation




```
alpha = 90   * pi/180;
beta  = 90.5 * pi/180;
nX = cos(alpha); nY = cos(beta);
nZ = sqrt(1 - nX^2 - nY^2);
refAmplitude = max(max(abs(objectWave)));
referenceWave = zeros(samplesY, samplesX);
for column = 1:samplesX
  for row = 1:samplesY
    x = (column-1) * Delta + cornerX;
    y = (row-1)    * Delta + cornerY;
    referenceWave(row,column) = refAmplitude * ...
            exp(1i*k*(x*nX + y*nY + hologramZ*nZ));
  end
end
```



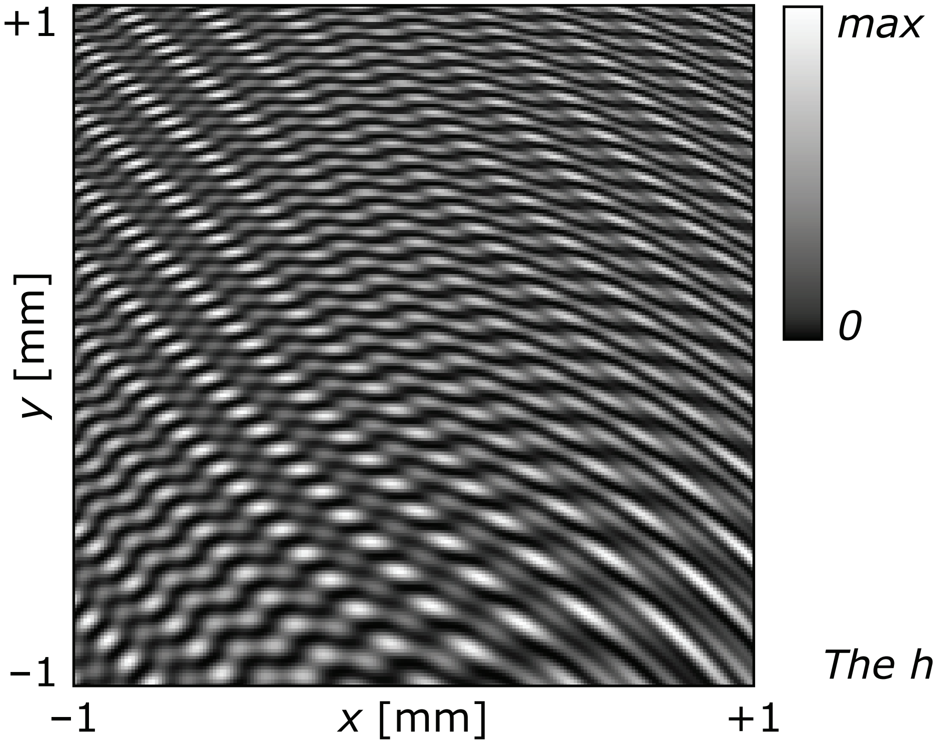
Real part of the object wave
(Just for information;
it has no physical meaning!)



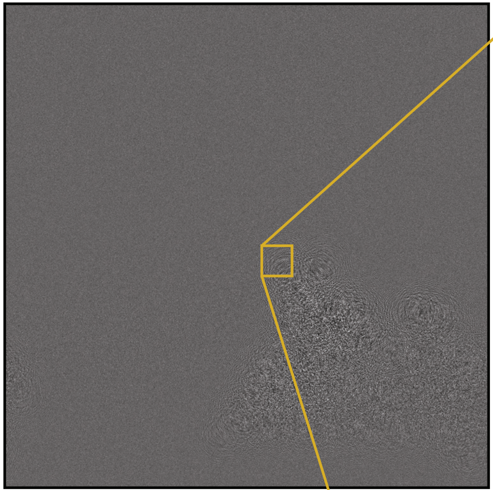
Intensity of the object wave
(Just for information;
it has no practical use!)

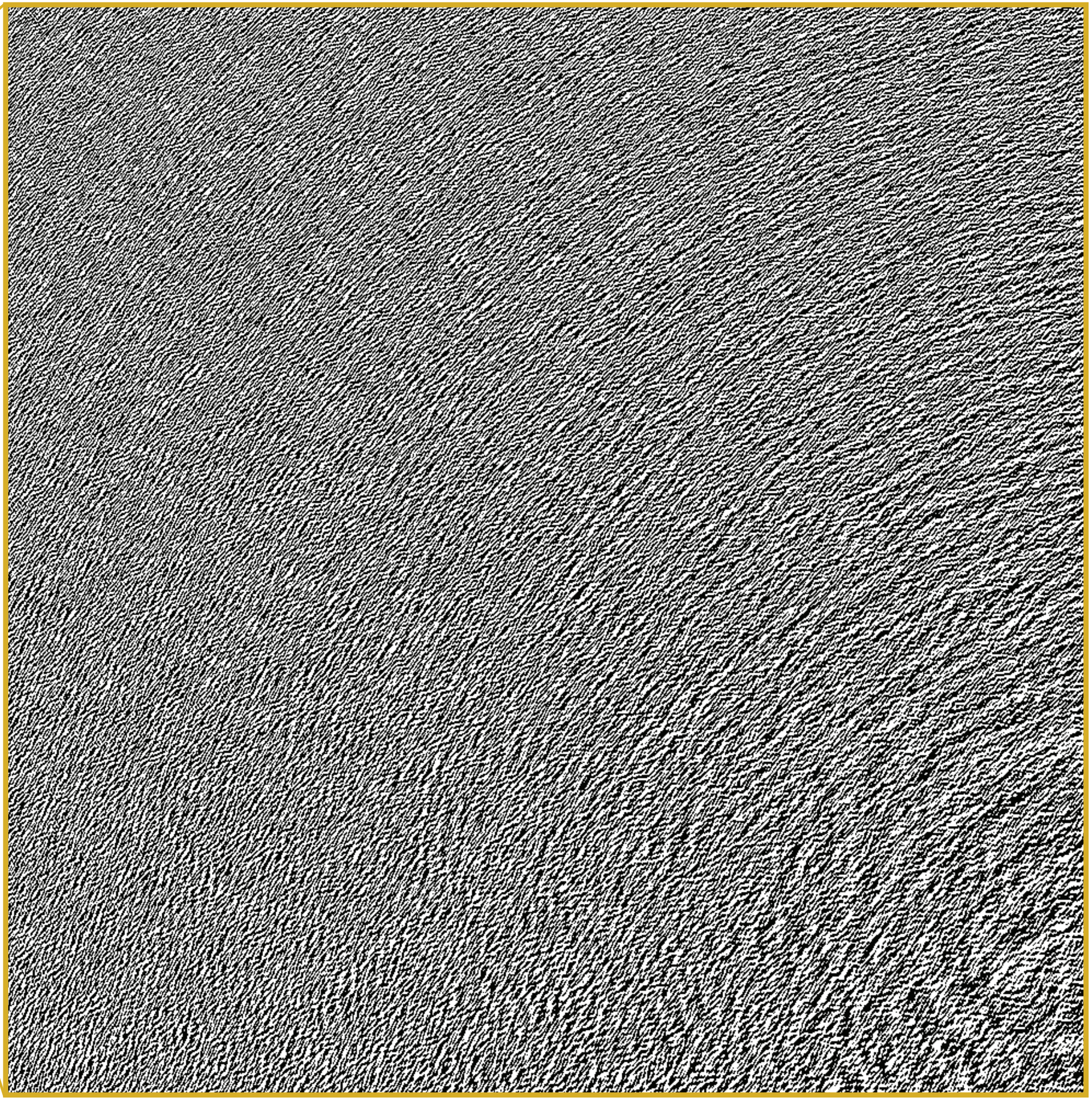
Hologram calculation

```
optField = objectWave + referenceWave;
hologram = optField .* conj(optField);
```



The hologram (intensity picture)





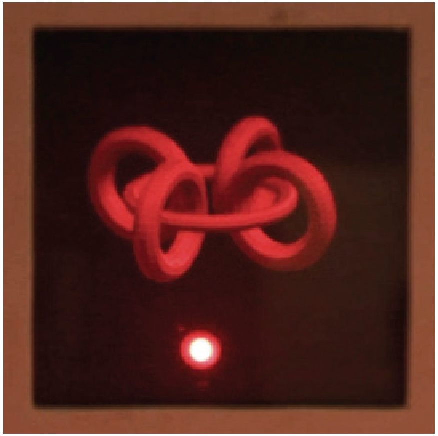
Computer generated hologram of a 3-D scene

6144 × 6144 pixels
Size 4,3 × 4,3 cm²
(resolution 3600 dpi ~ pixel size 7 µm)

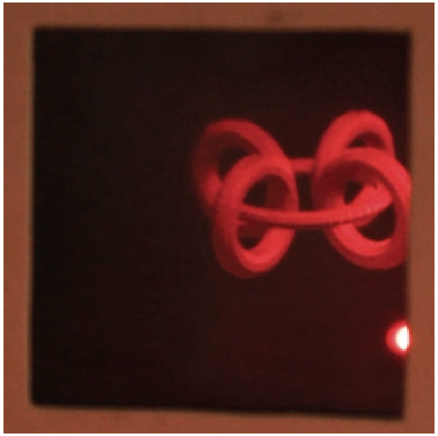
How to “print” a calculated hologram?

- electron beam lithography – very expensive
  - 0.05 μm details ⇒ diffraction up to 90°
  - size up to ~ 5 × 5 cm^2, recording 1 mm^2/min
- laser lithography – expensive
  - 1 μm details ⇒ diffraction up to 20°
  - size up to ~ 20 × 20 cm^2, recording 4 mm^2/min

left view



central view

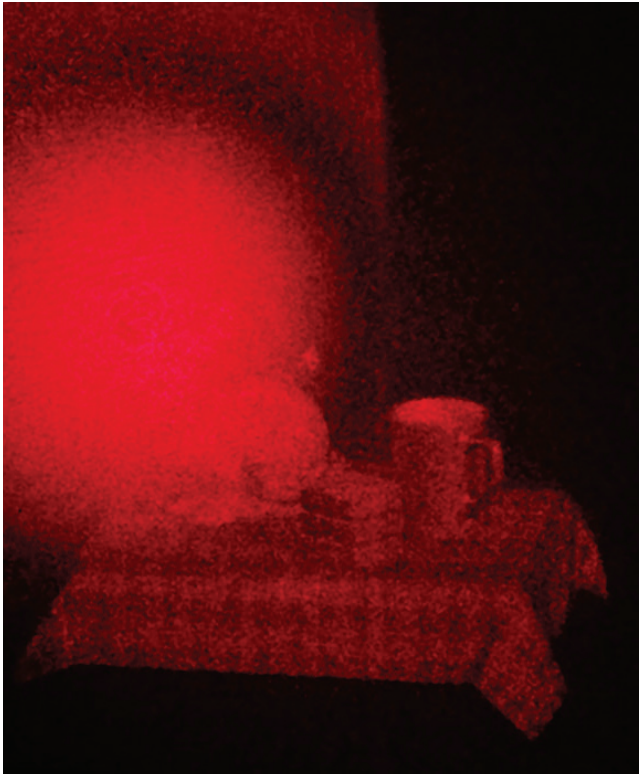


right view

Hologram by K. Matsushima

- imagesetter
  - 10 μm details
    ⇒ diffraction up to 2°
  - price ~ 5 € per A4
- laser printer
  - 100 μm details
    ⇒ diffraction up to 0.5°

- “holographic printers” do not usually print the calculated pattern



Hologram by I. Hanák, M. Janda

Electronic “holographic display”?

- microdisplays = spatial light modulators
- transmissive or reflective
- size up to 40 mm diagonal
- resolution up to 8K (7680 × 4320 pixels)
- pixel size down to ~ 4 μm
- usually LCD (liquid crystal display),
  DMD (digital micromirror device) or
  LCOS (liquid crystal on silicon)

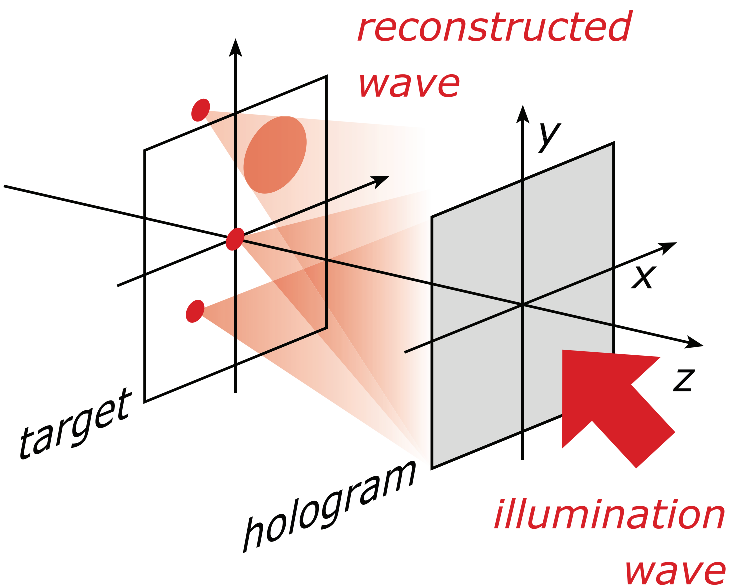
- small size, small diffraction angle (up to 5°)
  - tiling, multiplexing, additional optics to improve performance

Numerical simulation of hologram reconstruction

- useful for evaluation purposes
- the second basic building block of advanced algorithms of computer generated display holography
- allows to observe the real image
- to simulate the virtual image, it is necessary to add a lens simulation
  (easy, but not covered in the tutorial)

Real image calculation

- illuminate the hologram “from behind”– each point of the hologram becomes a light source
  $U(x, y, z_{\text{holo}}) = \mathit{hologram}(x, y) \times U_{\text{illum}}(x, y, z_{\text{holo}})$
- place “a target screen” somewhere
- calculate light from the hologram on the target screen
- constructive interference creates the real image



- sum of spherical waves
  - phasor of a spherical wave
    $K(x, y, z) = A\exp(\mathrm{j}kr) / r$
    $r = [(x - x_C)^2 + (y - y_C)^2 + (z - z_C)^2]^{1/2}$
  - each has its own amplitude A
  - their origins (x_C, y_C, z_C) are in the plane of the hologram, $z_C = z_{holo}$
  - the screen is usually parallel to the hologram in $z = z_{target}$
- it is possible to use convolution

$$U(x, y, z_{target}) = U(x, y, z_{holo}) \otimes \underbrace{K(x, y, z_{holo} - z_{target})}_{\textit{kernel}}$$

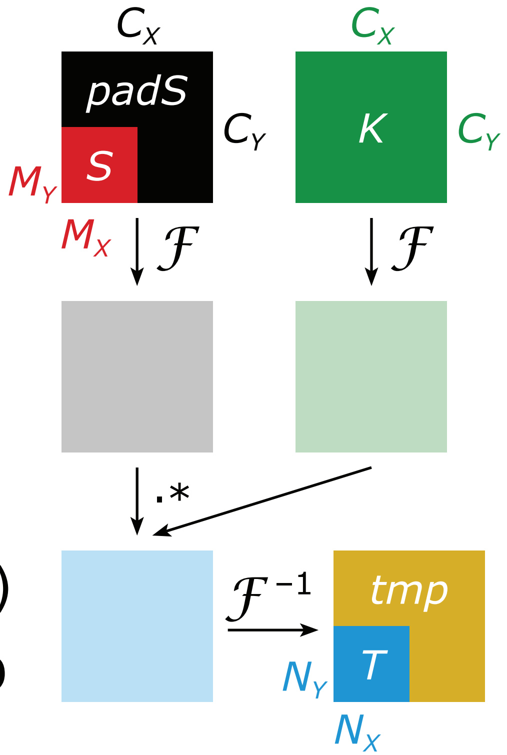
- convolution calculation using the Fourier transform

$$U(x, y, z_{\text{screen}}) = \mathcal{F}^{-1}\left\{\mathcal{F}\{U(x, y, z_{\text{holo}})\} \times \mathcal{F}\{K(x, y, z_{\text{holo}} - z_{\text{target}})\}\right\}$$

- efficient numerical algorithm: the fast Fourier transform (FFT)
- convolution calculation using FFT requires some additional steps

- to calculate $T = S \otimes K$
  S: source, $M_X \times M_Y$ samples
  T: target, $N_X \times N_Y$ samples
  K: kernel (spherical w. phasor), $C_X \times C_Y$ samples

1. pick any $C_X \geq M_X + N_X - 1$,
   $C_Y \geq M_Y + N_Y - 1$
2. add zero samples to S
   to get size $C_X \times C_Y$ *(zero padding)*
3. calculate $C_X \times C_Y$ samples of K
4. calculate
   `tmp = ifft2(fft2(padS) .* fft2(K))`
5. T = first $N_X \times N_Y$ samples from tmp



Actual code for hologram propagation

```
propagZ = 200e-3;
cX = 2*samplesX-1; cY = 2*samplesY-1;
auxX = ((0:cX-1) - (samplesX-1)) * Delta;
auxY = ((0:cY-1) - (samplesY-1)) * Delta;
[auxXX, auxYY] = meshgrid(auxX, auxY);
padS = zeros(cY, cX);
padS(1:samplesY, 1:samplesX) = ...
            hologram .* conj(referenceWave);
r = sqrt(auxXX.^2 + auxYY.^2 + propagZ^2);
kernel = k*1i*propagZ*exp(1i*k*r) ./ r.^2;
kernel = circshift(kernel, ...
           [-samplesY + 1, -samplesX + 1]);
tmp = ifft2(fft2(padS) .* fft2(kernel));
target = tmp(1:samplesY, 1:samplesX);
```

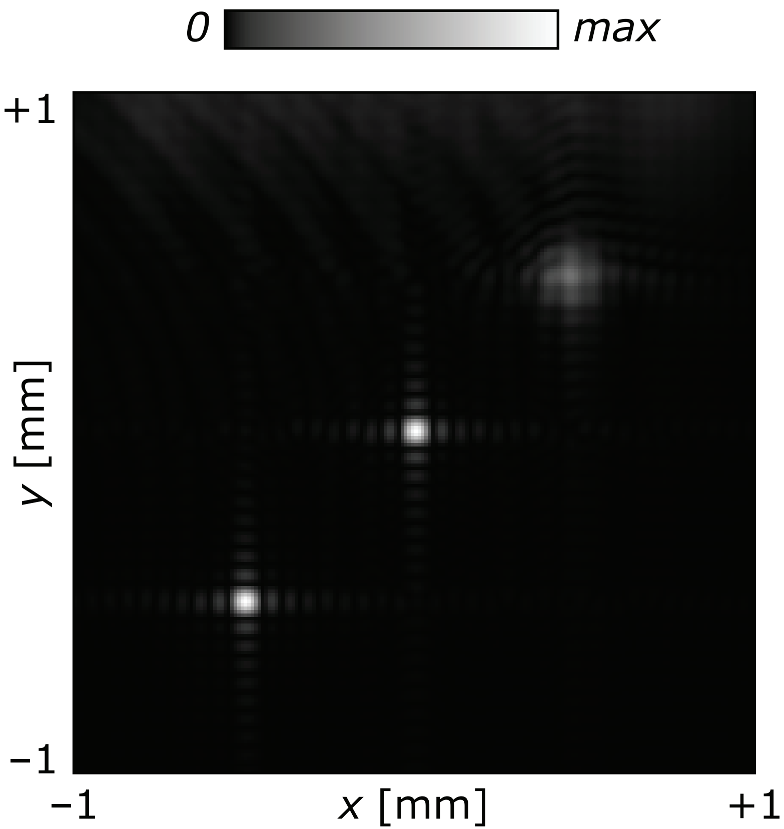
1. initialization

2. zero padding & illumination

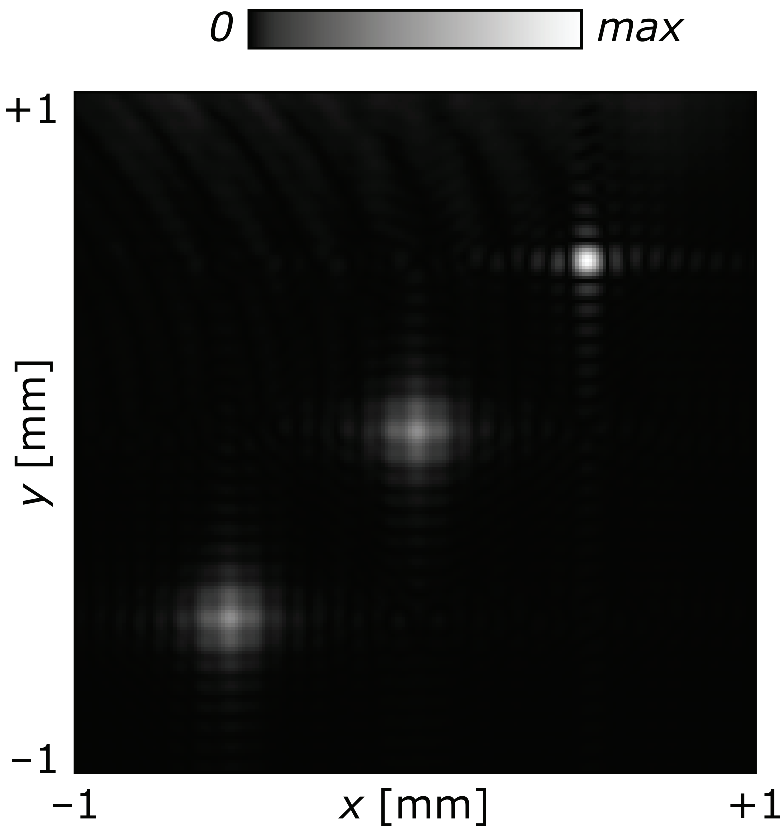
3. kernel calculation

4.

5.



The real image in z = −200 mm
(intensity picture)



The real image in z = −220 mm
(intensity picture)

- strictly speaking, we should use a proper propagation kernel instead of spherical waves
- Rayleigh-Sommerfeld propagation kernel
  - derived from the Maxwell's equations

$$K_{\mathrm{RS}}(x, y, z) = \underbrace{-\frac{1}{2\pi}\left(\mathrm{j}k - \frac{1}{r}\right)\frac{z}{r}}_{\substack{\textit{additional factor} \\ \textit{(can be often approximated or neglected)}}} \underbrace{\frac{\exp(\mathrm{j}kr)}{r}}_{\substack{\textit{spherical} \\ \textit{wave}}}$$

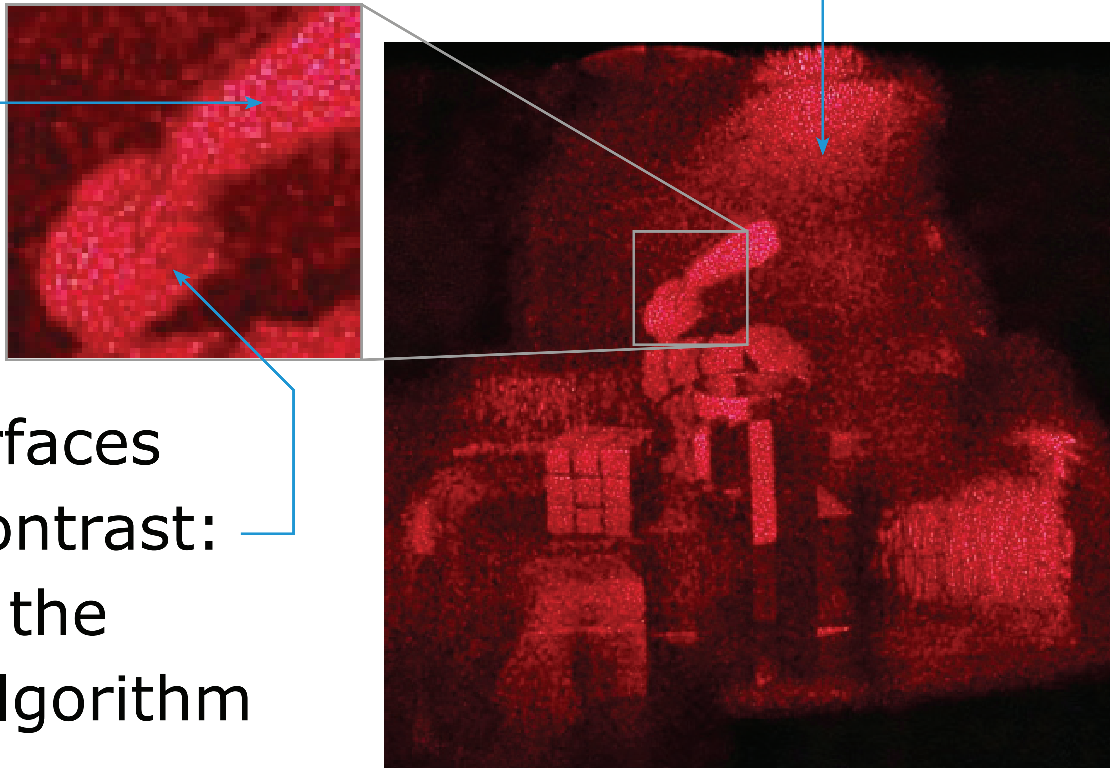
- Fresnel propagation kernel
  - follows from the Taylor approximation of r

$$K_{\mathrm{FS}}(x, y, z) = -\frac{\exp(\mathrm{j}kz)}{\mathrm{j}\lambda z}\exp\left(\mathrm{j}k\frac{x^2 + y^2}{2z}\right)$$

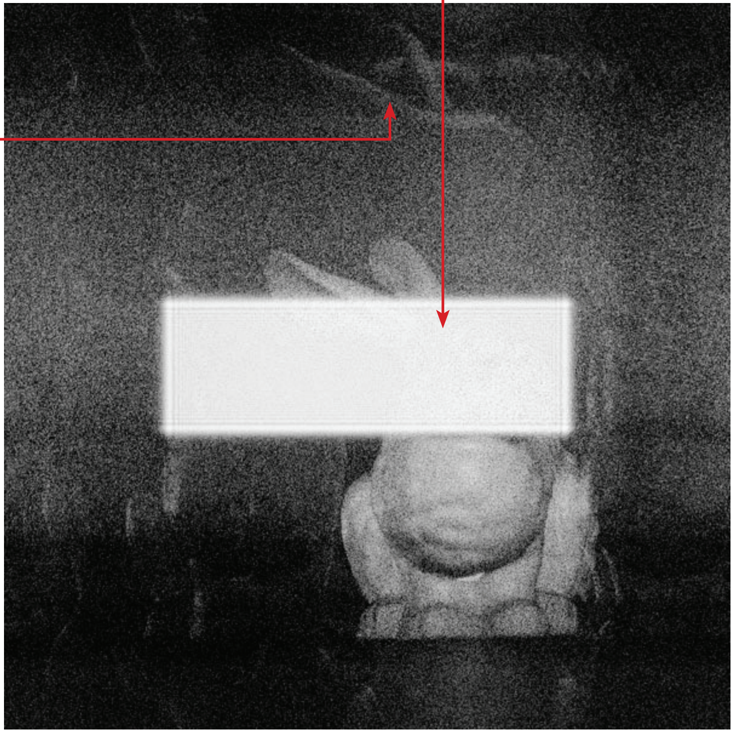
  - nice numerical properties ⇒ often employed

Are the calculations correct?

- physical hologram observation
  note various artifacts:
  - blurry ghost images: unwanted diffraction orders
  - speckle noise: property of coherent llumination of diffuse surfaces
  - unrealistic contrast: weakness of the generation algorithm



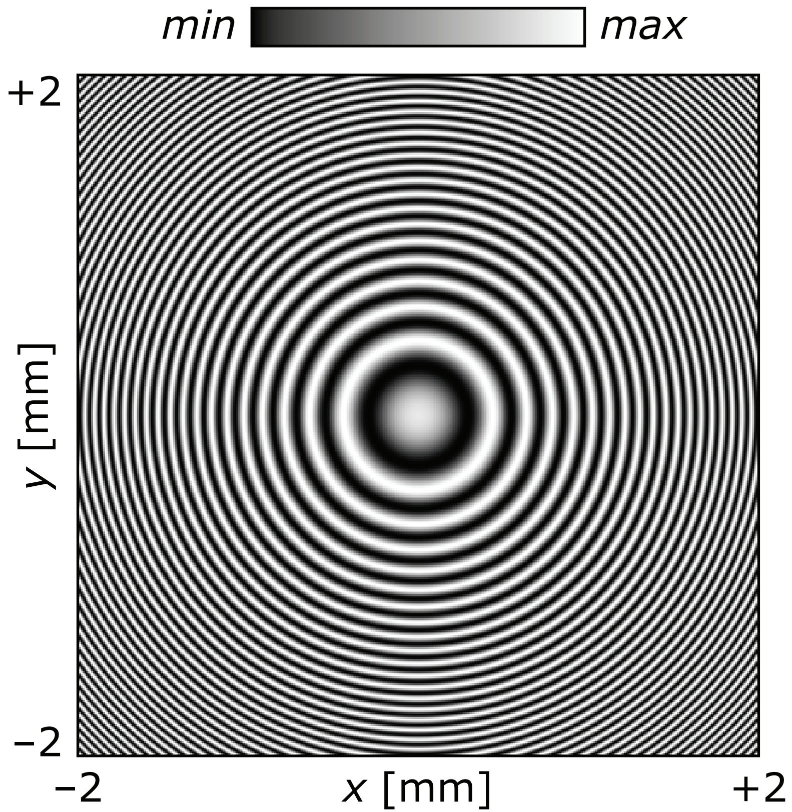
- simulated hologram observation:
  note various artifacts:
  - speckle noise: physically correct
  - 0th diffraction order: weakness of the simulation setup, but physically correct
  - replicas of the image: error of the algorithm



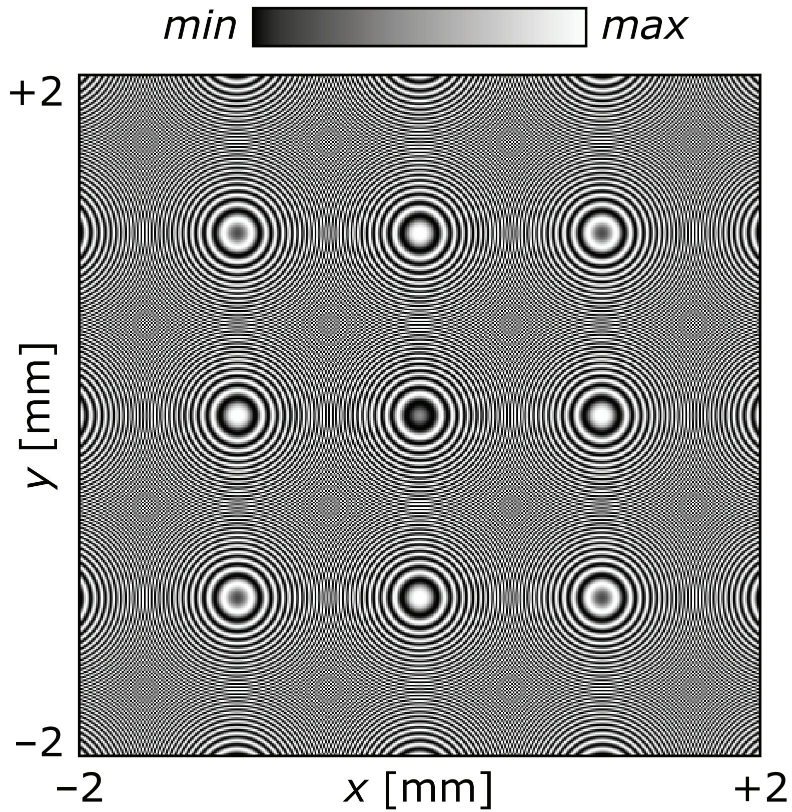
- it is not easy for a beginner to distinguish errors in algorithm from a physical phenomenon

Aliasing error

- difficult to recognize and fix for a beginner



real part of $K_{RS}(x, y, 200\text{ mm})$



real part of $K_{RS}(x, y, 20\text{ mm})$

(*sampling distance* 10 μm, λ = 532 nm)

Local frequency

- $\sin(2\pi fx)$ – frequency $f = \partial(fx) / \partial x$
- the most oscillatory part of $K_{RS}(x, y, z)$:
  $\exp(j\, 2\pi\, r/\lambda) = \exp(j\, 2\pi\, g)$
  $r = (x^2 + y^2 + z^2)^{1/2}$
- by analogy: local frequency

$$lf_X = \frac{\partial g}{\partial x} = \frac{1}{\lambda}\frac{\partial r}{\partial x} = \frac{x}{\lambda r} \qquad lf_Y = \frac{\partial g}{\partial y} = \frac{1}{\lambda}\frac{\partial r}{\partial y} = \frac{y}{\lambda r}$$

- due to sampling theorem, it must hold
  $lf_X \leq 0.5 \times$ sampling frequency (the same for lf_Y)
- big x, y or small z leads to high local frequency
- usually safe to set $K_{RS}(x, y, z) = 0$
  at points where the local frequency is too high

Angular spectrum decomposition

- propagation using convolution:

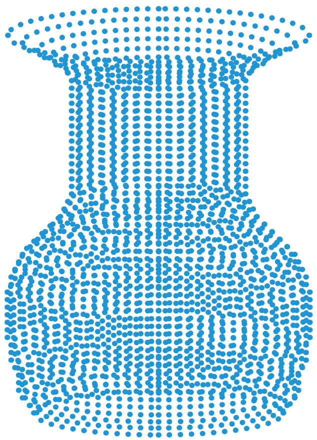
  $U(x, y, z_{\text{screen}}) =$
  $\mathcal{F}^{-1}\{\mathcal{F}\{U(x, y, z_{\text{holo}})\} \times \mathcal{F}\{K(x, y, z_{\text{holo}} - z_{\text{target}})\}\}$
- it holds

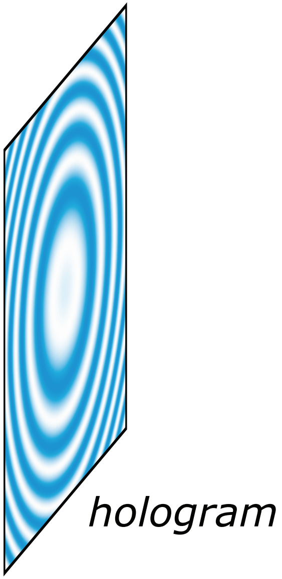
  $\mathcal{F}\{K_{RS}(x, y, z)\} = H_{RS}(f_X, f_Y, z) =$
  $\exp[\mathrm{j}\ 2\pi\, \mathrm{z}\,(\lambda^{-2} - f_x{}^2 - f_y{}^2)^{1/2}]$
- transfer function H_{RS} has low local frequency for small z ⇒ allows propagation calculation for small z
- usually safe to set $H_{\text{RS}}(f_x, f_Y, z) = 0$ if local frequency is too high
- also allows to calculate propagation between non-parallel planes

SELECTED BUILDING BLOCKS OF ADVANCED CGDH ALGORITHMS

Scene replacement with a point cloud

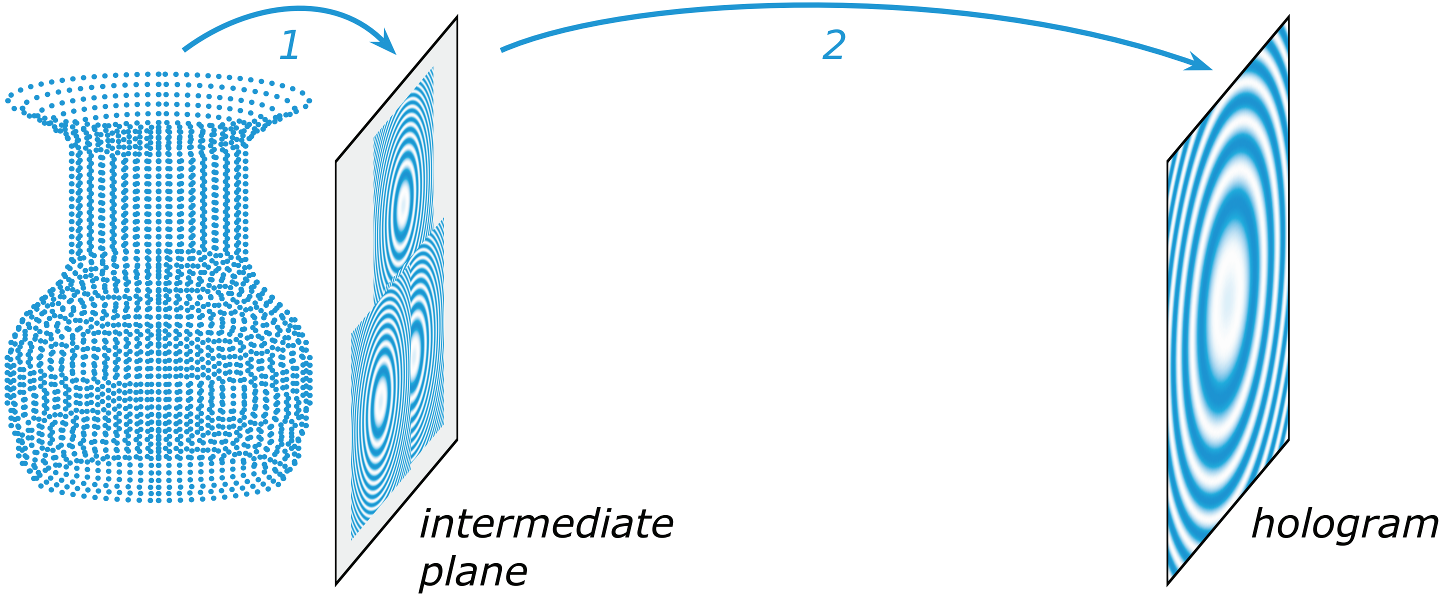
- extraordinary number of points needed
- each point illuminates the whole hologram
  ⇒ slow
- does not handle surface visibility
- easy parallelization ⇒ fast for thousands of points





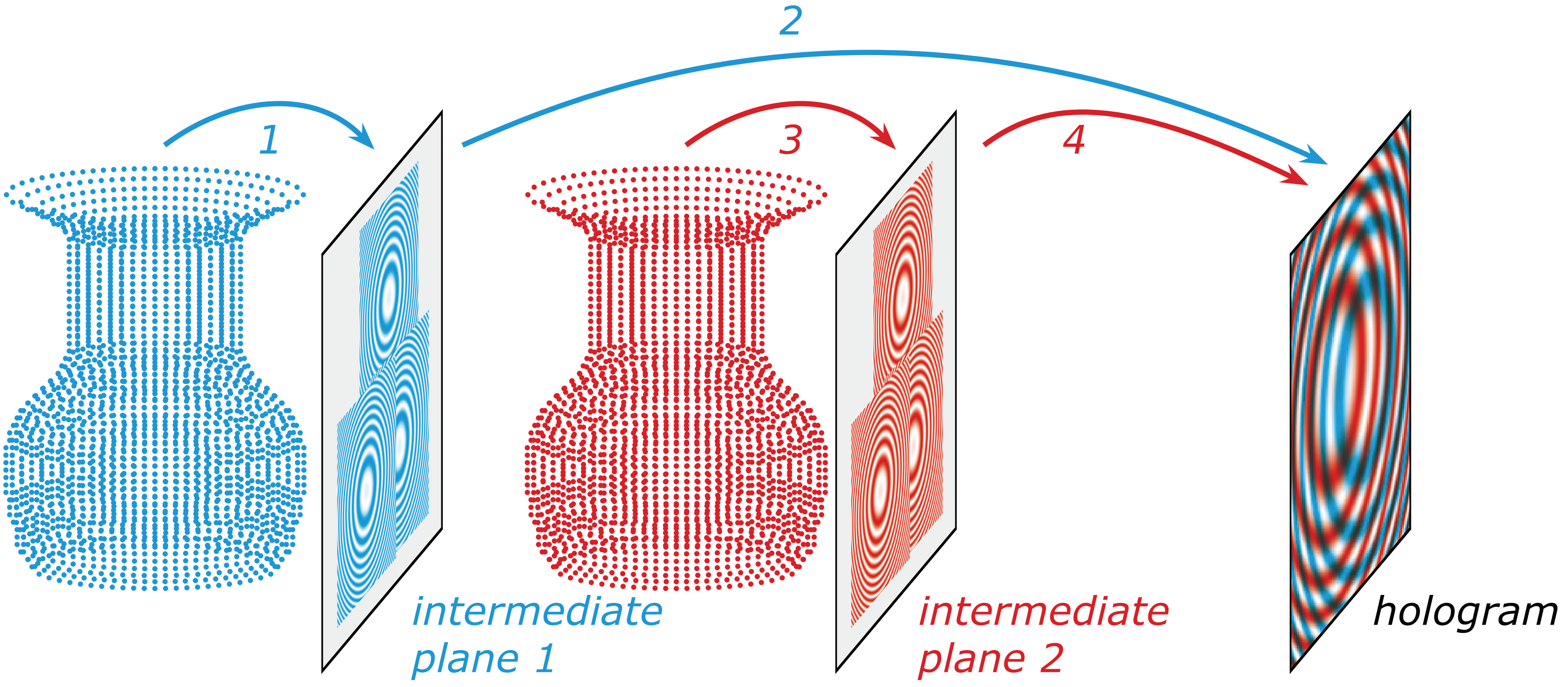
Restricted area of contribution

- intermediate plane close to the point cloud
  - point contributes to a small area only ⇒ fast
- propagation of intermediate plane to the hologram ⇒ fast
- works well for shallow scenes only



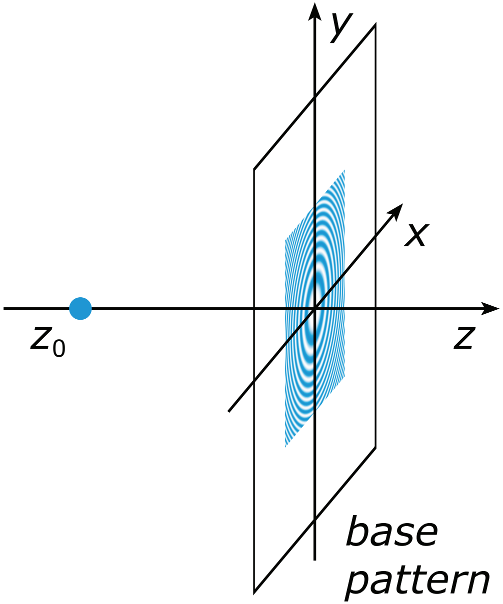
Multiple intermediate planes

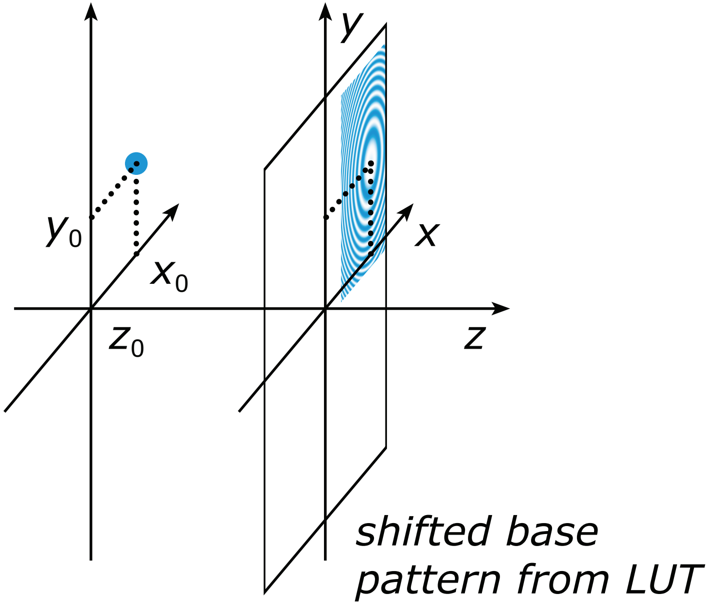
- propagation of a point to the nearest intermediate plane
- propagation of each intermediate plane to the hologram, summation of results



Look-up tables (LUT)

- pre-calculation of propagation from $(0, 0, z_0)$ to the plane $z = 0 \Rightarrow$ base patterns for many z_0
- propagation from (x_0, y_0, z_0) to the plane $z = 0$ is a shifted base pattern
  $\Rightarrow$ "copy & paste" instead of the pattern calculation





Separable look-up tables

- Fresnel approximation of a spherical wave:

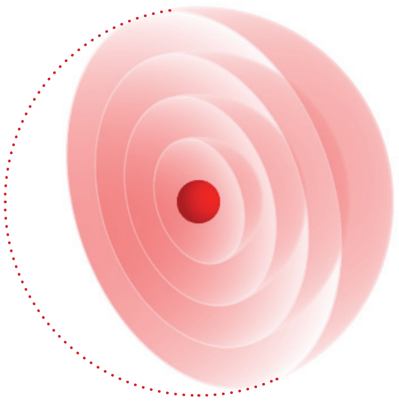
$$K_{\mathrm{FS}}(x, y, z) = -\frac{\exp(\mathrm{j}kz)}{\mathrm{j}\lambda z}\exp\left(\mathrm{j}k\frac{x^2}{2z}\right)\exp\left(\mathrm{j}k\frac{y^2}{2z}\right)$$

- separable
  - only 1-D patterns have to be pre-calculated: $\exp(\mathrm{j}kx^2/2z)$ and $\exp(\mathrm{j}ky^2/2z)$
  - look-up tables small (they are just 1-D)
- multiplication on the fly

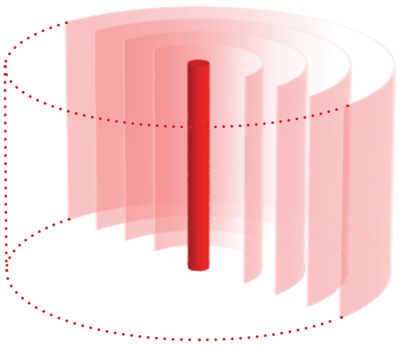
Other primitive objects

- point ⇒ spherical wave
- line ⇒ cylindrical wave
- line segment ⇒
  "clipped cylindrical wave"
- triangle ⇒
  wave can be also written
  in the closed form

- problem: flat triangle
  ⇒ constant phase on the surface
  ⇒ emits light in one direction
  ⇒ hard to simulate diffuse surfaces



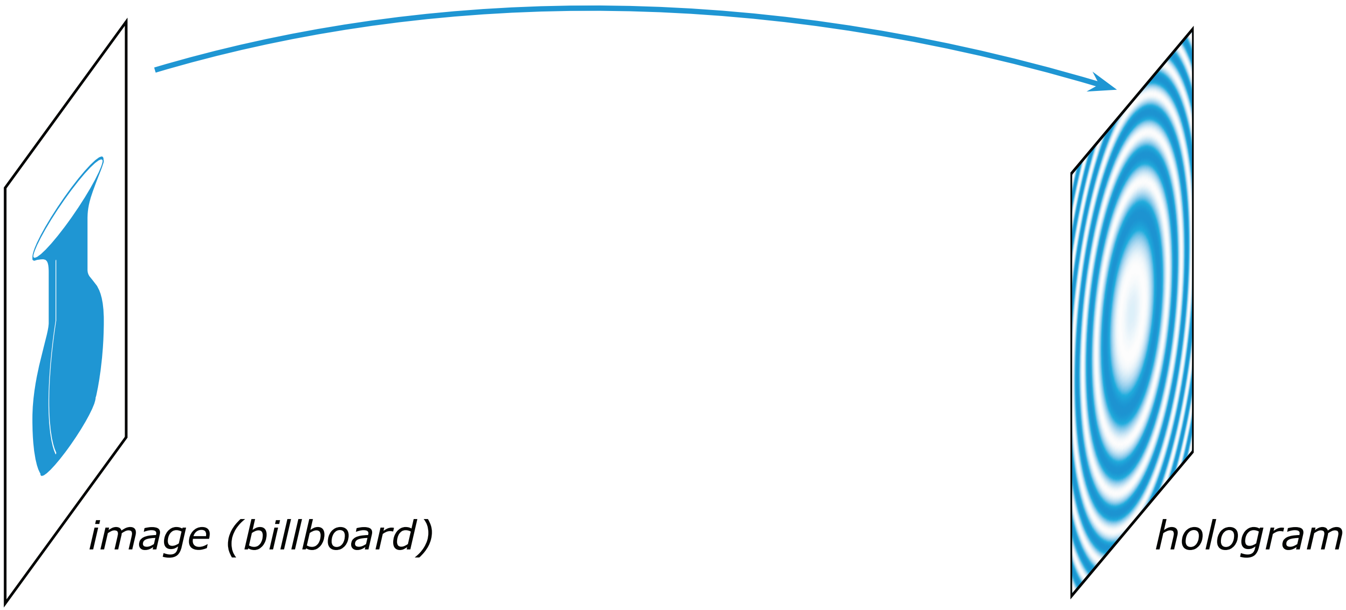
spherical wave



cylindrical wave clipped on the top and the bottom of the line

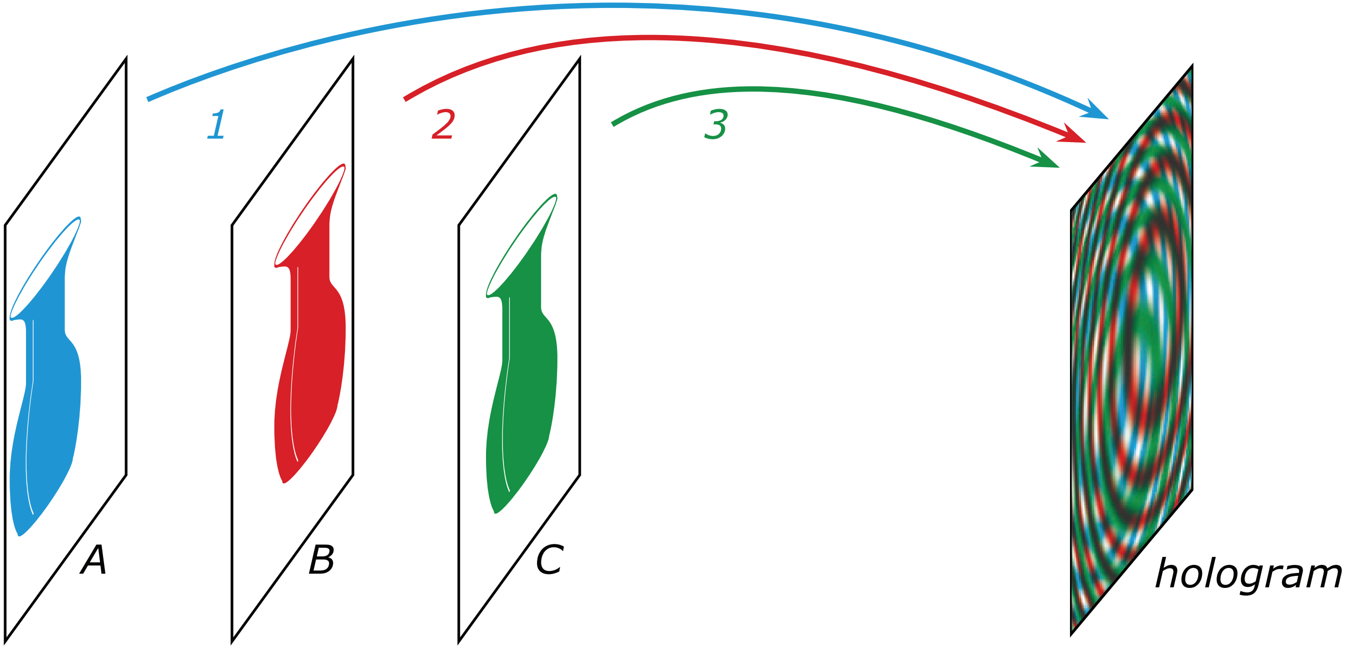
Single billboard

- object replacement with a flat image (billboard)
- propagation of the billboard to the hologram
- hidden surface elimination solved by computer graphics when rendering the image
- reconstructed scene is not 3-D



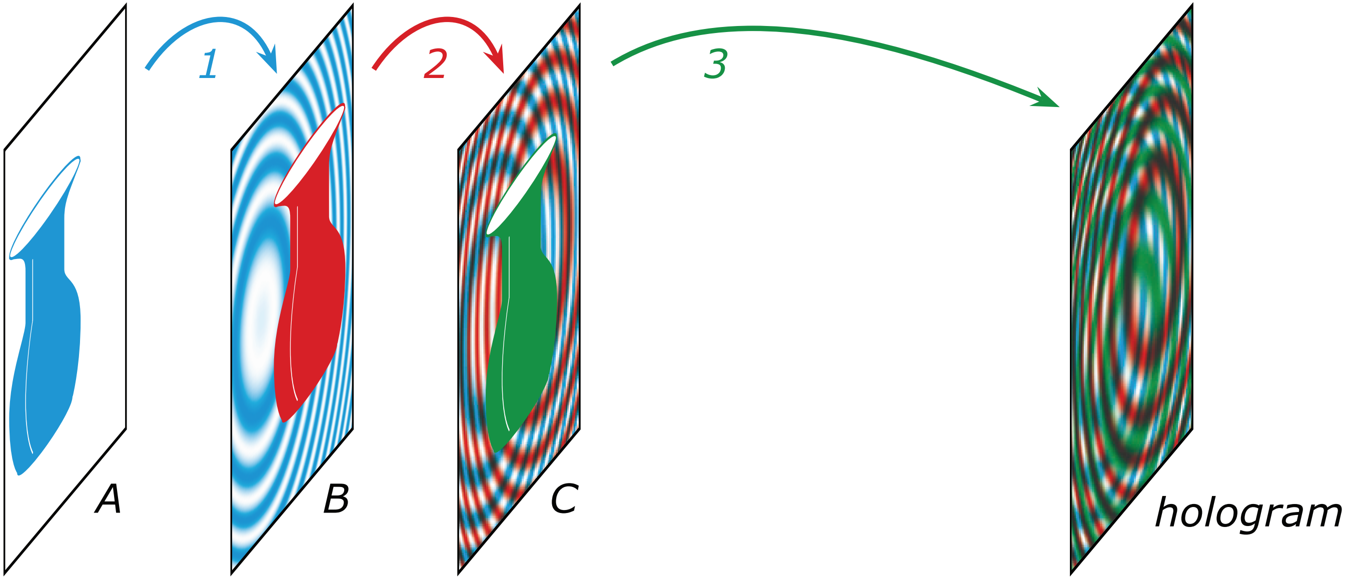
Multiple billboards

- scene replacement with several billboards
- propagation A → H, B → H, C → H, …, summation
- reconstructed scene is 3-D
- does not handle hidden surface elimination between billboards



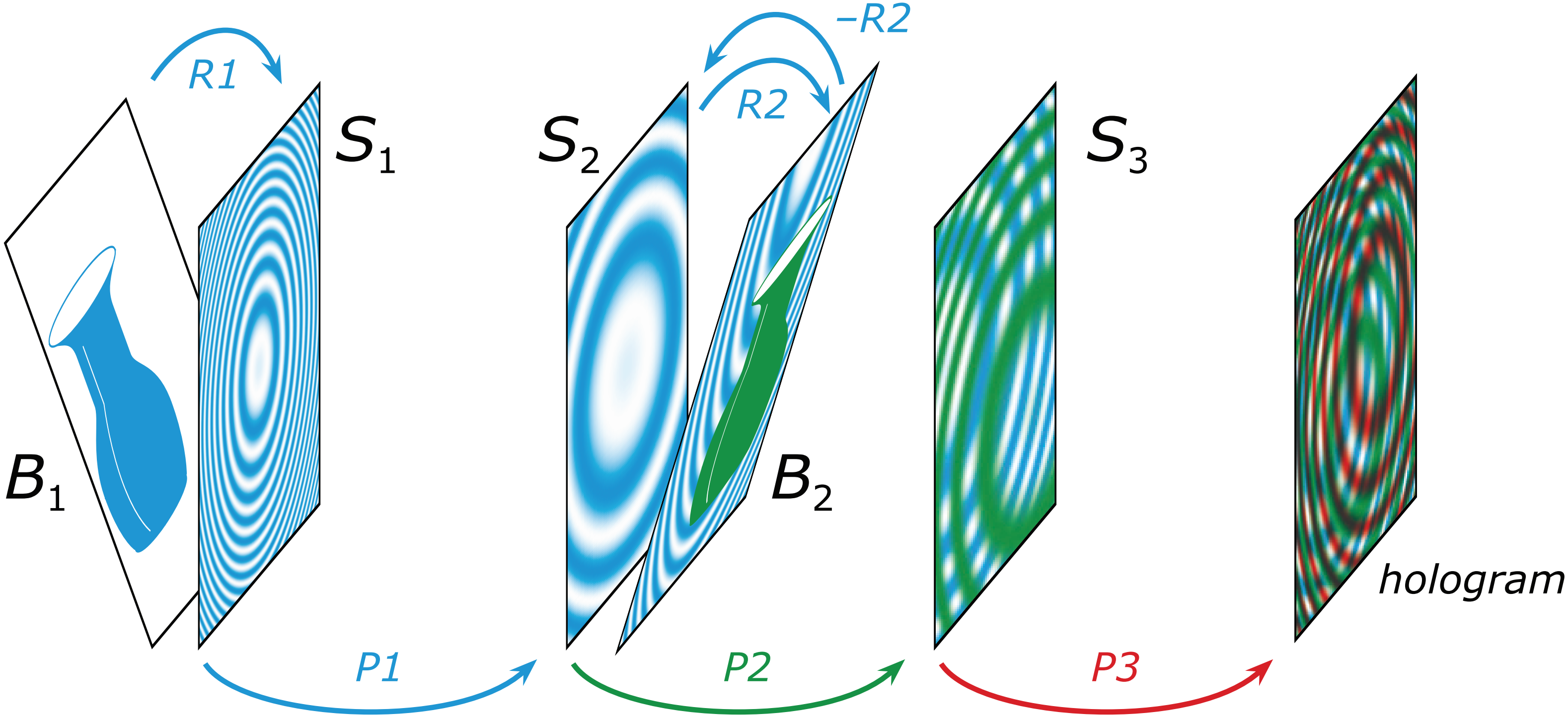
Sweep propagation plane

- propagation A → B, masking,
  B → C, masking, …, C → H
- solves hidden surface elimination
- reconstructed scene is 3-D, but sliced
  (many billboards – better quality, slower)



Generalized sweep propagation plane

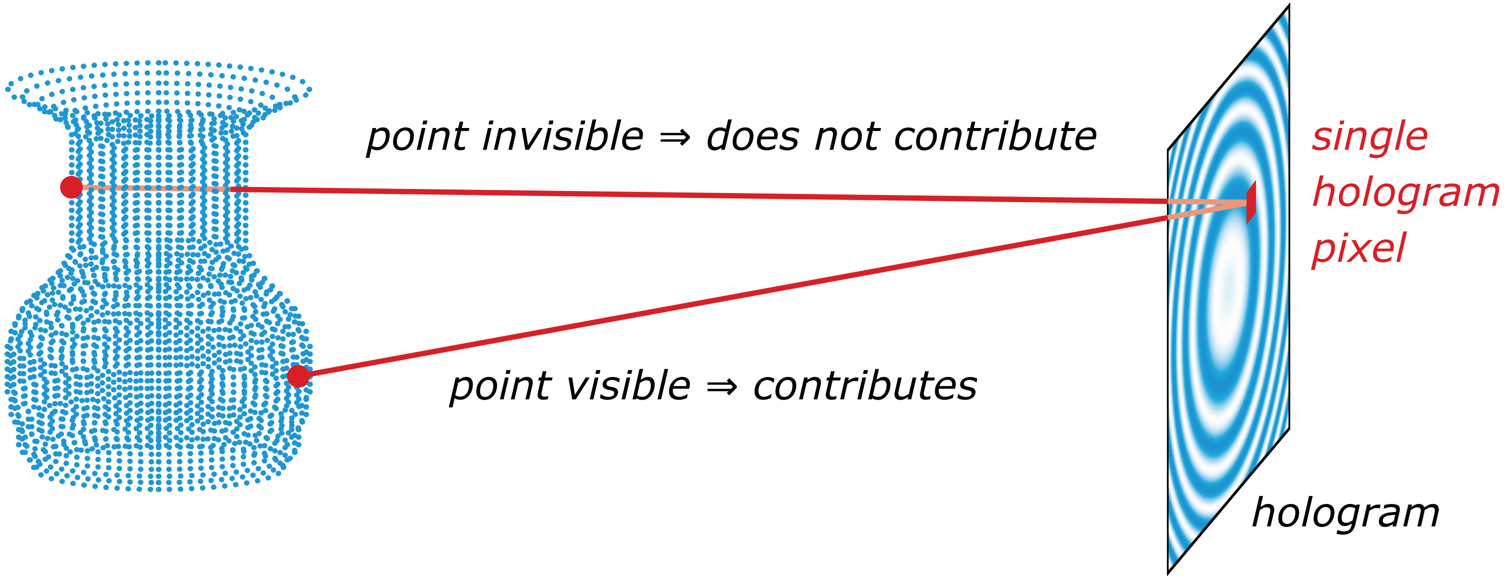
- scene split to rotated billboards
- optical rotation of billboard 1 to sweep plane 1
  propagation $S_1 \rightarrow S_2$, rotation $S_2 \rightarrow B_2$, masking,
  rotation $B_2 \rightarrow S_2$, propagation $S_2 \rightarrow S_3$, …



- natural solution for triangular meshes
  - every textured triangle is a billboard
- good hidden surface elimination
  - however, it has the same problems as the painter's algorithm
- some steps can be simplified for faster calculation
- allows to calculate high resolution holograms in reasonable time (~ 4 Gpixels, several hours)

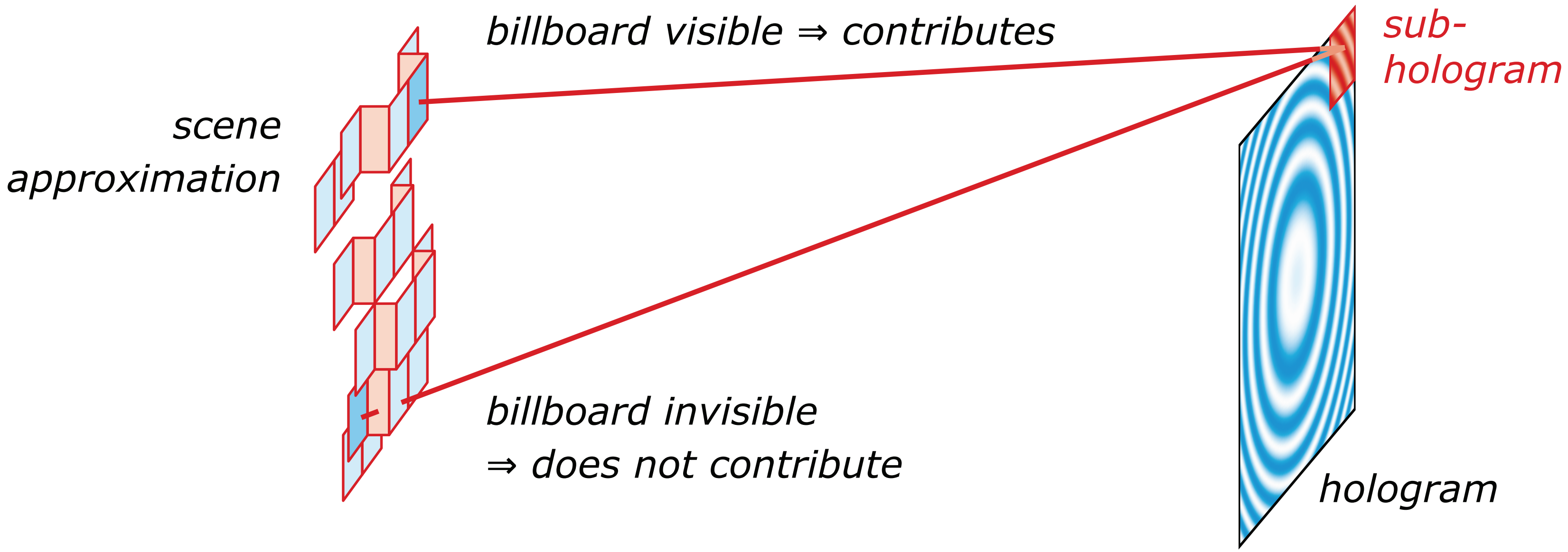
Basic ray casting

- point cloud rendering enhanced with ray casting for visibility testing
- extremely slow



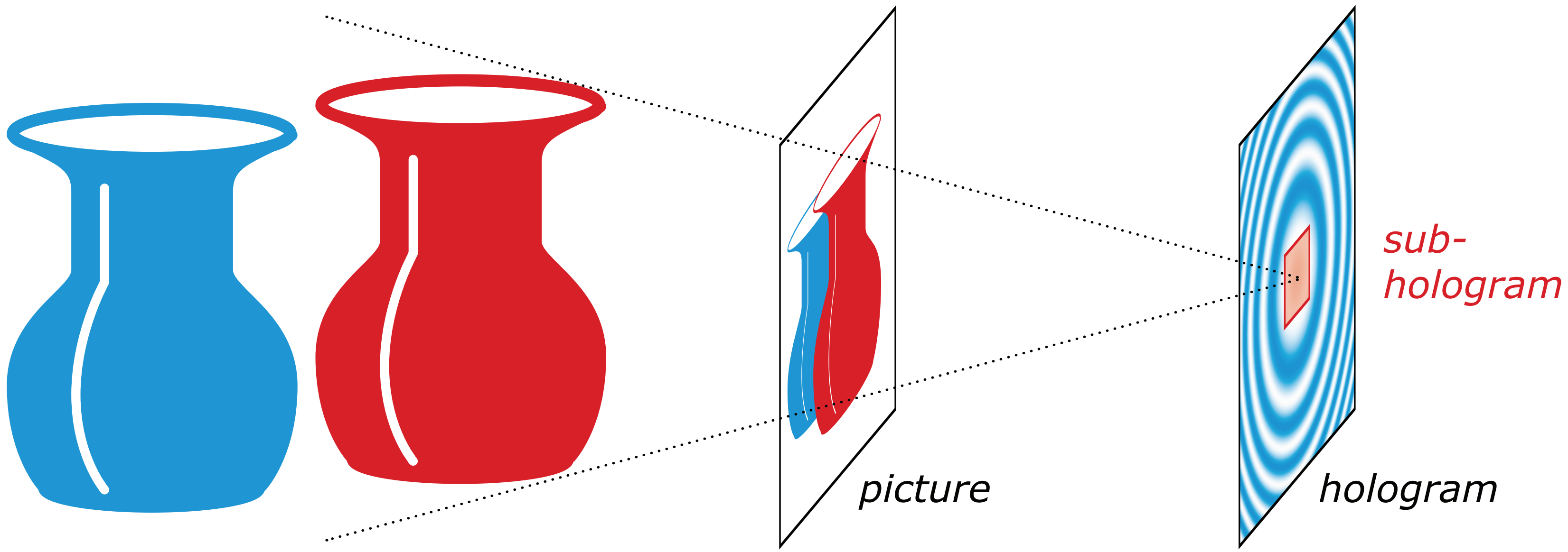
Billboards & ray casting

- scene decomposition to rectangular patches
- hologram decomposition to subholograms
- ray casting between billboards and subholograms only
- billboard → subhologram propagation fast



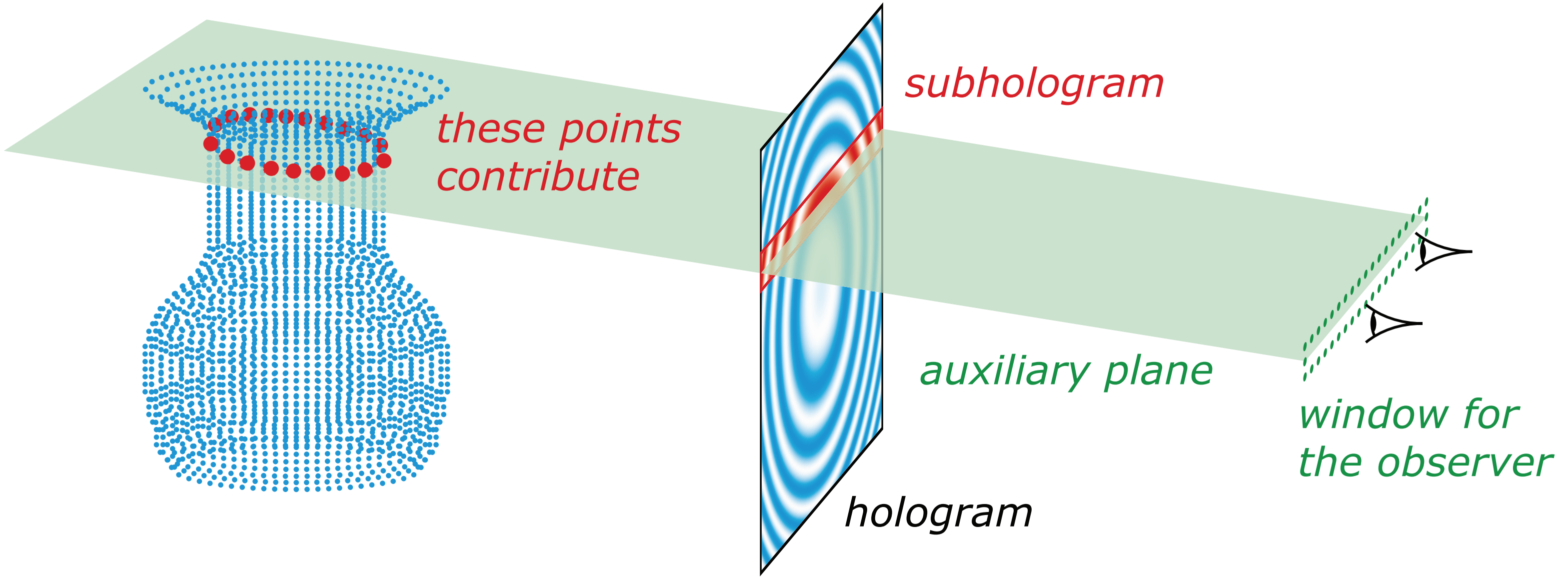
Holographic stereogram

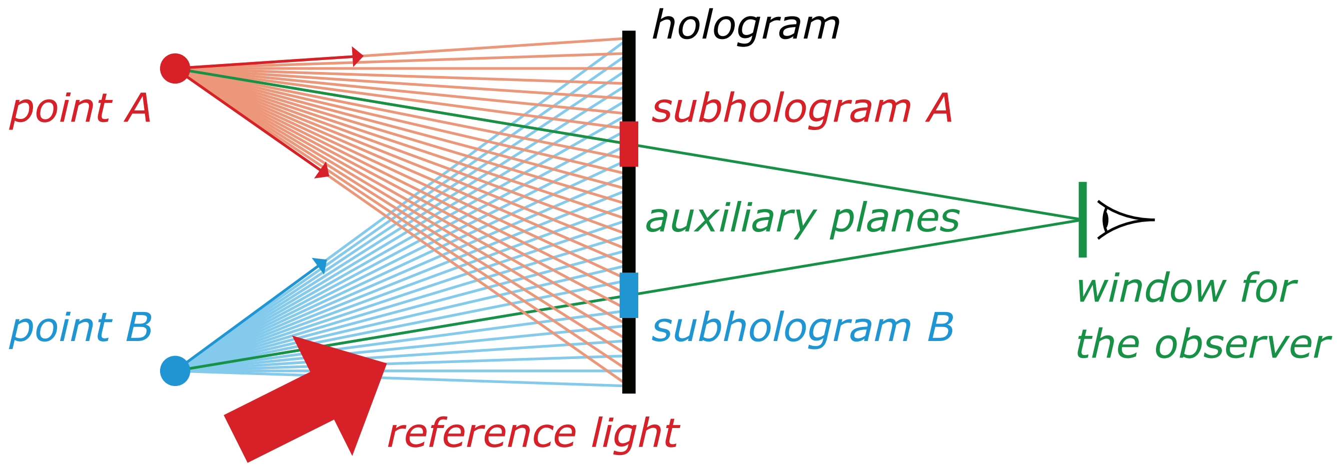
- for each subhologram
  - take a picture from its centre (i.e. record light intensity coming from various directions)
  - make a diffractive structure that replicates light rays – can be done using one Fourier transform



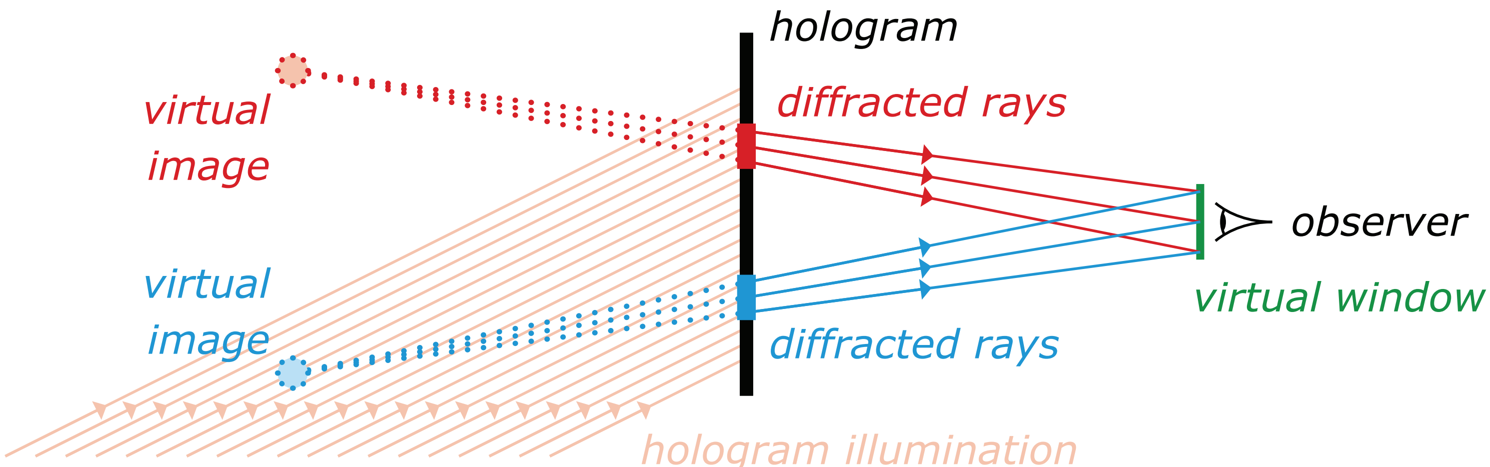
Horizontal parallax only rainbow hologram

- the image should be visible from “a window” only
- hologram decomposition to subholograms (“slits”)
- determine which points of the scene contribute to the subhologram
- calculate the subhologram

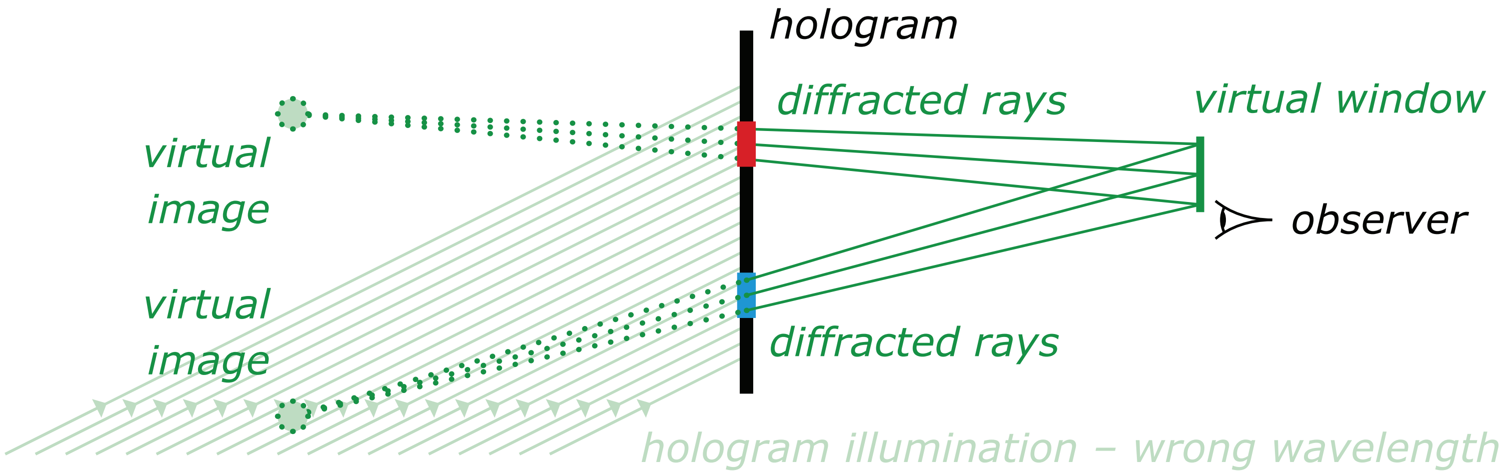




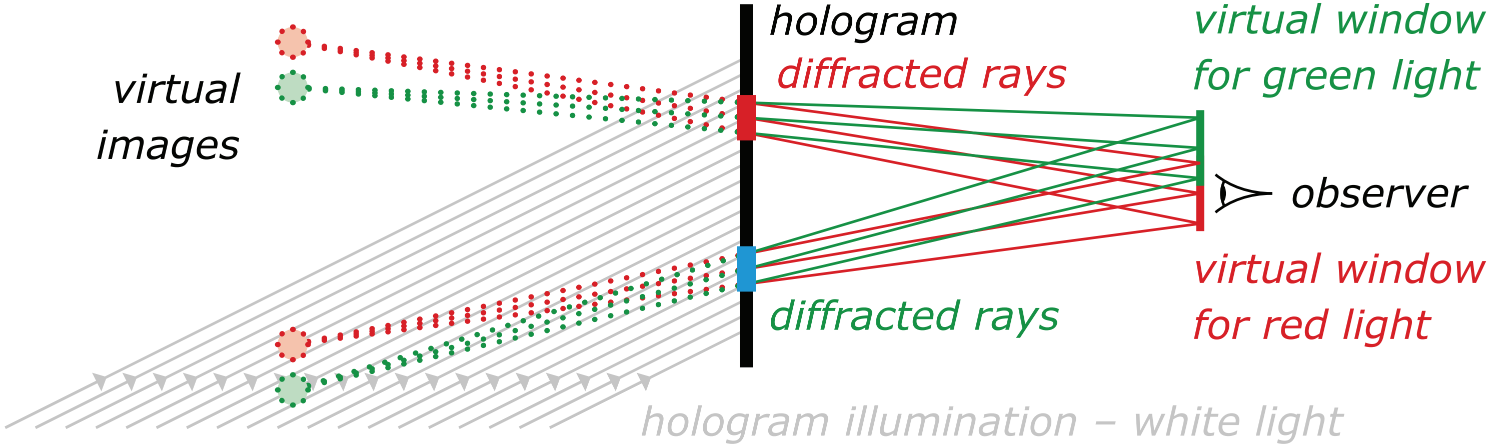
Hologram recording: *subhologram A records light from a point A only.*



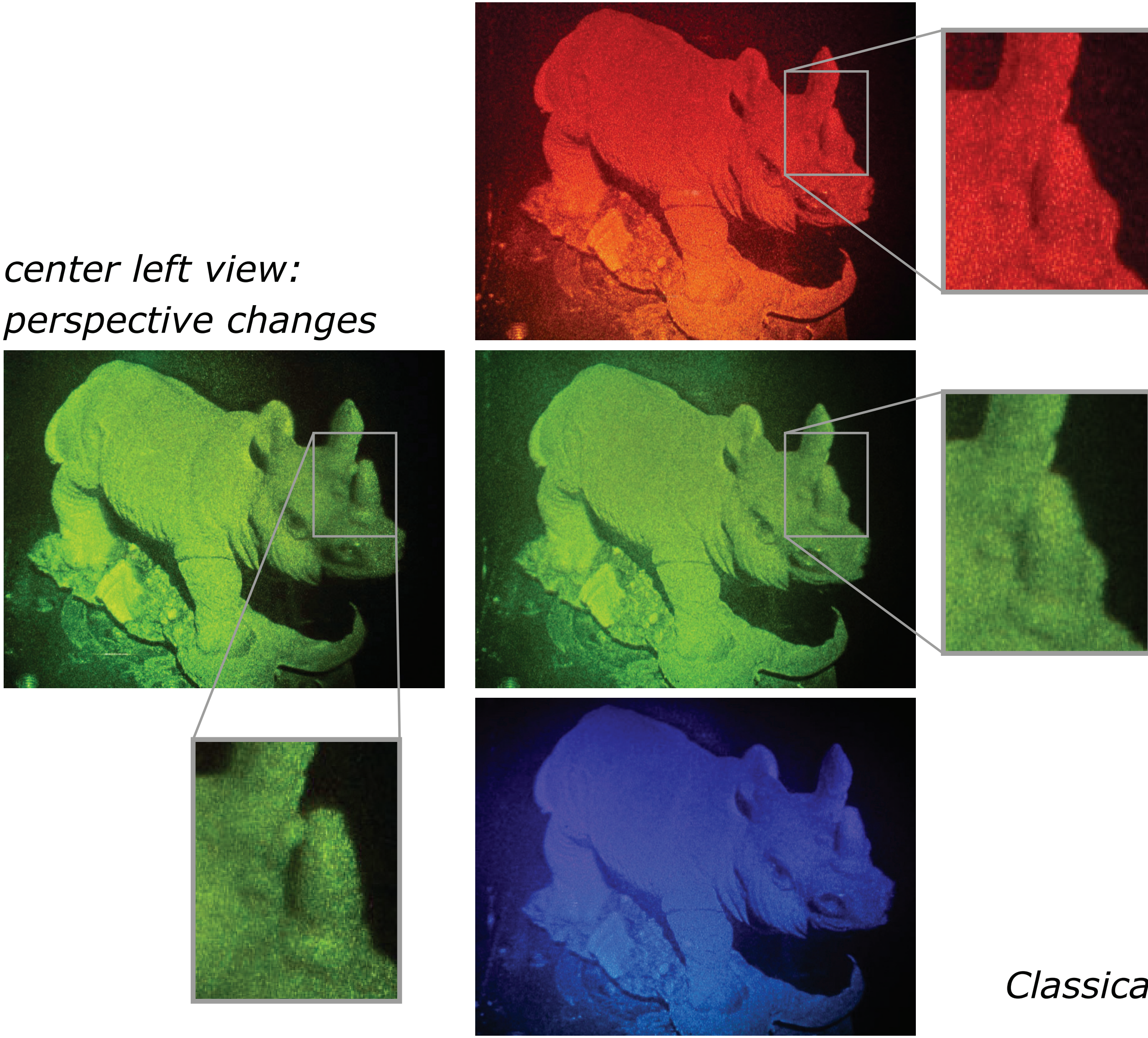
Hologram reconstruction: *both points can be seen in the window only*



Hologram reconstruction with wrong (shorter) wavelength: image shifts, virtual window shifts ⇒ observer does not see the image



Hologram reconstruction with white light: many virtual windows



Classical rainbow hologram
by Š. Němcová

Diffraction specific algorithms

- try to make a diffractive structure that creates desired light distribution in the target distance
- for example iterative:
  1. initialize “hologram” **H** to random numbers
  2. propagate to target distance: **H’** = propag(**H**)
  3. compare with desired light distribution **L**:

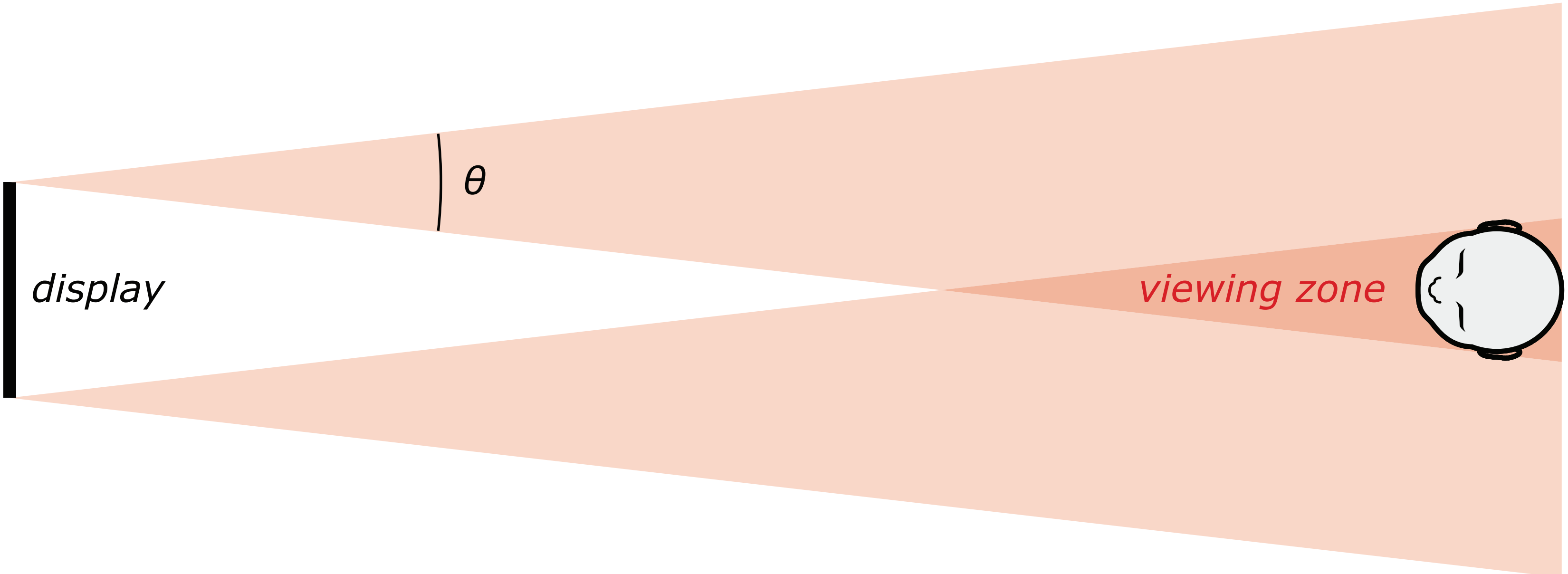
     if **H’** ≈ **L** then **DONE**
  4. modify **H’** so that it is closer to **L**
  5. propagate back to the plane of the hologram:

     H = backpropag(**H’**)
  6. apply constraints to **H** (e.g. 0≤transmittance≤1)
  7. go to step 2

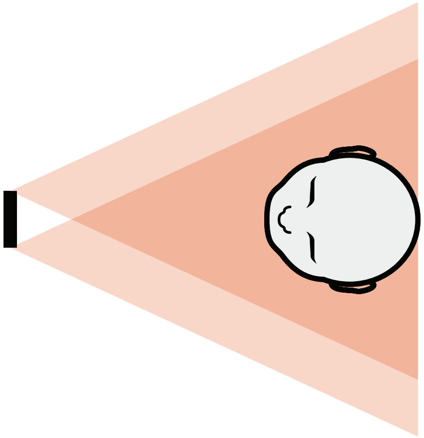
HOLOGRAPHIC DISPLAYS

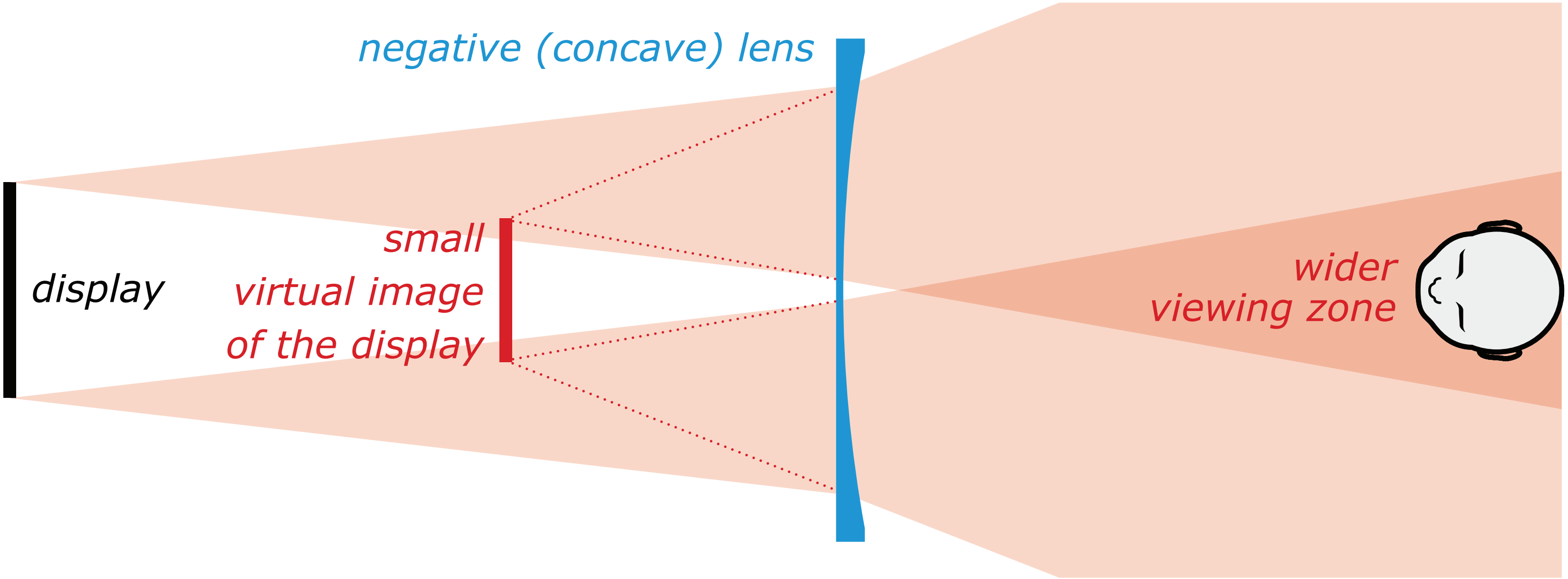
Space-bandwidth product

- a hologram (diffraction pattern)
  - spatial extent (width × height)
  - frequency range f_{MIN} to f_{MAX} in X/Y axis
  - frequency f is related to diffraction angle θ:
    $\sin\theta = \lambda f$
- for pixelated holographic displays:
  spatial extent W × frequency range F
  = 2 × number of samples N
  (space-bandwidth product)
- example: $\lambda = 0.5\ \mu\mathrm{m}$, $\theta = 30°$, size $100 \times 100\ \mathrm{mm}^2$
  $N = 2W\sin(\theta) / \lambda \quad \Rightarrow \quad 200\,000 \times 200\,000$ samples
- contemporary 8K displays: 8000 × 4000 samples



- space-bandwidth product constant:
  - either large display,
    narrow viewing zone
  - or small display,
    wide viewing zone

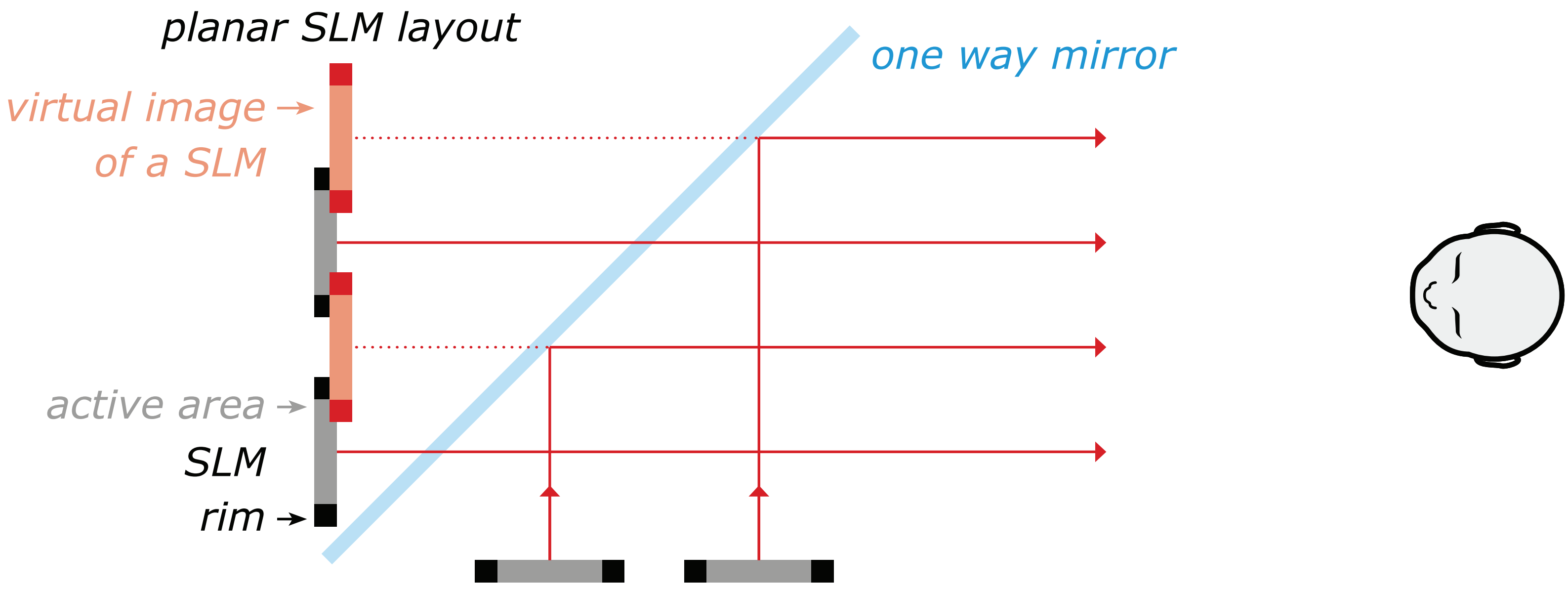




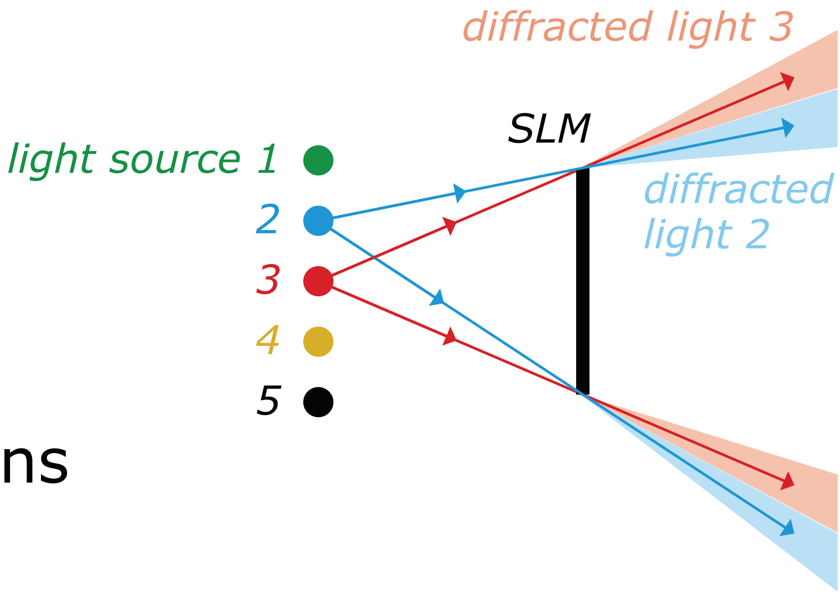
- use a lens to enlarge the angle?
  ⇒ virtual image of the display gets smaller
- space-bandwidth product is hard to cheat!

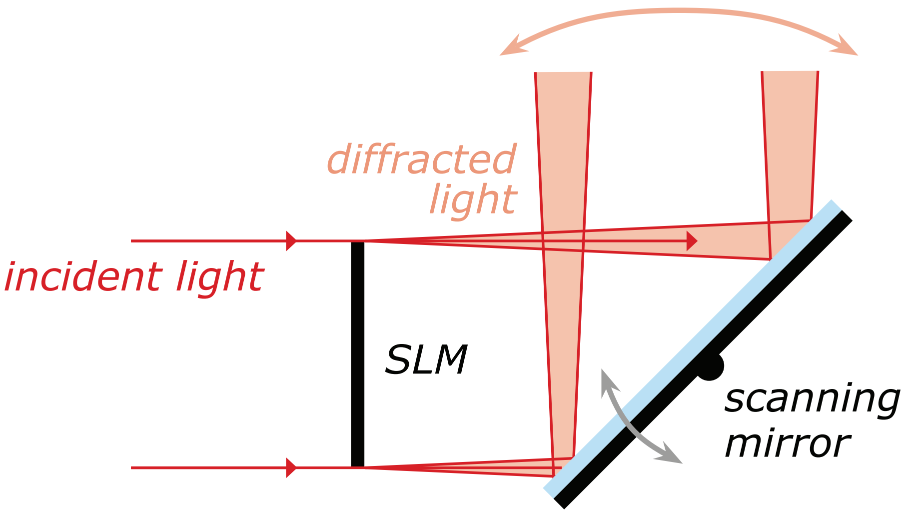
SLM based holographic display systems

- multiple spatial light modulators
  - planar layout – does not increase viewing angle
  - circular layout – increases horizontal view. angle
  - additional optical elements (e.g. one way mirror) for seamless stitching

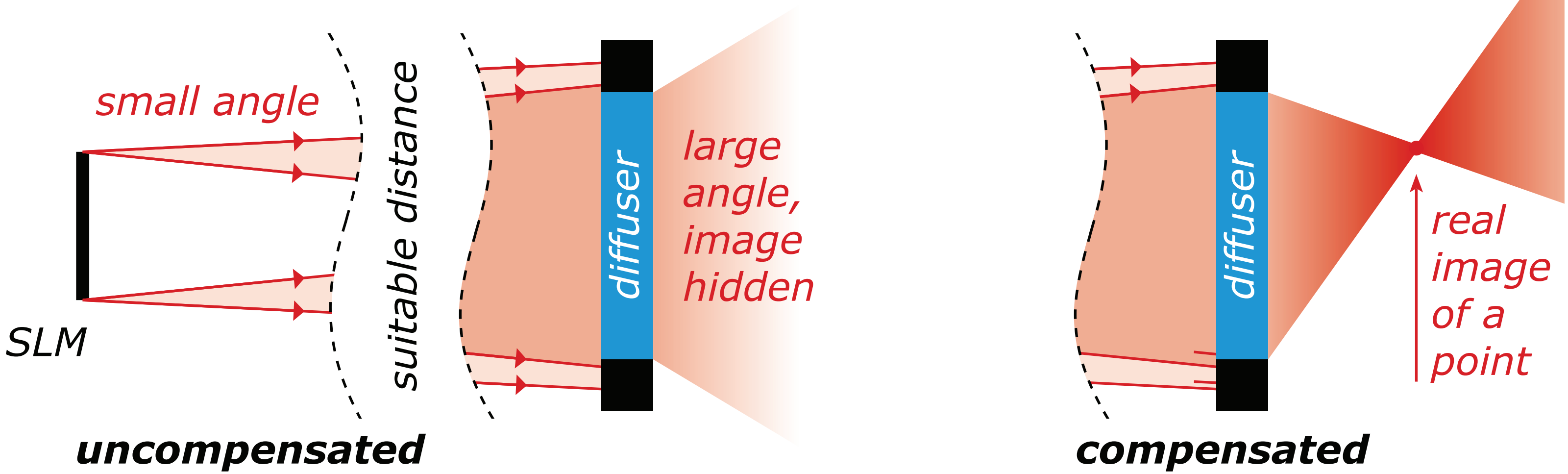


- time multiplexing
  - sequential illumination of the spatial light modulator (SLM) from various directions
  - beam steering (horizontal, vertical or both)





- speckle field
  - diffuser changes phase randomly at each point
  - ⇒ display size = diffuser size (good), viewing angle = diffusing angle (very good), obscures image due to phase breakup (bad)
  - phase breakup can be compensated at SLM
  - ⇒ big display, big viewing angle
  - trades space-bandwidth product for contrast



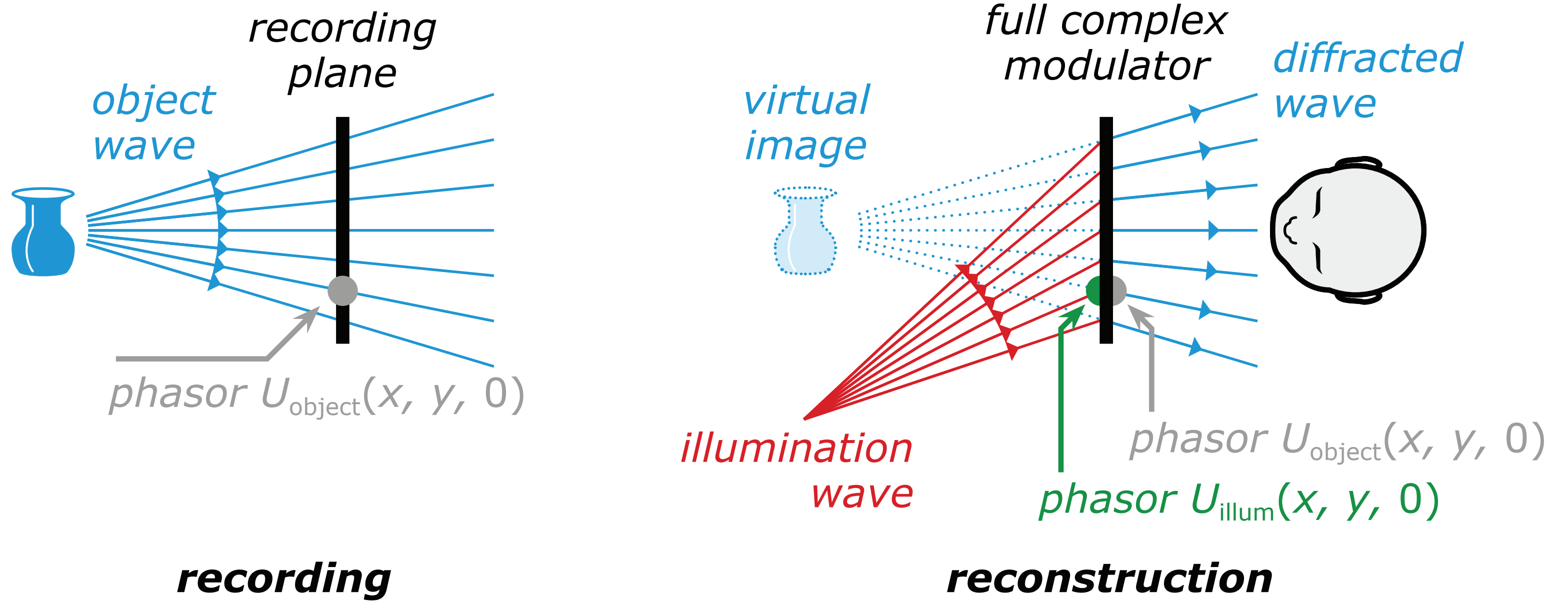
Modulation types

- original holography: amplitude modulation (transmittance of a point of a hologram $\in [0, 1]$)
  - calculate phasors U_{object}, $U_{\text{reference}}$
  - transmittance $\propto |U_{\text{object}} + U_{\text{reference}}|^2$
  - attenuates light $\Rightarrow$ low brightness
- bipolar intensity: alternative amplitude modulation
  - transmittance $\propto \text{Re}\{U_{\text{object}}\, U^*_{\text{reference}}\}$
  - does not contain fringes that form unwanted diffraction

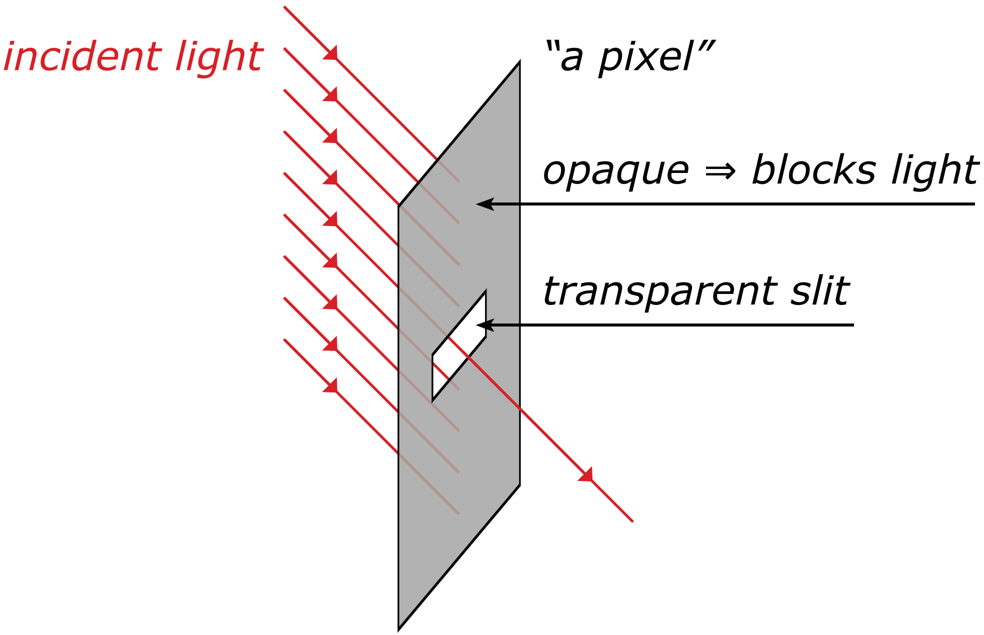
- bleached holograms: phase modulation (transmittance of a point of a hologram = exp(jφ))
  - $I = |U_{\text{object}} + U_{\text{reference}}|^2$
    transmittance $\propto$ exp(jI)
  - much brighter than amplitude hologram, more noise
- kinoform: alternative phase modulation
  - calculate the object wave only (complex values)
  - set amplitude = 1, keep phase
    transmittance = $U_{\text{object}} / |U_{\text{object}}|$

  $\Rightarrow$ phase only modulation
  - similar properties as bleached holograms
  - physically (almost) impossible, calculation easy

- full complex modulation
  - both amplitude and phase modulation at once
  - "recording" does not require reference wave
  - transmittance = $U_{\text{object}} / U_{\text{illumination}}$
  - perfect image – exact copy of the object wave
  - we don't know how to make it well

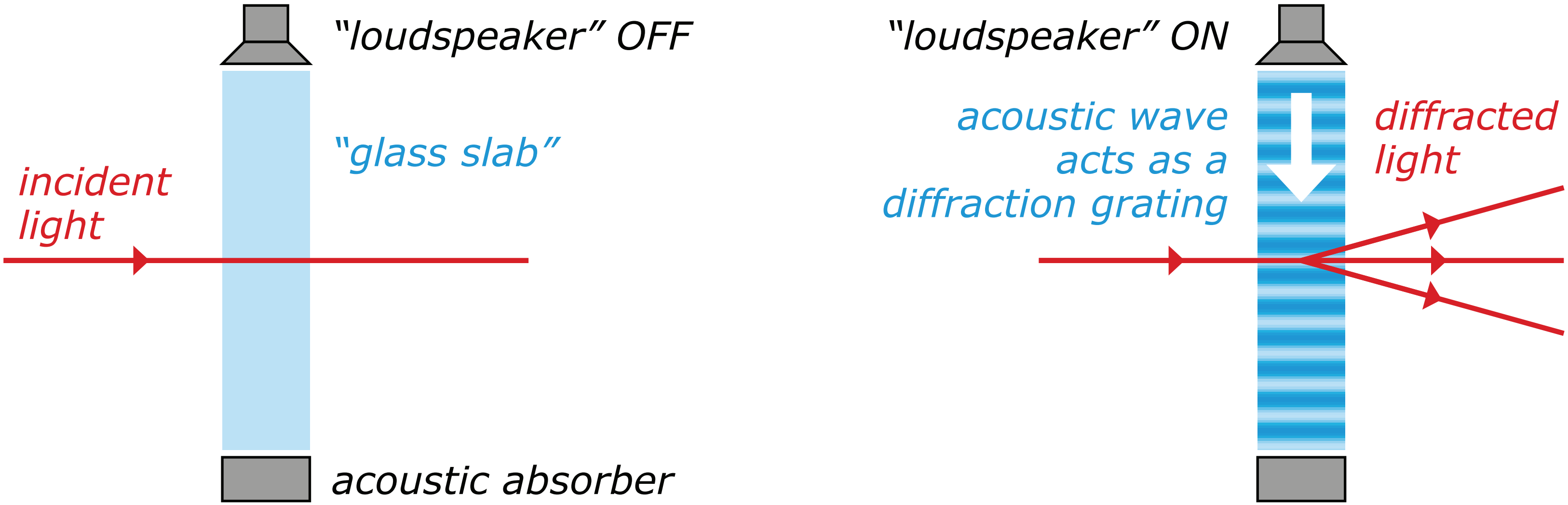


- phase detour
  - early approximation of the complex modulation
  - width of the slit modulates light intensity
    ⇒ amplitude modulation
  - position of the slit picks light with proper phase
    ⇒ phase modulation
  - dim image, requires very high resolution
  - other, more advanced versions exist



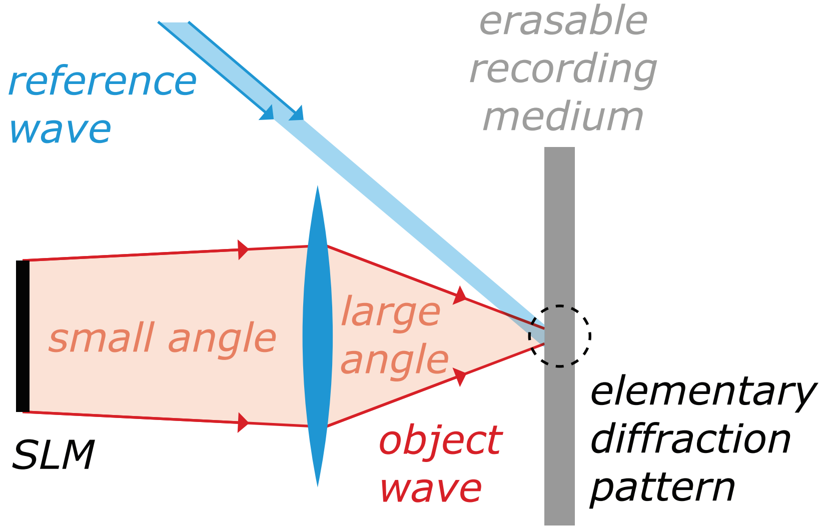
Acousto-optical modulator

- does not display the hologram “pixel by pixel”
- displays an elementary diffractive grating at once
- incident sound wave ⇒ frequencies in the grating
- limited to 1-D gratings
  ⇒ suitable for horizontal parallax only displays

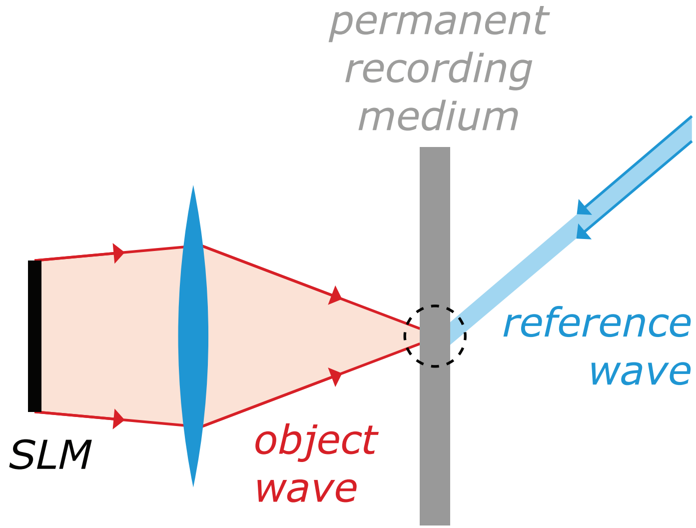


True holographic modulator

- elementary diffraction pattern – physically recorded hologram
- recording medium erasable ⇒ holographic display
- recording medium permanent
  ⇒ holographic printer

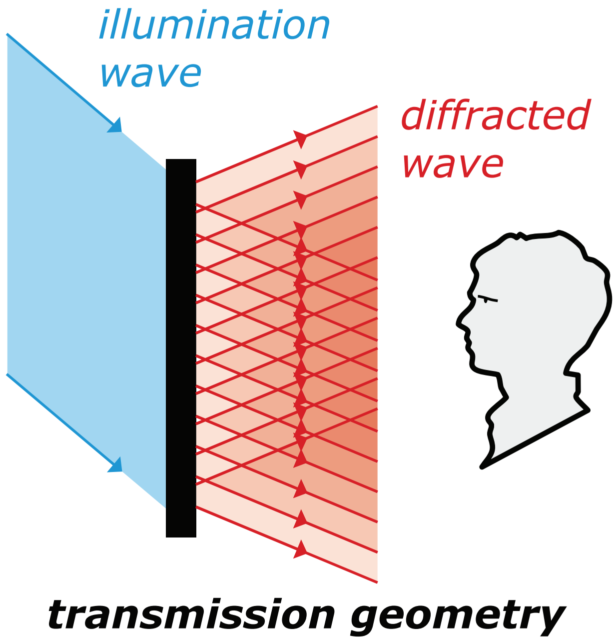


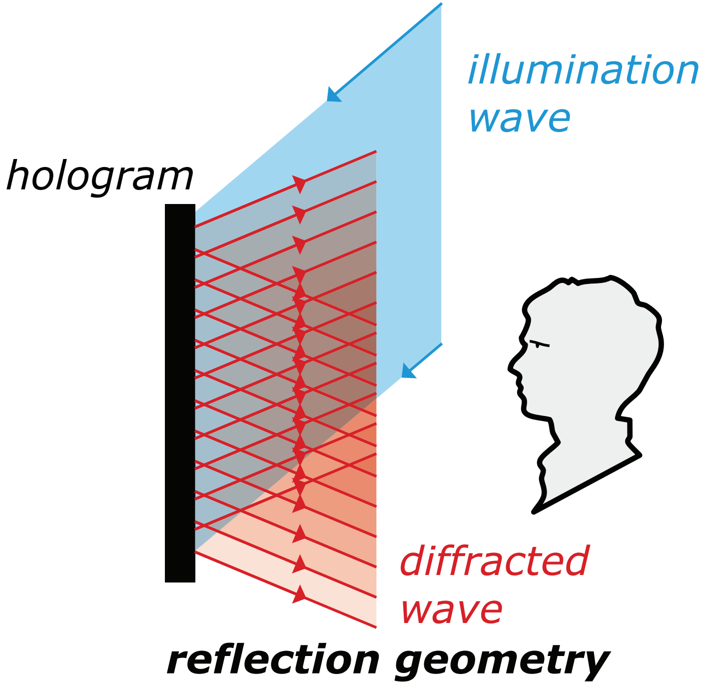
transmission geometry



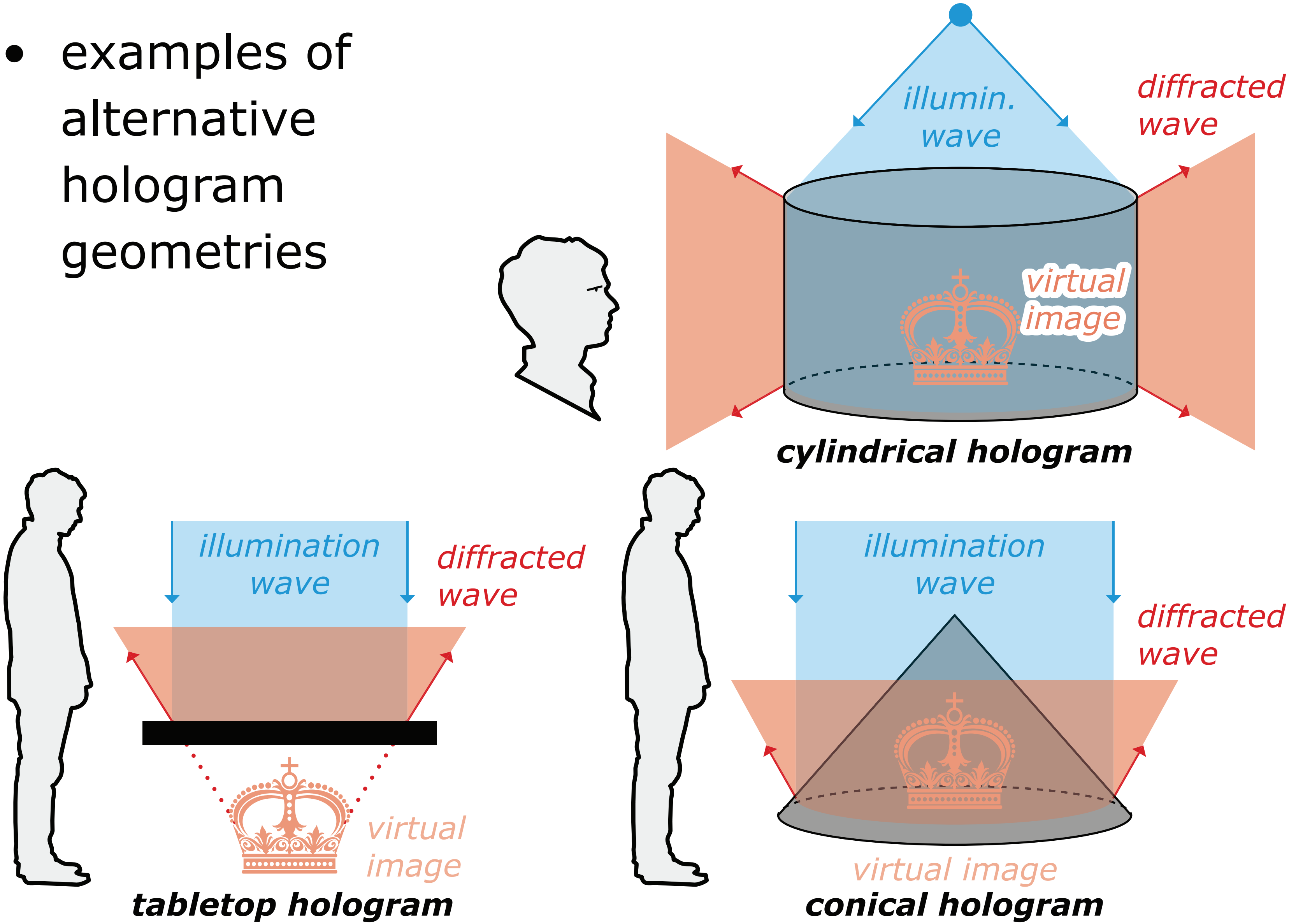
reflection geometry

- transmission geometry: requires back illumination
  - more suitable for an enclosed device
- reflection geometry: requires front illumination
  - more suitable for prints
  - allows white light illumination and full colour reconstruction





- examples of alternative hologram geometries



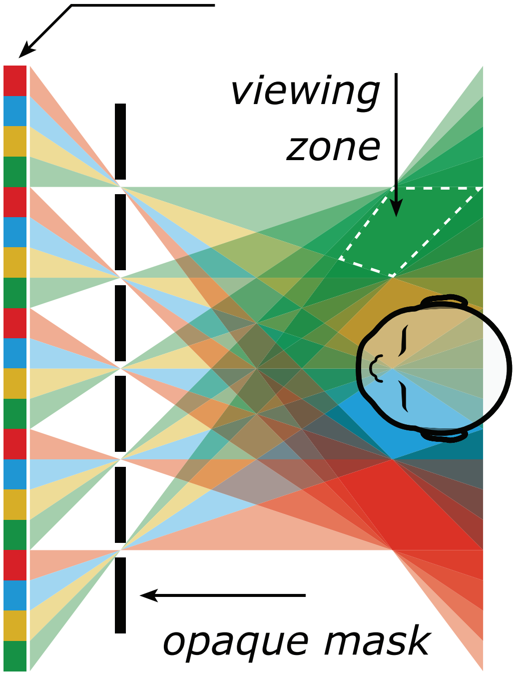
Holographic displays in general

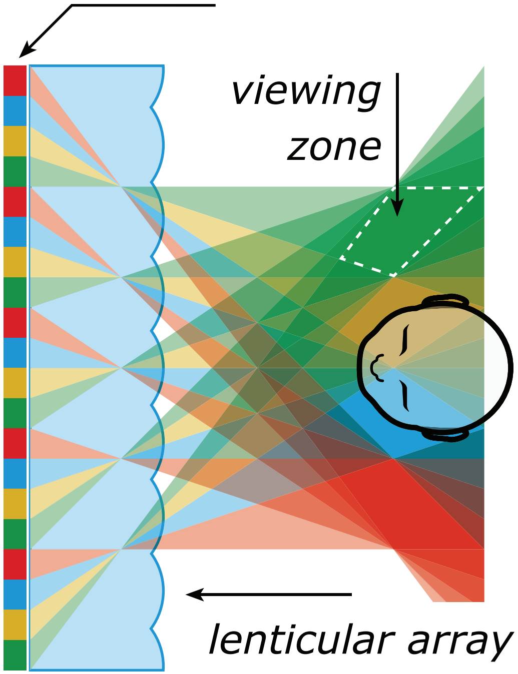
- pros:
  - offer great 3-D experience,
    hardly distinguishable from reality
- cons:
  - require high space-bandwidth product
  - hologram calculation is slow
  - require coherent light,
    which causes speckle noise
  - technology for both display and calculation
    is far from optimal

COMPETING TECHNOLOGIES

Light field displays

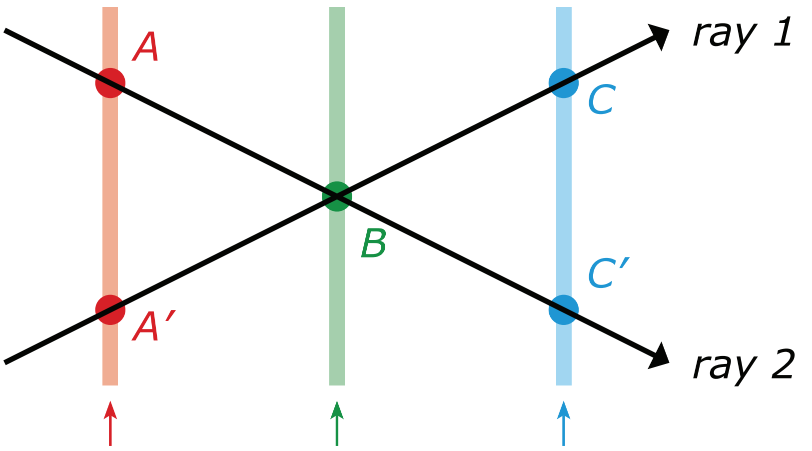
- parallax barrier displays, lenticular displays
  - established technology
  - problematic viewing outside viewing zones





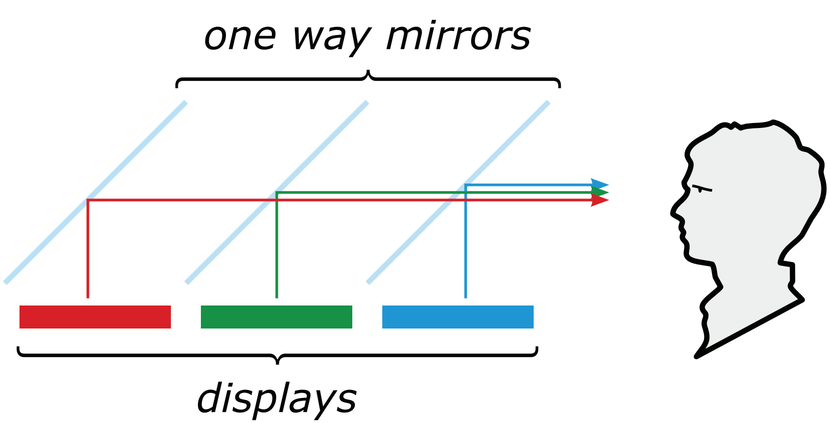
- idea of “light field” L(**x**, **n**):
  light ray from a point **x** (usually in $z = 0$)
  in a direction **n** has luminance L(**x**, **n**)
  ⇒ light field is 4-D (at least)
- based on ray optics
  - very intuitive
  - easy to define light field that cannot exist (e.g. a single ray)
  - does not work well with small distances and angles that are required for high fidelity display
- it is not clear how to sample L(**x**, **n**)
  ⇒ limited resolution and depth of field

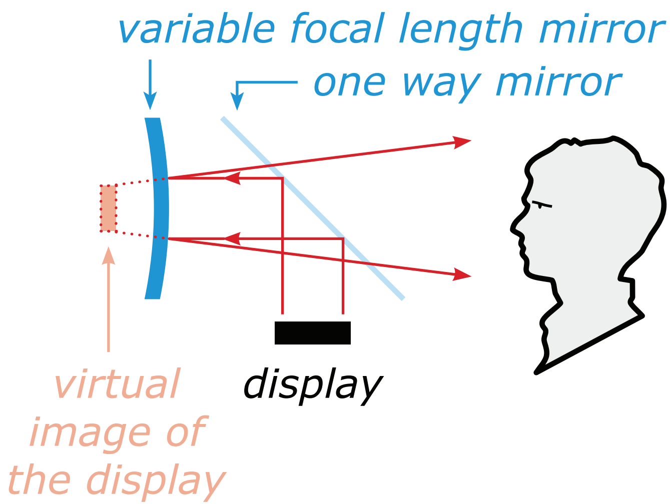
- multilayer (tensor) displays
  - attempts to approximate light field directly
  - ray 1 (luminance L_1) crosses *A′*, *B*, *C*
    ray 2 (luminance L_2) crosses *A*, *B*, *C′*
    …
    system of equations
  - how to set transparency at points *A*, *B*, *C*, …?
    ⇒ solve the system of equations
  - combines well with other technologies, affordable
  - solution is only approximate
    ⇒ ghosting



Multiplane displays

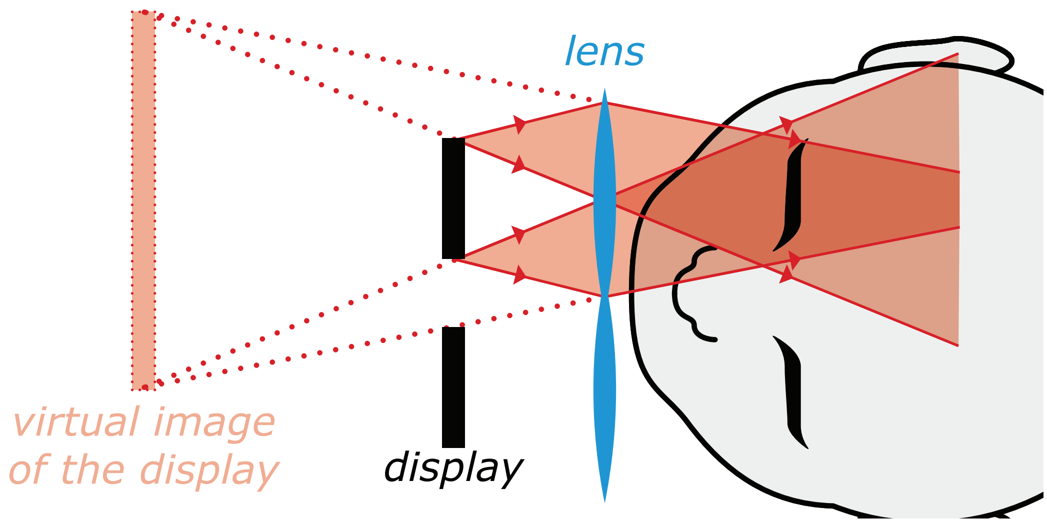
- decomposition of a scene to planar slices
- display slices on real or virtual places in space
- no vergence-accommodation conflict
- problems with hidden surface elimination
  ⇒ usually requires user tracking





Near eye display

- for augmented / virtual reality
- requires precise head tracking and fast response
- only small display area required
- should provide accommodation eye response
  - natural with holography
  - possible with light field displays



RECOMMENDED READING

General holography

- **Holographic Imaging**

  S. A. Benton, V. M. Bove Jr.; Wiley 2008

  excellent introduction to general holography and display holography, touches digital holography a bit

- **Optical Holography**

  R. J. Collier, C. B. Burckhardt, L. H. Lin; Academic Press 1971

  classic textbook, holography in depth; maybe not suitable as a first book on holography you read, but definitely worth reading after gaining some experience

Display holography

- **Three-Dimensional Imaging Techniques**

  T. Okoshi; Academic Press 1976

  considerations on 3-D imaging, both holography and integral imaging; still very relevant book

- **Practical Holography**

  G. Saxby, S. Zacharovas; CRC Press 2015

  a must for anyone making classical display holograms, also covers holographic printers

- **Ultra-Realistic Imaging**

  H. Bjelkhagen, D. Brotherton-Ratcliffe; CRC Press 2013

  full colour holography and holographic printing in depth

Fourier optics

- **Introduction to Fourier Optics**

  J. W. Goodman; Roberts and Company Publishers 2004

  classic textbook, diffraction and related phenomena including holography in (reasonable) depth; every digital holographer should have it at hand

- **Computational Fourier Optics: A MATLAB Tutorial**

  David G. Voelz; SPIE Press 2011

  nice and short introduction to the topic, works well as a supplement to the Goodman's book

Digital holography

- **Digital Holography**
  P. Picart, J.-C. Li; Wiley-ISTE 2012
- **Digital Holography and Digital Image Processing**
  L. Yaroslavsky; Springer 2004
- **Digital Holography and Wavefront Sensing**
  U. Schnars, C. Falldorf, J. Watson, W. Jüptner; Springer 2015
- **Introduction to Modern Digital Holography**
  T.-C. Poon; Cambridge University Press 2014

each book contains general introduction, then focuses on different aspects and applications

Journals

- Optics Express, Applied Optics, Optics Letters
  - *most CGDH articles is published here nowadays*
- Journal of the Optical Society of America A, Optical Engineering, Optics Communications, Journal of Display Technology
  - *worth checking regularly*

Conferences

- International Symposium on Display Holography
  - *mostly display holography, art and technology, both classical and digital*
- Practical Holography (SPIE)
  - *art and technology, both classical and digital*
- Digital Holography & 3-D Imaging (OSA)
  - *general digital holography, 3-D imaging*
- The Holography Conference (Reconnaissance)
  - *mostly security holography and packaging*

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Thank you for your attention

QUESTIONS?

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download example scripts and course notes:

http://holo.zcu.cz

Computer generated display holography

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