HOLOGRAPHY & COMPUTER SCIENCE

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ZČU / Matematické modelování
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Unrelated to holography:
Lecture contents

1 Non-holographic technologies
2 Principle of holography
3 Applications of holography
4 Introduction to computer generated holography
5 Recommended reading
Star Wars: A New Hope (directed by G. Lucas, 1977)
Microsoft HoloLens: visualization of augmented reality
Kagamine Rin & Len at a Hatsune Miku concert
Cheoptics 360™ by viZoo
Lenticular sheet

Light rays in the lenticular sheet

Interlaced image below the lenticular sheet

Sample images A, B, C

Lenticular imaging
Integral (light field) display
(nVidia near-to-eye prototype)
360° Light Field Display
University of Southern California
Plasma volumetric display by Burton Inc.
PRINCIPLE OF HOLOGRAPHY
observer

object

light reflected
off the object
mirror

object
virtual image of the object

mirror

observer

object
virtual image of the object

observer
virtual image of the object

light source

observer
Light diffraction

- depends on frequency $f = 1 / d$ of the pattern
- output angle of the rays: grating equation
  \[ \sin \theta_{\text{out}} = m\lambda / d + \sin \theta_{\text{in}} \]

\[ m = \pm 1 \]
\[ m = 0 \]
\[ m = \pm 1 \]
Image formation by means of diffraction

- grating equation: \( \sin \theta_{\text{out}} = m \lambda / d + \sin \theta_{\text{in}} \)
- example: \( \lambda = 0,5 \, \mu m \quad d = 10 \, \mu m \quad \theta_{\text{in}} = 0 \quad m = 1 \)
  \[ \Rightarrow \theta_{\text{out}} = 2,87^\circ \]
Diffraction pattern formation using interference

- An interference pattern can be recorded and subsequently used as a diffraction pattern.
Light interference

- two “coherent” light beams “interfere”: create a pattern of light and dark stripes

\[
d = \frac{\lambda}{\sin \theta_A - \sin \theta_B}
\]

example: \( \lambda = 0.5 \, \mu m \)

\( \theta_A = 45^\circ \)
\( \theta_B = -45^\circ \)

\( \Rightarrow \) \( d = 0.35 \, \mu m \)
Principle of holography

- interference equation: \( d = \frac{\lambda_1}{\sin \theta_A - \sin \theta_B} \)
- grating equation: \( \sin \theta_{\text{out}} = m\frac{\lambda_2}{d} + \sin \theta_{\text{in}} \)
- after substitution of \( d \): the \( \sin \theta \) equation
  \[
  \sin \theta_{\text{out}} = m \frac{\lambda_2}{\lambda_1} (\sin \theta_A - \sin \theta_B) + \sin \theta_{\text{in}}
  \]
- for \( m = 1 \), \( \lambda_1 = \lambda_2 \), \( \sin \theta_B = \sin \theta_{\text{in}} \Rightarrow \sin \theta_{\text{out}} = \sin \theta_A \)
• hologram: the interference pattern of
  – an object wave: \( \theta_{\text{obj}} (= \theta_A), \lambda_1 = \lambda_{\text{ref}} \)
  – a reference wave: \( \theta_{\text{ref}} (= \theta_B), \lambda_1 = \lambda_{\text{ref}} \)
• hologram observation: illuminate it by
  – an illumination wave: \( \theta_{\text{ill}} (= \theta_{\text{in}}), \lambda_2 = \lambda_{\text{ill}} \)
• \( \sin \theta_{\text{out}} = m \frac{\lambda_{\text{ill}}}{\lambda_{\text{ref}}} (\sin \theta_{\text{obj}} - \sin \theta_{\text{ref}}) + \sin \theta_{\text{ill}} \)
• example: \( \lambda_{\text{ill}} = \lambda_{\text{ref}}, \theta_{\text{ill}} = \theta_{\text{ref}} = 0 \)

\( \theta_{\text{ref}} = 0 \quad \theta_{\text{obj}1} \quad \theta_{\text{obj}2} \quad \theta_{\text{ill}} = 0 \quad \theta_{\text{out}1} \quad \theta_{\text{out}2} \quad \theta_{\text{ill}} = 0 \quad \theta_{\text{out}1} \quad \theta_{\text{out}2} \)

light sensitive material virtual image hologram hologram recording reconstruction \( (m = +1) \) reconstruction \( (m = -1) \) real image
Virtual image formation

- illuminate a hologram with a light source
- light beams diffract on the interference pattern
- diffracted rays are the same as the rays from the original object

![Diagram of hologram and light rays](image-url)
Real image formation

- output angle of the rays: $\sin \theta_{\text{out}} = \frac{m \lambda}{d} + \sin \theta_{\text{in}}$
- for $m = -1$, rays can create real image of the scene
- both rays for $m = +1$ and $-1$ appear at once
  ⇒ no need to distinguish between them

![Diagram showing light, hologram, virtual and real images](image-url)
Classical holography

- capturing the interference pattern of laser lights using a photosensitive material
  - requires high quality lasers
  - requires high resolution recording materials (currently up to 10,000 lines/mm)
  - requires vibration-free environment
  - usually requires chemical processing
- reconstructing the hologram using light source
  - custom lighting setup required
- properly recorded and illuminated holograms provide ultra realistic image
Digital holography (DH)

- light sensitive sensor (e.g. CCD or CMOS) instead of photochemical light sensitive material
  - very fast
  - cannot capture high spatial frequencies (currently about 250 lines/mm)
- numerical simulation of the hologram reconstruction
- digital processing of the captured hologram instead of its visual inspection
  - automatic evaluation
  - allows processing hard to achieve in classical holography
Computer generated holography (CGH)

- numerical simulation of the **hologram recording process** (“sort of”)
- electronic display of a hologram
  - e.g. microdisplays with very fine pixels (**SLM** – spatial light modulator), currently up to 130 lines/mm
- “printing a hardcopy”
  - laser lithography
    - expensive, up to 600 lines/mm
  - electron beam lithography
    - very expensive, up to 10000 lines/mm
Computer generated display holography (CGDH)

- computer generated hologram of a 3-D scene for display purposes
- computer graphics
  - makes a digital image to be displayed on a common electronic display
- computer generated display holography
  - makes a pattern to be displayed on a holographic display
- combination approaches are common, e.g., computer graphics for image rendering, subsequent classical holography for making an interference pattern
Basic hologram recording setups

- on-axis (Gabor) hologram
  - mostly for transparent objects (restrictive)
  - image damaged by the 0th order,
    ±1st orders overlap (bad)
  - low spatial frequencies (100 lines/mm – good)
• off-axis transmission (Leith-Upatnieks) hologram
  – for both opaque and transparent objects
  – clear image (good)
  – high spatial frequencies (1000 lines/mm – bad)
  – visible in laser light only (uncomfortable)
• reflection (Denisyuk) hologram
  – the simplest setup (good)
  – visible in white light (good)
  – simply allows colour imaging (very good)
  – high spatial frequencies (4000 lines/mm – bad)
  – the diffraction pattern is volumetric, i.e., 3-D, not planar, i.e., 2-D (very bad)
APPLICATIONS OF HOLOGRAPHY
• cultural heritage conservation
  – holograms instead of real exhibits
  – the exhibit too valuable or fragile, multiple exhibitions at once, multiple views of the same exhibit at once
  – almost perfect image of the exhibit, scale 1 : 1

A full colour Denisyuk hologram of the “15th anniversary Fabergé Easter egg”, A. Sarakinos, HIH, 2015.
• microscopy, visual inspection
  1. perfect recording of light
     (from a biological sample, a bubble chamber, ...)
  2. hologram examination
     (unlimited time of observation, examination in safe environment, holograms can be archived, ...)

Holography & computer science
• digital holographic microscopy
  – acquisition of a digital hologram
  – numerical reconstruction
  ⇒ signal filtering, unwanted diffraction removal, numerical analysis, ...

![Diagram of holographic microscopy setup]

- **laser**
- **CCD**
- **lens**
- **pinhole**
- **mirror**
- **splitter**
- **microscope objective**
- **sample**
- **mirror**
• enhancing electron microscopy
  – original D. Gabor idea behind holography
    (although in fact, it never worked)
  – hologram recording with electron beam
    ($\lambda$ is $100\,000\times$ smaller than for visible light)
  – hologram enlargement, visible light illumination
    $\Rightarrow$ image $100\,000\times$ bigger

in the $\sin \theta$ equation: $\frac{\lambda_{\text{ill}}}{\lambda_{\text{ref}}} = 100\,000$

\[
\sin \theta_{\text{out}} = m \frac{\lambda_{\text{ill}}}{\lambda_{\text{ref}}} (\sin \theta_{\text{obj}} - \sin \theta_{\text{ref}}) + \sin \theta_{\text{ill}}
\]
holographic optical elements (HOE)
- mimicking any optical element
- cheaper, easier aberration correction, ...
- also called diffractive optical elements (DOE)
  (the difference between HOE and DOE is subtle)
- example: holographic optical element (waveguide coupler) for augmented reality head-up displays
- non-destructive testing
  - double object recording on one hologram: shifts between recordings smear hologram fringes
  - taking a hologram of a vibrating object: vibration causes loss of hologram fringes
  ⇒ no fringes = no image = black strips on the object

• surface metrology
  – digital hologram of a real object
  – numerical reconstruction of a hologram
  – reconstructed phase ~ surface bumpiness

(Schnars et al.: Digital Holography and Wavefront Sensing)
• remote digital holographic interferometry
  – hologram of a master sample (A)
  – reconstruction of a real image of a master over a tested object B
  – contours ~ objects differences

(Schnars et al.: Digital Holography and Wavefront Sensing)
• embossed holograms
  – bumpy surface diffracts light
  ⇒ surface relief hologram:

  – making a master stamp expensive
  – making embossed copies cheap
  – can contain hidden features
  ⇒ hard to counterfeit
  ⇒ suitable as a security element
• holographic memory
  – images show just the principle, not the actual implementation

  **splitter**  **SLM**  **hologram**

  ![Diagram](image)

  **several exposures in a single hologram**

  **selective reconstruction by reconstruction wave change**
• holographic memory (continued)
  - spatial light modulator (SLM) A: data
  - SLM B: address

![Diagram showing holographic memory components]

- **SLM B**
- **SLM A**
- **splitter**
- **laser**
- **hologram**

- **multiple exposure of single hologram**
- **selective reconstruction by reconstruction wave change**
• holographic cryptography
  – SLM A: data, SLM B: key
  – wrong key reconstruction: scrambled output
  – images show just the principle, not the actual implementation
signal processing
  - conversion between plane and spherical waves: convex lens of focal length $f$

wave representation

lens

point sources

wave representation

focused points

ray representation

$f$
signal processing (continued)
- a wave has \textbf{an amplitude} \( A \) and \textbf{a phase} \( \varphi \)
- complex amplitude: \( A \exp(j\varphi) \)
- complex amplitude of a plane wave in the plane \( z = 0 \):
  \[
  U(x, y) = A_{ab} \exp(-j[ax + by])
  \]
- \( a, b \) depend on wave inclination
- illumination with many plane waves:
  \[
  U(x, y) = \int_a \int_b A_{ab} \exp(-j[ax + by]) \, da \, db
  \]
  \( \Rightarrow \) can be seen as the Fourier transform of \( A_{ab} \)

----

\textbf{Fourier transform (\textit{not a proper definition!})}: \[
\text{FT}\{A(a, b)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_{ab} \exp(-j[ax + by]) \, da \, db
\]
• signal processing (continued)
  – 2f system – optical Fourier transform unit
  – 4f system – optical filtering system
INTRODUCTION TO COMPUTER GENERATED HOLOGRAPHY
Nature of light

- force interaction between (oscillating) point charges
- a point source of light: $A \cos(\varphi - \omega t)$
- movement up and down: $A \cos(\varphi - \omega t)$
- optical field ($\sim$ electromag. force) at a distance $r$:
  $$u(r, t) = \frac{A}{r} \cos \left[ \varphi - \omega \left( t - \frac{r}{c} \right) \right] = A'(r) \cos(\varphi'(r) - \omega t)$$

- period of oscillation $T = 1.7 \times 10^{-15}$ s
- (time) frequency $f = 1/T = 600$ THz
- angular frequency $\omega = 2\pi/T$
- speed of light $c$
- wave length $\lambda = cT = 0.5 \mu m$
- wave number $k = 2\pi/\lambda = 1.2 \times 10^7$ m$^{-1}$
We are looking for the optical field here.

\[
\frac{A}{r_1} \cos(kr_1 - \omega t) + \frac{A}{r_2} \cos(kr_2 - \omega t) \\
\approx 2 \frac{A}{r_1} \cos \left( \frac{k(r_1 - r_2)}{2} \right) \cos \left( \frac{k(r_1 + r_2)}{2} - \omega t \right)
\]

\[
u(r_1 - r_2, t)
\]

Optical field at different times.
• photographic film reacts on time average of light intensity $\propto (A')^2$

⇒ cannot distinguish close “dimmer” light from distant “brighter” light

• constructive $\times$ destructive interference

intensity of light
Phasor arithmetic

- \( j^2 = -1 \)
- \( e^{jx} = \cos x + j \sin x \)

- \( u(r, t) = A(r) \cos[\varphi(r) - \omega t] = \text{Re}\{A(r) \ e^{j[\varphi(r) - \omega t]}\} = \text{Re}\{A(r) \ e^{j\varphi(r)} e^{-j\omega t}\} \)

- phasor (complex amplitude):
  \( U(r) = A(r) \ e^{j\varphi(r)} \)

- light amplitude: \( A = |U| \)
  light phase: \( \varphi = \text{arg}(U) \)

- light intensity:
  \( I = |U|^2 = UU^* = A \ e^{j\varphi} A \ e^{-j\varphi} = A^2 \)
Advantage of phasor arithmetic

- optical field – time dependent function:
  \[ u(r, t) = A(r) \cos(\varphi(r) - \omega t) \]

- its phasor (complex amplitude):
  \[ U(r) = A(r) \exp[j \varphi(r)] \]

- sum of optical fields:
  \[ A_1(r) \cos(\varphi_1(r) - \omega t) + A_2(r) \cos(\varphi_2(r) - \omega t) + \ldots = ? \]

- in phasor arithmetic:
  \[ A_1(r) \exp[j \varphi_1(r)] + A_2(r) \exp[j \varphi_2(r)] + \ldots = U_{\text{total}}(r) \]

- optical field (if needed):
  \[ u_{\text{total}}(r, t) = \text{Re}\{U_{\text{total}}(r) e^{-j\omega t}\} \]
Hologram recording simulation

- assume hologram in the plane $z = 0$
- calculation of a hologram of a synthetic scene: for every point $(x, y, 0)$ of the hologram:
  - get the complex amplitude $U_{\text{obj}}$ of the object wave at $(x, y, 0)$
  - get the complex amplitude $U_{\text{ref}}$ of the reference wave at $(x, y, 0)$
  - calculate captured intensity at $(x, y, 0)$

$$I(x, y, 0) = |U_{\text{obj}} + U_{\text{ref}}|^2$$
Computer generated hologram of a point cloud

- the simplest algorithm in CGH
- basic building block of advanced algorithms of computer generated display holography
• spherical wave
  – light emitted by a point light source
  – \( r \): distance from the light source
  – complex amplitude:
    \[
    U(r) = \frac{A}{r} \exp(j[kr + \varphi])
    \]
  – locally resembles a plane in a big distance
  – rays: “directions perpendicular to wavefronts”
• plane wave
  - light emitted by a point light source located in a direction \(-\mathbf{n}\) far away, \(|\mathbf{n}| = 1\)
  - \(\mathbf{n}\) is a direction of light propagation and the normal vector of the wavefronts
  - point in space \(\mathbf{x} = (x, y, z)\)
  - wavefront plane equation \(\mathbf{n} \cdot \mathbf{x} = \text{const.}\)
  - wavefronts separation \(\lambda\)
  - complex amplitude:
    \[ U(\mathbf{x}) = A \exp(j[k\mathbf{n} \cdot \mathbf{x} + \varphi]) \]
Really unoptimized Matlab (Octave) code

Initialization

\[
\begin{align*}
\text{lambda} & = 532 \times 10^{-9}; \\
\text{hologramHeight} & = 2 \times 10^{-3}; \\
\text{hologramWidth} & = 2 \times 10^{-3}; \\
\text{hologramZ} & = 0; \\
\text{Delta} & = 10 \times 10^{-6}; \\
\text{samplesX} & = \frac{\text{hologramWidth}}{\text{Delta}}; \\
\text{samplesY} & = \frac{\text{hologramHeight}}{\text{Delta}}; \\
\text{cornerX} & = -\frac{\text{hologramWidth}}{2}; \\
\text{cornerY} & = -\frac{\text{hologramHeight}}{2}; \\
\text{points} & = \begin{bmatrix}
0, & 0, & -0.2; \\
-\text{hologramWidth}/4, -\text{hologramHeight}/4, & -0.2; \\
\text{hologramWidth}/4, \text{hologramHeight}/4, & -0.22
\end{bmatrix};
\end{align*}
\]
Object wave calculation

\[ k = \frac{2 \pi}{\lambda}; \]

```
objectWave = zeros(samplesY, samplesX);
for s = 1:rows(points)
    for column = 1:samplesX
        for row = 1:samplesY
            x = (column-1) * Delta + cornerX;
            y = (row-1)    * Delta + cornerY;
            r = sqrt((x - points(s, 1))^2 ...
                     + (y - points(s, 2))^2 ...
                     + (hologramZ - points(s, 3))^2);
            objectWave(row,column) += exp(1i*k*r) / r;
        end
    end
end
```
Real part of the object wave
(Just for information; it has no physical meaning!)

Intensity of the object wave
(Just for information; it has no practical use!)
Reference wave calculation

alpha = 90 * pi/180;
beta  = 90.5 * pi/180;
nX = cos(alpha); nY = cos(beta);
nZ = sqrt(1 - nX^2 - nY^2);
refAmplitude = max(max(abs(objectWave))); 
referenceWave = zeros(samplesY, samplesX);
for column = 1:samplesX
    for row = 1:samplesY
        x = (column-1) * Delta + cornerX;
        y = (row-1)    * Delta + cornerY;
        referenceWave(row,column) = refAmplitude * ...
            exp(1i*k*(x*nX + y*nY + hologramZ*nZ));
    end
end
Real part of the object wave
(Just for information; it has no physical meaning!)

Intensity of the object wave
(Just for information; it has no practical use!)
Hologram calculation

\[
\text{optField} = \text{objectWave} + \text{referenceWave};
\]
\[
\text{hologram} = \text{optField} \cdot \text{conj(optField)};
\]

\[x \text{[mm]} \quad y \text{[mm]}\]

\[-1 \quad 0 \quad +1\]

\[max\]

The hologram (intensity picture)
Computer generated hologram of a 3-D scene

6144 × 6144 pixels
Size 4.3 × 4.3 cm²
(resolution 3600 dpi
~ pixel size 7 μm)
How to “print” a calculated hologram?

- electron beam lithography – very expensive
  - 0.05 μm details ⇒ diffraction up to 90°
  - size up to ~ 5 × 5 cm², recording 1 mm²/min
- laser lithography – expensive
  - 1 μm details ⇒ diffraction up to 20°
  - size up to ~ 20 × 20 cm², recording 4 mm²/min

Hologram by K. Matsushima
• imagesetter
  – 10 μm details
    ⇒ diffraction up to 2°
  – price ~ 5 € per A4
• laser printer
  – 100 μm details
    ⇒ diffraction up to 0.5°
• “holographic printers”
  do not usually print
  the calculated pattern

Hologram by I. Hanák, M. Janda
Electronic “holographic display”?

- microdisplays = spatial light modulators
- transmissive or reflective
- size up to 40 mm diagonal
- resolution up to $7680 \times 4320$ px
- pixel size down to $\sim 4 \, \mu m$
- LCD (liquid crystal display), LCOS (liquid crystal on silicon) or DMD (digital micromirror device)
- small size, small diffraction angle (up to $5^\circ$)
  - additional techniques to improve performance

DMD chip by Texas Instruments
RECOMMENDED READING
General holography

- **Holographic Imaging**
  
  *S. A. Benton, V. M. Bove Jr.; Wiley 2008*

  excellent introduction to general holography and display holography, touches digital holography a bit

- **Optical Holography**
  
  *R. J. Collier, C. B. Burckhardt, L. H. Lin; Academic Press 1971*

  classic textbook, holography in depth; maybe not suitable as a first book on holography you read, but definitely worth reading after gaining some experience
Display holography

• **Three-Dimensional Imaging Techniques**
  *T. Okoshi; Academic Press 1976*
  
  considerations on 3-D imaging, both holography and integral imaging; still very relevant book

• **Practical Holography**
  *G. Saxby, S. Zacharovas; CRC Press 2015*
  
  a must for anyone making classical display holograms, also covers holographic printers

• **Ultra-Realistic Imaging**
  *H. Bjelkhagen, D. Brotherton-Ratcliffe; CRC Press 2013*
  
  full colour holography and holographic printing in depth
Fourier optics

- **Introduction to Fourier Optics**
  
  *J. W. Goodman; Roberts and Company Publishers 2004*

  classic textbook, diffraction and related phenomena including holography in (reasonable) depth; every digital holographer should have it at hand

- **Computational Fourier Optics: A MATLAB Tutorial**
  
  *David G. Voelz; SPIE Press 2011*

  nice and short introduction to the topic, works well as a supplement to the Goodman’s book
Digital holography

- **Digital Holography**
  
  *P. Picart, J.-C. Li; Wiley-ISTE 2012*

- **Digital Holography and Digital Image Processing**
  
  *L. Yaroslavsky; Springer 2004*

- **Digital Holography and Wavefront Sensing**
  
  *U. Schnars, C. Falldorf, J. Watson, W. Jüptner; Springer 2015*

- **Introduction to Modern Digital Holography**
  
  *T.-C. Poon; Cambridge University Press 2014*

Each book contains a general introduction, then focuses on different aspects and applications.
Journals

- Optics Express, Applied Optics, Optics Letters
  - *most CGDH articles is published here nowadays*
  - *worth checking regularly*
Conferences

- International Symposium on Display Holography
  - *mostly display holography, art and technology, both classical and digital*

- Practical Holography (SPIE)
  - *art and technology, both classical and digital*

- Digital Holography & 3-D Imaging (OSA)
  - *general digital holography, 3-D imaging*

- The Holography Conference (Reconnaissance)
  - *mostly security holography and packaging*
Thank you for your attention

QUESTIONS?

Download example scripts and the course notes for the tutorial “Computer generated display holography” at

http://holo.zcu.cz

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