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The Usual Method : Coupled Wave Theory

The propagation of light in holographic gratings is described by the Helmholtz equation.

This equation can either be solved rigorously as in RIGOROUS COUPLED WAVE THEORY or...

An approximate mathematical solution to this equation can be found (in the case of a harmonic index profile) if an assumption is made that only a single reference wave and a single signal wave exist in the grating (KOGELNIK'S THEORY).



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D. Brotherton-Ratcliffe, "A treatment of the general volume holographic grating as an array of parallel stacked mirrors," *J. Mod. Optics*, 59, 1113-1132 (2012)

Parallel Stacked Mirrors

A More Intuitive Method - PSM

The PSM method starts with the principle of Fresnel Reflection.

Fresnel reflection was discovered by Augustin Jean Fresnel (1788-1827) and provides simple mathematical formulae for the amount of reflection and transmission of light occurring at an interface of two media having different refractive index.



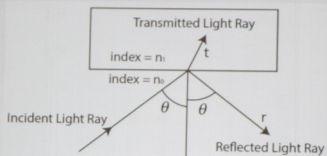
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Augustin Jean Fresnel (1788-1827)

The Starting Point : Fresnel Reflection

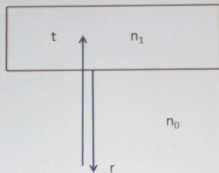




The very simplest Case : Normal Incidence

$$t = \frac{2n_1}{n_0 + n_1}$$

$$r = \frac{n_1 - n_0}{n_0 + n_1}$$





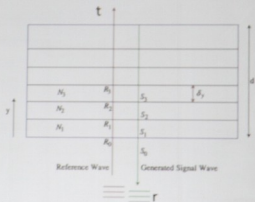
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Normal Incidence LOSSLESS PSM Model

An unslanted reflection grating is modelled by many different layers, each having a slightly different refractive index. A single reference (or illuminating) wave and a single reflected (or signal) wave is assumed.





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Normal Incidence PSM Model

We start by illuminating the grating with a plane wave

$$R^{ext} = e^{j\beta y}$$

$$\beta = \frac{2\pi n_0}{\lambda_c}$$

This is an exact solution of Maxwell's equations in the exterior region



Normal Incidence PSM Model

We can now use the Fresnel formulae to write down recurrence relations for the reference and signal waves at each grating depth...

$$R_J = 2e^{j\beta s y} \frac{N_{J-1}}{N_J + N_{J-1}} R_{J-1} + e^{j\beta s y} \frac{N_{J-1} - N_J}{N_J + N_{J-1}} S_J$$

$$S_J = 2e^{j\beta s y} \frac{N_{J+1}}{N_{J+1} + N_J} S_{J+1} + e^{j\beta s y} \frac{N_{J+1} - N_J}{N_{J+1} + N_J} R_J$$

$$\beta = \frac{2\pi n_g}{\lambda_c}$$



Normal Incidence PSM Model

And now we can let the number of layers go to infinity so that we model the grating essentially perfectly. When we do this the recurrence relations turn into differential equations

$$X_{j-1} = X_j - \frac{dX}{dy} \delta y - \dots \quad \left. \vphantom{X_{j-1}} \right\} \begin{aligned} \frac{dR}{dy} &= \frac{R}{2} \left(2i\beta - \frac{1}{n} \frac{dn}{dy} \right) - \frac{1}{2n} \frac{dn}{dy} S \\ \frac{dS}{dy} &= -\frac{S}{2} \left(\frac{1}{n} \frac{dn}{dy} + 2i\beta \right) - \frac{1}{2n} \frac{dn}{dy} R \end{aligned}$$

And these equations are an exact solution of Maxwell's equations...



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The Case of a Harmonic grating

Most simple holographic reflection gratings consist of a harmonic index modulation. This can be expressed mathematically as

$$n = n_0 + n_1 \cos\left(\frac{4\pi n_0}{\lambda_r} y\right) = n_0 + \frac{n_1}{2} e^{\frac{4i\pi n_0}{\lambda_r} y} + e^{-\frac{4i\pi n_0}{\lambda_r} y}$$

The PSM equations have approximate (but usually very accurate) analytic solutions in this case.



Analytic Solutions

PSM

$$\frac{dR}{dy} = i\alpha\kappa\hat{S}$$

$$\frac{d\hat{S}}{dy} + 2i\beta(1-\alpha)\hat{S} = -i\alpha\kappa R$$

$$\alpha = \frac{\lambda_c}{\lambda_r}$$

$$\kappa = \frac{\pi n_1}{\lambda_c}$$

KOGELNIK

$$\frac{dR_K}{dy} = i\kappa\hat{S}_K$$

$$\frac{d\hat{S}_K}{dy} + 2i\beta\alpha(1-\alpha)\hat{S}_K = -i\kappa R_K$$

$$\beta = \frac{2\pi n_0}{\lambda_c}$$



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Diffraction Efficiency

Boundary Conditions for Reflection Gratings

$$R(y=0) = R_K(y=0) = 1$$

$$\hat{S}(y=d) = \hat{S}_K(y=d) = 0$$

Diffraction Efficiency

$$\eta = S(0)S^*(0)$$

$$\eta = \frac{1}{1 - \frac{C_R C_S}{\kappa^2} \square^2 \operatorname{csh}^2(d \square)}$$

$$\square^2 = -\frac{\beta^2}{4C_S^2} - \frac{\kappa^2}{C_R C_S}$$

$$C_{R(\text{PM})} = 1/\alpha$$

$$C_{S(\text{PM})} = -1/\alpha$$

$$\beta_{(\text{PM})} = 2\beta(1-\alpha)/\alpha$$

$$C_{R(\text{KDD})} = 1$$

$$C_{S(\text{KDD})} = 2\alpha - 1$$

$$\beta_{(\text{KDD})} = 2\alpha\beta(1-\alpha)$$

$$\alpha = \frac{\lambda_x}{\lambda_y}$$

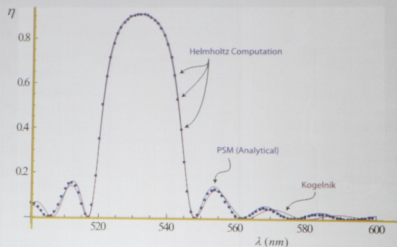


Normal Incidence Unslanted Reflection Grating

Recording Wavelength 532nm

Comparison of PSM vs Kogelnik

Comparison of the predictions of the PSM analytical model, Kogelnik's coupled wave theory and a Runge–Kutta numerical solution of the Helmholtz equation for a typical normal incidence reflection grating of a thickness of 7 mm and with $n_0 = 1.5$, $n_1 = 0.045$. The grating was recorded at 532 nm.

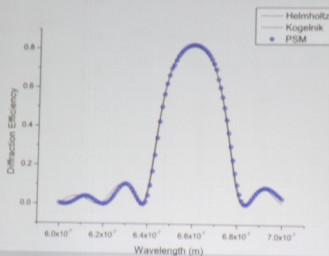




Normal Incidence Unslanted Reflection Grating

Recording Wavelength 660nm

Comparison of PSM vs Kogelnik





Multi-Coloured Reflection Gratings

$$n = n_0 + n_1 \cos(2\alpha_1 \beta y) + n_2 \cos(2\alpha_2 \beta y) + \dots$$

$$= n_0 + \frac{n_1}{2} \left\{ e^{2i\beta\alpha_1 y} + e^{-2i\beta\alpha_1 y} \right\} + \frac{n_2}{2} \left\{ e^{2i\beta\alpha_2 y} + e^{-2i\beta\alpha_2 y} \right\} + \dots$$

$$\alpha = \frac{\lambda_c}{\lambda_r}$$

$$\beta = \frac{2\pi n_0}{\lambda_c}$$



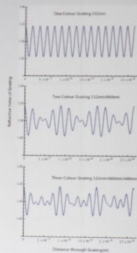
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Multi-Coloured Reflection Gratings

Multi-colour grating index profiles for a modulation of $n_1 = 0.045$





Multi-Coloured Reflection Gratings

$$\frac{dR}{dx} = -S \sum_{j=1}^N \kappa_j \alpha_j e^{2i\beta x(\alpha_j - 1)}$$

$$\frac{dS}{dx} = R \sum_{j=1}^N \kappa_j \alpha_j e^{-2i\beta x(\alpha_j - 1)}$$

Once again the PSM equations have approximate (but usually very accurate) analytic solutions for the multi-colour case .



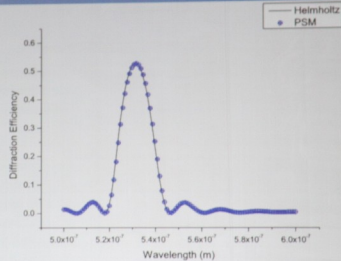
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Normal incidence Unslanted

Multi-Coloured Reflection Gratings

A normal incidence reflection grating recorded using two wavelengths: 532 nm and 660 nm. A value of $n_1 = n_2 = 0.045/2$ was used giving a total index modulation of 0.045. $n_0 = 1.5$. The grating thickness is 7 mm. (a) Replay near 660 nm and (b) replay near 532 nm.



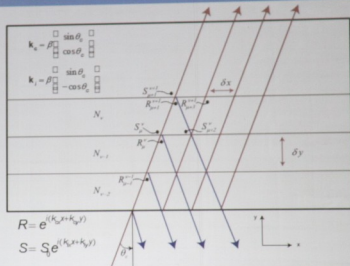


Oblique incidence Unslanted
Sigma Polarisation

Oblique Incidence Reflection Gratings

$$r_{k,m+1} = \frac{N_{k+1} \sqrt{1 - \frac{n_b^2}{N_{k+1}^2} \sin^2 \theta_c} - N_k \sqrt{1 - \frac{n_b^2}{N_k^2} \sin^2 \theta_c}}{N_{k+1} \sqrt{1 - \frac{n_b^2}{N_{k+1}^2} \sin^2 \theta_c} + N_k \sqrt{1 - \frac{n_b^2}{N_k^2} \sin^2 \theta_c}}$$

$$t_{k,m+1} = \frac{2 N_k \sqrt{1 - \frac{n_b^2}{N_k^2} \sin^2 \theta_c}}{N_{k+1} \sqrt{1 - \frac{n_b^2}{N_{k+1}^2} \sin^2 \theta_c} + N_k \sqrt{1 - \frac{n_b^2}{N_k^2} \sin^2 \theta_c}}$$





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Oblique Incidence Unslanted
Sigma Polarisation

Oblique Incidence

As before we simply write down the recurrence relations from the ray diagram – e.g.

$$\begin{aligned}
 R_{\nu+1}^{-1} &= e^{i(m_\nu f x + i \cos \theta_\nu f y)} R_\nu \\
 &+ e^{i(m_\nu f x + i \cos \theta_\nu f y)} S_\nu
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2N_{\nu-1} \sqrt{1 - \frac{n_b^2}{N_{\nu-1}^2} \sin^2 \theta_c}}{N_{\nu-1} \sqrt{1 - \frac{n_b^2}{N_{\nu-1}^2} \sin^2 \theta_c} + N_\nu \sqrt{1 - \frac{n_b^2}{N_\nu^2} \sin^2 \theta_c}} \\
 & \frac{N_{\nu-1} \sqrt{1 - \frac{n_b^2}{N_{\nu-1}^2} \sin^2 \theta_c} - N_\nu \sqrt{1 - \frac{n_b^2}{N_\nu^2} \sin^2 \theta_c}}{N_{\nu-1} \sqrt{1 - \frac{n_b^2}{N_{\nu-1}^2} \sin^2 \theta_c} + N_\nu \sqrt{1 - \frac{n_b^2}{N_\nu^2} \sin^2 \theta_c}}
 \end{aligned}$$



Oblique Incidence

Then we convert again to differential form by letting the number of layers go to infinity and using Taylor expansions:

$$R_{\mu+1}^{-1} = R_{\mu}^{-1} + \frac{\partial R_{\mu}^{-1}}{\partial x} \delta x + \frac{\partial R_{\mu}^{-1}}{\partial y} \delta y + \dots$$

$$S_{\mu+1}^{-1} = S_{\mu}^{-1} + \frac{\partial S_{\mu}^{-1}}{\partial x} \delta x - \frac{\partial S_{\mu}^{-1}}{\partial y} \delta y + \dots$$

$$N_{\nu+1} = N_{\nu} - \frac{\partial N_{\nu}}{\partial y} \delta y + \dots$$

We also make the assumption of constant ray direction

This gives a more general set of PSM equations:

$$\left\{ \begin{array}{l} \frac{k_x}{\beta} \square R = \sin \theta_s \frac{\partial R}{\partial x} + \cos \theta_s \frac{\partial R}{\partial y} = \frac{R}{2} \square 2i\beta - \frac{1}{2ncos\theta_s} \frac{\partial n}{\partial y} - \frac{S}{2ncos\theta_s} \frac{\partial n}{\partial y} \\ \frac{k_x}{\beta} \square S = \sin \theta_s \frac{\partial S}{\partial x} - \cos \theta_s \frac{\partial S}{\partial y} = \frac{S}{2} \square 2i\beta + \frac{1}{ncos\theta_s} \frac{\partial n}{\partial y} + \frac{R}{2ncos\theta_s} \frac{\partial n}{\partial y} \end{array} \right.$$



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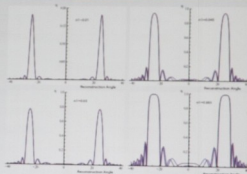
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Oblique Incidence Unslanted
Sigma Polarisation

Oblique Incidence

These partial differential equations can now be simplified to ordinary differential equations and then very accurate simple analytic solutions can be found for both single and multi-colour gratings.

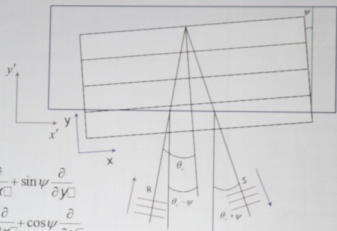


Four graphs showing the predicted diffractive response (sigma-polarisation) versus replay wavelength of typical unslanted single-colour reflection gratings using the PSM analytical model (blue lines) and Kogelnik's theory (red lines). Recording wavelength: $\lambda_{mr} = 532 \text{ nm}$; recording angle: $\theta_r = 20 \text{ degrees}$; replay angle: $\theta_c = 20 \text{ degrees}$; grating thickness: $d = 7 \text{ mm}$; $n_0 = 1.5$; index modulations shown on graphs.



Oblique Incidence in Slanted Gratings

The PSM equations which describe slanted gratings are derived by simply rotating the unslanted PSM partial differential equations.



$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x} \frac{\partial}{\partial x'} + \frac{\partial y'}{\partial x} \frac{\partial}{\partial y'} = \cos \psi \frac{\partial}{\partial x'} + \sin \psi \frac{\partial}{\partial y'}$$

$$\frac{\partial}{\partial y} = \frac{\partial y'}{\partial y} \frac{\partial}{\partial y'} + \frac{\partial x'}{\partial y} \frac{\partial}{\partial x'} = -\sin \psi \frac{\partial}{\partial x'} + \cos \psi \frac{\partial}{\partial y'}$$



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Slanted Gratings

The procedure of rotating the unslanted PSM (reflection) equations to describe a general slanted grating leads to accurate analytic expressions for diffraction efficiency of both reflection and transmission gratings.

As Ardie Osanlou et al show (see poster session) the PSM analytic expressions generally provide slightly better answers than Kogelnik's theory for most reflection gratings but the converse is true for most transmission gratings.



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D. Brotherton-Ratcliffe, "Analytical treatment of the polychromatic spatially multiplexed volume holographic grating," *Appl. Opt.* 51, 7188-7199 (2012).

Multiplexed Reflection Gratings

The PSM theory may be extended to describe spatially multiplexed (and multi-colour) gratings and gives analytical results very similar to N-Coupled Wave theory.

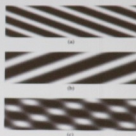


Fig. 3. Example of a spatially multiplexed phase reflection grating. The grating, whose (x, y) index distribution is shown in (c), is formed by the sequential recording of the two single gratings shown in (a) and (b). Each diagram shows a section of size $0.5 \mu\text{m}$ by $3 \mu\text{m}$. Each single grating has been recorded with a reflection beam angle of $\theta_r = 30^\circ$ and with a wavelength of 532 nm . One grating has a slope of $\alpha_x = 30^\circ$ and the other has a slope of $\alpha_x = -30^\circ$. Note that the form of the multiplexed grating in (c) is fundamentally different from the characteristic linear shape of its component single gratings of (a) and (b). Note also that identical index modulations for each of the two component gratings have been assumed in (c).

20 October 2012 / Vol. 51, No. 30 / APPLIED OPTICS 7183

Multiplexed Reflection Gratings

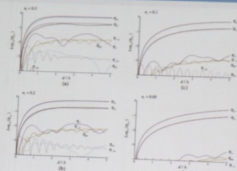


Fig. 4. Color index diffraction efficiency, versus normalized grating thickness, if it is produced by the PSM model and by an RCW structure for the case of the zero-multiplexed reflection grating of Fig. 2(a) as Bragg resonator. The grating is irradiated using light of $n_1 = 1.5$ in air, $n_2 = 1.0$ to 1.3 , $\Lambda = 0.2$ in μm and $n_1 = 1.5$ to 1.3 in μm . In each case the average index inside and outside the grating has been taken to be $n_1 = 1.5$. The dashed lines indicate the 0^{th} and 1^{st} orders of the PSM model and the solid lines indicate the orders of the grating but have been subtracted. The most prominent RCW modes are the 0^{th} and 1^{st} modes, which correspond to the 0^{th} and 1^{st} modes in PSM, from that n_2 values in the larger of the two grating periods, i.e., in that of the component grating shown in Fig. 2(b).

PSM and Colour Holograms

The Multi-Colour N-PSM theory can be used to analytically describe diffraction in colour holograms...

8. Polarizability Reflection Hologram

In the limit that $N \rightarrow \infty$ — the results of Section 5 lead to formulas for the diffraction efficiency of the polarizability hologram:

$$\eta_{\pm 1,0}(\Phi, \Psi) = \frac{\epsilon_1^2(\Phi)}{\epsilon_0} \frac{1}{\cos \Phi} \tanh^2 \left(\epsilon_1 \sqrt{\frac{\epsilon_0}{\cos \Phi}} \right)$$

$$\eta_{\pm 1,0} = \frac{1}{\Delta \Phi} \int_{\epsilon_0}^{\epsilon_1(\Phi)} \frac{\epsilon_1^2(\Phi)}{\cos \Phi} \tanh^2 \left(\epsilon_1 \sqrt{\frac{\epsilon_0}{\cos \Phi}} \right) d\Phi$$

$$= \tanh^2 \left(\epsilon_1 \sqrt{\frac{\epsilon_0}{\cos \Phi}} \right) \quad (8)$$

where

$$\epsilon_0 = \frac{1}{\Delta \Phi} \int_{\epsilon_0}^{\epsilon_1(\Phi)} \epsilon_1(\Phi) d\Phi \quad (8')$$

and where Φ is the replay image angle and $\Delta \Phi$ is the total reconstructed image angle range. It is important to point out that these approximations are strictly valid when the diffracted light from the hologram radiates freely in the x and y dimensions but is essentially collimated in the z dimension. This restriction comes about because we have developed the N-PSM model in this paper using an ignorable z coordinate. This has allowed us to consider the z -polarization only for all Fresnel reflections with the electric field vector pointing only in the x direction. Of course the basic PSM model is capable of treating either the x or z -polarizations with equal ease and fully three-dimensional (3D) equations for the arbitrarily tilted single gratings have already been published (5). But it is not a completely trivial matter to maintain the required N²-PSM theory, which describes diffraction from multiplexed gratings whose component grating vectors are no longer restricted to the (x, y) plane. Nevertheless, energy conservation suggests that we can expect Eq. (8) to provide a good estimate of diffraction for a reflection hologram illuminated by collimated light with its electric field in the y direction and which radiates three-dimensionally under the condition that the field of view in the x dimension remains small. And Shieh's law will of course extend readily to higher angles as the exterior field of view of a hologram is always much larger than the effective field of view within the emulsion layer as typical detectors have an index substantially greater than air.

Under the assumption

$$\epsilon_1^2(\Phi) = \epsilon_1^2 \cos \Phi \quad (8'')$$

Eq. (8) reduces to the simpler form

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$$\eta_{\pm 1,0} = \tanh^2 \left(\epsilon_1 \sqrt{\frac{\epsilon_0}{\cos \Phi}} \right) \quad (8''')$$

If, on the other hand, we assume a flat distribution for $\epsilon_1^2(\Phi)$ between $-\Delta \Phi$ and $+\Delta \Phi$ by giving a field of view of $2\Delta \Phi$ within the hologram, which is of course rather unrealistic, then Eq. (8) reduces to

$$\eta_{\pm 1,0} = \tanh^2 \left(1.08 \frac{\epsilon_1 \sqrt{\epsilon_0}}{\cos \Phi} \right) \quad (8''')$$

In general the form of $\epsilon_1^2(\Phi)$ will produce a form factor which is usually quite close to unity as long as the field of view is not very large and we can therefore adjust Eq. (8''') for most practical purposes. Shieh's law also holds here as the steepness angle within the hologram.

Figure 5 shows graphically the results of Eq. (8) for several typical three-color recording parameter sets. In Fig. 5(a) we assume an index modulation of $n_1 = 0.21$ for each wavelength. This is typical of a silver halide emulsion such as PPS-100/3N, which has a thickness of between 9 and 10 μm . We can therefore expect a diffraction efficiency for each color in the region of 20% for reflection holograms made in this type of material. In Fig. 5(b) we use an index modulation of 0.03 for each chromatic component.

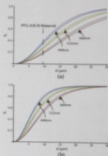


Fig. 5. Color-mixed diffraction efficiency versus grating thickness for typical incident three-color reflection volume phase holograms recorded in the N-PSM theory at polarizations. All holograms are recorded at 480 nm, 550 nm, and 640 nm. Shaded lines indicate $\pm 1^{\text{st}}$ reflection diffraction orders. Solid lines indicate the sum of normal incidence efficiencies. (a) $n_1(480) = n_1(550) = n_1(640) = 0.21$ and (b) $n_1(480) = n_1(550) = n_1(640) = 0.03$.



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D. Brotherton-Ratcliffe, A. Ovanlou and P. Excell, "Using the parallel-stacked mirror model to analytically describe diffraction in the planar volume reflection grating with finite absorption", *Applied Optics* Vol. 54, pp. 3700-3707 (2015)

PSM and Lossy Gratings

PSM can be extended to describe gratings with finite loss

$$n = (n_0 + i\chi_0) + \frac{1}{2}(n_1 + i\chi_1) \{ e^{Kx} + e^{-Kx} \}$$

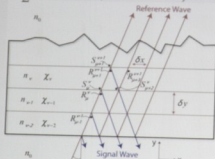


Fig.1 The Reply of an unslanted reflection grating, as treated by the PSM model, showing infinitesimally thick dielectric layers and the Signal and Reference fields present at the index discontinuities.

$$\begin{aligned} \Omega = & \begin{bmatrix} (n_{k+1} + i\chi_{k+1}) \sqrt{1 - \frac{n_0^2}{(n_{k+1} + i\chi_{k+1})^2} \sin^2 \theta_c} \\ (n_k + i\chi_k) \sqrt{1 - \frac{n_0^2}{(n_k + i\chi_k)^2} \sin^2 \theta_c} \end{bmatrix} \\ \begin{bmatrix} t \\ r \end{bmatrix}_{k,k+1} = & \begin{bmatrix} 2(n_k + i\chi_k) \sqrt{1 - \frac{n_0^2}{(n_k + i\chi_k)^2} \sin^2 \theta_c} \\ \frac{1}{\Omega} \begin{bmatrix} (n_{k+1} + i\chi_{k+1}) \sqrt{1 - \frac{n_0^2}{(n_{k+1} + i\chi_{k+1})^2} \sin^2 \theta_c} \\ -(n_k + i\chi_k) \sqrt{1 - \frac{n_0^2}{(n_k + i\chi_k)^2} \sin^2 \theta_c} \end{bmatrix} \end{bmatrix} \end{aligned}$$



PSM and Lossy Gratings

Differential equations can then be derived in exactly the same fashion as before

$$\frac{1}{\beta} \begin{bmatrix} \mathbf{k}_c \\ \mathbf{k}_i \end{bmatrix} \begin{bmatrix} R \\ S \end{bmatrix} = \begin{bmatrix} v - \rho \\ \rho \end{bmatrix} \begin{bmatrix} -\rho \\ v + \rho \end{bmatrix} \begin{bmatrix} R \\ S \end{bmatrix}$$

$$v = \frac{i\beta}{n_0}(n + i\chi)$$

$$\rho = \frac{1}{2(n + i\chi)\cos\theta_c} \left[\frac{\partial n}{\partial y} + i \frac{\partial \chi}{\partial y} \right]$$



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PSM and Lossy Gratings

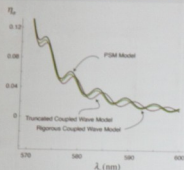


Fig. 1 Magnified section of Fig. 2(b) showing the typically better fit obtainable with PSM to the rigorous coupled wave solution than that offered by the truncated coupled wave approach.

$$D_0 \approx \frac{\mu_0 \alpha \sigma_0 d}{2 \cos \theta_c \sqrt{\epsilon_0}}$$

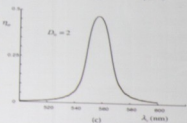
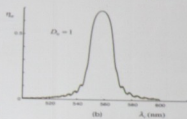
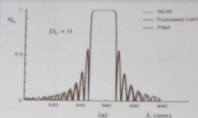


Fig. 2 Diffraction efficiency versus replay wavelength showing the effect of increasing the average loss, D_0 . The graphs pertain to gratings recorded at an (unreduced) incidence angle of 20 degrees and replayed at 10 degrees. The recording wavelength is 620nm and the grating has a thickness of 20 microns. The index modulation is real with a value of n_1 of 0.045, $n_2^2=1.5$. On each plot there are three lines corresponding to PSM (black), Rigorous Coupled Wave theory (green) and Truncated Coupled Wave theory (red).



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Absorption Gratings

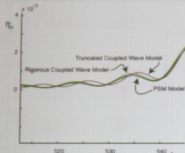


Fig 6 Magnified section of Fig 5 showing the typically better fit obtainable (for absorption gratings) with PIM to the rigorous coupled wave solution than that offered by the truncated coupled wave approach.

$$\eta_0 = \frac{1}{4 + 4\sqrt{3} \coth \left[\frac{D_0 \sqrt{3}}{2} \right] + 3 \coth \left[\frac{D_0 \sqrt{3}}{2} \right]} = \frac{1}{7 + 4\sqrt{3}} \approx 0.071797$$

$$D_1 \approx \frac{2\pi d \chi_1}{\lambda_c \cos \theta_c}$$

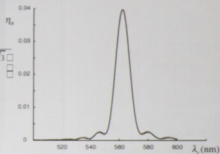


Fig 5 Diffraction efficiency versus replay wavelength for an unslanted absorption grating of thickness 10 microns with D_0 and D_1 equal to 0.15. Recording Wavelength 532nm. Recording incidence angle 35 degrees. Replay incidence angle 30 degrees. $n_w=1.5$. Green line indicates rigorous coupled wave calculation. Red line indicates truncated coupled wave model. Black line represents the PIM model.



Mixed Phase Absorption Gratings

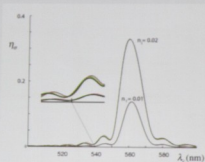


Fig. 7 Mixed phased absorption grating. The plot is identical to Fig. 6 except that a finite real harmonic index has been added. Green line indicates rigorous coupled wave calculation. Red line indicates truncated coupled wave model. Black line represents the POM model.



D. Brotherton-Ratcliffe, H. Bjelkhagen, A. Osanlou, and P. Excell, "Diffraction in volume reflection gratings with variable fringe contrast", *Applied Optics* Vol. 54, pp.5057-5064 (2015)

PSM & Variable Phase Contrast Gratings

At Bragg Resonance

$$n = n_0 + \frac{n_1}{2} \left\{ e^{+iagly} + e^{-iagly} \right\} \gamma(y)$$

$$\eta = \left| \frac{C_S}{C_R} \right| S(0)S'(0) = \tanh^2 \left[\kappa \int_0^d \gamma(y) dy \right]$$

$$\eta = \tanh^2 \left[\kappa d_{\text{eff}} \right]$$

$$d_{\text{eff}} = \int_0^d \gamma(y) dy$$

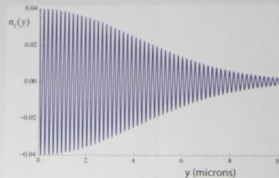


Fig 2 Harmonic grating profile with Gaussian decaying fringe contrast function (equation 1). Recording wavelength 532nm. $\kappa_0=0.04$, $d=10$ microns.



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Away from Bragg Resonance

$$\gamma(y) = \frac{1}{1 + ay}$$

This contrast function gives an analytic solution with PSM:

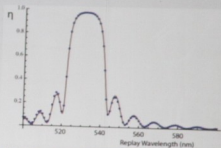


Fig 7. Diffraction efficiency versus replay wavelength for the hyperbolically decaying fringe contrast function of equation and Fig 4. Recording wavelength 532nm. $n_0=1.5$, $n_1=0.06$. Red solid line is the analytic result of equation and blue circles represent a Runge-Kutta integration of the Helmholtz equation.



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PSM in Quantum Physics, Neutron Optics and Acoustics

The PSM model may be utilised to describe particle diffraction from quantum periodic structures as the time independent Schrödinger equation for a harmonic potential is analytically identical to the corresponding Helmholtz equation describing optical diffraction from a harmonic index. Potential applications include the analysis of neutron super-mirrors that have been recorded using holographic techniques (13,14). Finally the results may also be useful in the study of acoustic diffraction from harmonic structures where the transfer matrix approach is well known (15).

13. M.Fally et al, Diffraction gratings for neutrons from polymers and holographic polymer-dispersed liquid crystals 2009 J. Opt. A: Pure Appl. Opt. 11 024019

14. J.Kleppetal, "Holographic Gratings for low-neutron optics", Materials, 5, 2788-2815; doi:10.3390/ma5122788 (2012)

15. M.Abid et al, "Acoustic Response of a Multilayer Panel with Viscoelastic Material", International Journal of Acoustics and Vibration, Vol. 17, No.2, pp82-89 (2012)



End

The PSM Theory is described in full detail in the follow references:

- 1) D. Brotherton-Ratcliffe, "A treatment of the general volume holographic grating as an array of parallel stacked mirrors," *J. Mod. Optics*, 59, 1113-1132 (2012).
- 2) D. Brotherton-Ratcliffe, "Analytical treatment of the polychromatic spatially multiplexed volume holographic grating," *Appl. Opt.* 51, 7188-7199 (2012).
- 3) D. Brotherton-Ratcliffe, "A new type of coupled wave theory capable of analytically describing diffraction in polychromatic spatially multiplexed holographic gratings," *Journal of Physics, Conference Series* 415 (2013) 012034 doi:10.1088/1742-6596/415/1/012034.
- 4) H. Bjelkhagen and D. Brotherton-Ratcliffe, *Ultra-realistic imaging – advanced techniques in analogue and digital colour holography*, (CRC- Taylor and Francis 2012).
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- 6) D. Brotherton-Ratcliffe, L. Shi, A. Osanlou and P. Excell, "A comparative study of the accuracy of the PSM and Kogelnik models of diffraction in reflection and transmission holographic gratings," *Opt. Express* 22 (26) 32384-32405 (2014).
- 7) D. Brotherton-Ratcliffe, A. Osanlou and P. Excell, "Using the parallel-stacked mirror model to analytically describe diffraction in the planar volume reflection grating with finite absorption", *Applied Optics* Vol. 54, pp. 3700–3707 (2015)
- 8) D. Brotherton-Ratcliffe, H. Bjelkhagen, A. Osanlou, and P. Excell, "Diffraction in volume reflection gratings with variable fringe contrast", *Applied Optics* Vol. 54, pp.5057-5064 (2015)