3D optical intensity modulation on curved surfaces by optimization method and its application to fabricate arbitrary patterns

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1. Introduction

♦ DOEs on curved surface

Diffractive optical elements (DOEs) is a flexible tool for creating arbitrary desired intensity distribution. It has many advantages such as small size, light weight, high diffractive efficiency etc. Usually DOEs is fabricated on a flat basement. But in recent years, the DOEs fabricated on curved surface (CS-DOEs) gains much improvement and is found many special applications in different fields.

Fig. 1 DOEs on flat surface

![Fig. 1 DOEs on flat surface](image1)

Fig. 2 DOEs on curve surface

![Fig. 2 DOEs on curve surface](image2)

The methods to fabricate micro/nano-structures on curved surfaces (CS) can be applied to produce many useful devices such as artificial compound eyes and thin-film transistors.

Fig. 3 Artificial compound eyes

Fig. 4 Thin-film transistor
There have been several methods to fabricate CS-DOEs including the ruling engine, the ion beam proximity lithography and the laser direct writing. However, these methods require expensive equipment and the fabricating process tends to be time-consuming.

Fig. 5 Ruling engine[1]

Fig. 6 Ion beam proximity lithography[2]

Fig. 7 Laser direct writing[2]

Our previous work

In our previous work, we researched the CS-DOEs based on the interference lithograph. The target CS-DOEs can be obtained by the interference of two pure phase holograms.

Fig. 8 Two pure phase interference lithograph

Details can be found in the reference:
The interference lithograph is a simple method to fabricate micro patterns in large areas. The modulation of the three-dimensional (3D) optical intensity with arbitrary distribution on curve surface is a key problem for fabricating 3D desired patterns.

- This technique employs two pure phase distribution (uploaded to the spatial light modulators, SLM).
- **The two SLM must be aligned precisely in micrometer dimension, or the output will be polluted by much noise.**

**Hard for the fabrication!**

Fig. 8 Two pure phase interference lithograph
2. Basic principle and our optimization method

- **The optimization method**

The point-based propagation holography and the modified phase retrieval method are both employed to design the pure-phase hologram for realizing the 3D intensity modulation on curved surface.

The purpose of our method is to:

- simplify the fabrication process;
- avoid the alignment of the two phases;

In the scalar diffraction domain, the propagation from one wavefront to another can be expressed by Huygens diffraction, while on the contrary, it will be the inverse Huygens diffraction.

**Huygens diffraction (HuF)**

\[ U(X_2) = \frac{1}{j\lambda \alpha} \iiint U(X_1) \frac{\exp(jkr)}{r} \cos \theta \, d\sigma \]

**Inverse Huygens diffraction (HuF⁻¹)**

\[ U(X_1) = \frac{i}{\lambda \alpha} \iiint U(X_2) \frac{\exp(-jkr')}{r'} \cos \theta' \, d\sigma' \]

where \( U(X_1) \) and \( U(X_2) \) are the wave-front on plane \( P_1 \) and curve surface \( P_2 \), \( \lambda \) is the wavelength, \( r^2 = (x-x')^2 + (y-y')^2 + (z-l(z))^2 \) and \( l(z) \) varies with different points of the curved surface.

Fig. 9 Schematic of the wave propagation

Algorithm to produce the pure-phase hologram

Initial distribution $\tilde{u}_e(x_1) = A_e \exp(j\theta_e)$

$P_2$

$u_e(x_1) = A_e \exp(j\theta_e)$ $\xrightarrow{\text{Huf}^{-1}(\cdot)}$ $Huf^e(\cdot)$ $\xrightarrow{\text{Huf}^e(\cdot)}$ $u_e(x_1) = B_e \exp(j\phi_e)$

Constrain: $A_e = M [A_0 + k (A_0 - A_e)] + \gamma (1 - M) A_e$

$\tilde{u}_e(x_2) = A_e \exp(j\theta_e)$ $\xrightarrow{\text{Huf}^{-1}(\cdot)}$ $Huf^e(\cdot)$ $\xrightarrow{\text{Huf}^e(\cdot)}$ $\tilde{u}_e(x_2) = A_e \exp(j\theta_e)$

Constrain: $B_e = 1$

$P_1$

$u_e(x_1) = \exp(j\phi_e)$ $\xrightarrow{\text{Huf}^{-1}(\cdot)}$ $Huf^e(\cdot)$ $\xrightarrow{\text{Huf}^e(\cdot)}$ $u_e(x_1) = \exp(j\phi_e)$

$\tilde{u}_e(x_1) = A_e \exp(j\theta_e)$

Insert the criterion
- Control the design errors
- Limiting the iterations $n \leq n_{\text{max}}$

$\tilde{u}_e(x_1) \in \rho$ Hologram

$A_0$: ideal intensity;
$A_e$: intensity of the curved surface after $n$th iteration;
k: feedback parameter $(0-1)$;
$\gamma$: noise suppression parameter $(0-1)$;
$M$: 0 at the zero padding point, while 1 at the imaging pixels.

Fig. 10 Flow chart of the algorithm

3. Results

♦ Simulations

The used curved surface is a cylindrical surface. The sampling pixel of the cylindrical surface is shown in Fig. 11, the top view is shown in Fig. 11(a), and the center green part is divided into many grids with equal areas as shown in Fig. 11(b).

(a)  
(b)  

$L$: side length of the DOE  
$R$: radius of the cylindrical surface  
$2\alpha$: field angle

Fig. 11 (a) Top view and (b) Sampling pixels of cylindrical surface

As shown in Fig. 12(a), the ideal 3D intensity $I$ is a four-Chinese-character on a cylindrical surface. The output pattern and the convergence curve in the iterations are shown in Fig. 12 (b) and (c), respectively. **The reconstructed pattern has good quality and the RE declines to 2.91% after 35 iterations.**

Other parameters are $L=12\text{mm}$, $R=51.852\text{mm}$ and the diffraction distance $d=433\text{mm}$.

$$RE = \sum \sum \left( \frac{|I_r(m,n) - I_0(m,n)|}{|I_0(m,n)|} \right)^2 \times 100\%$$

$I_r$: reconstructed

$I_0$: ideal

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Fig. 12 Numerical simulation for 3D binary pattern, (a) an ideal pattern and (b) the reconstructed pattern, (c) relative error with iterations.

To better demonstrate this method, we then study on a 3D gray level pattern. As shown in Fig. 13(a), the 3D gray level badge pattern of Beijing Institute of Technology is used as the ideal pattern. The output results are given in Fig. 13(b) and (c), the reconstructed pattern is good as well and the $RE$ is 1.66% after 35 iterations.

The other parameters are as same as the above example.

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Fig. 13 Numerical simulation for 3D gray level badge pattern, (a) an ideal pattern, (b) the reconstructed pattern, (c) relative error with iterations.

We also simulated more complicated curved surface. Its mathematical formula is expressed as $z = \sin(bx)\sin(cy)$, where parameters $a$, $b$, and $c$ are constant. In the example, $a = 0.1$, $b = c = 1$.

The results are shown in Fig. 14(b) and (c). **It is seen that the reconstructed pattern is achieved with high quality and the RE reduces to 1.02% after 20 iterations.**

*Fig. 14 Numerical simulation for 3D gray level pattern “cameraman” ($a = 1/10$, $b = 1$, $c = 1$), (a) an ideal pattern, (b) the reconstructed pattern, and (c) relative error with iterations.*

Optical experiment

Laser wavelength: $\lambda=532\mu m$; SLM: BNS XY series, $512 \times 512$ pixels, the active area is $7.68\times 7.68\mu m$; The 4-f lens and high-pass filter are used to eliminate unwanted noise. Other parameters are $L=12\mu m$, $R=51.852\mu m$, $d=2633\mu m$.


Fig. 15 Optical setup of the experiment.
The 3D binary pattern with four Chinese characters and the 3D gray level badge of Beijing Institute of Technology are tested as the ideal patterns. Figure 15(a) and (c) are captured by 500X Series Digital Microscope, while the magnified partial image Fig. 15(b) and (d) are captured by OLYMPUS BX51 microscope.

Fig. 16 Photograph of the (a) 3D binary pattern and (c) gray level badge fabricated on the cylindrical lens, respectively. (b) and (d) Enlarged picture of the corresponding parts, respectively.

4. Conclusion

- The phase optimization method is proposed to design the pure-phase hologram on the plane for realizing the 3D intensity modulation on target curved surface.
- Both the numerical simulations and the optical experiments are performed with high quality.
- The method simplifies the fabrication process by using only a single SLM. It can be applied to fabricate arbitrary 3D patterns on curved surface.
- It may also find applications in some other fields such as image processing, holographic projection, and 3D optical manipulation.
Thank you for your attention!

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