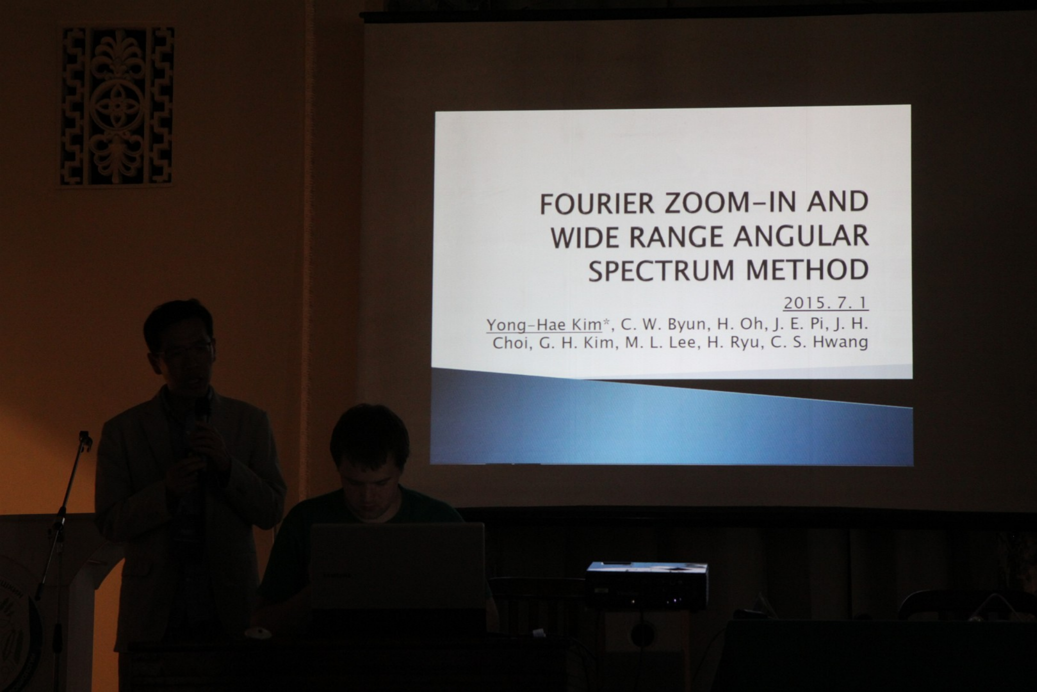
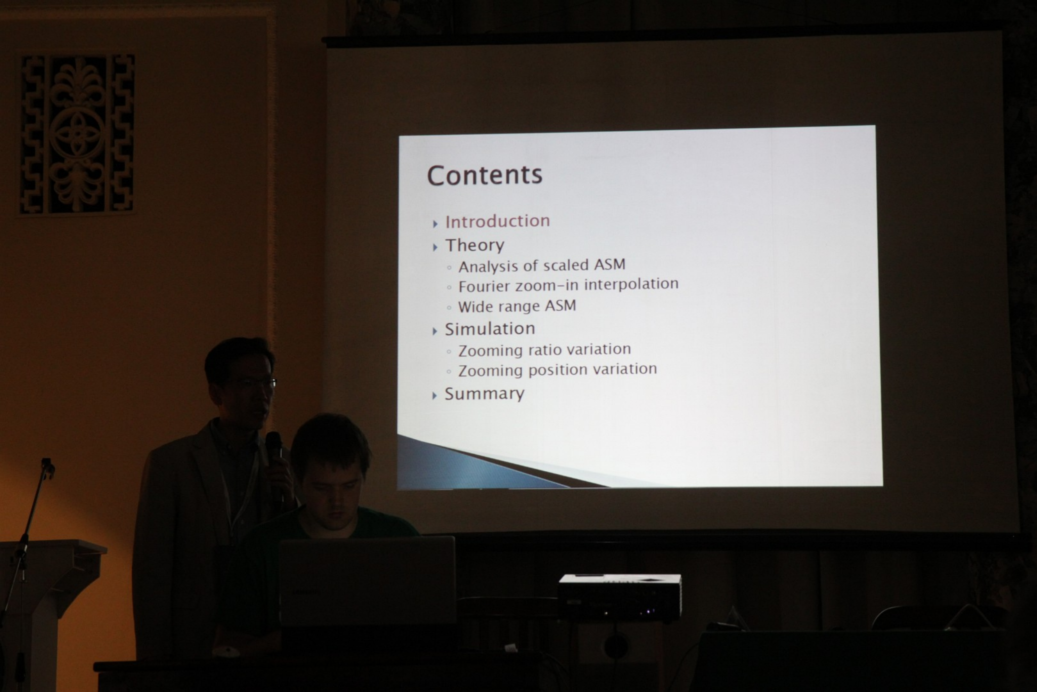

FOURIER ZOOM-IN AND WIDE RANGE ANGULAR SPECTRUM METHOD
2015. 7. 1
Yong-Hae Kim*, C. W. Byun, H. Oh, J. E. Pi, J. H. Choi, G. H. Kim, M. L. Lee, H. Ryu, C. S. Hwang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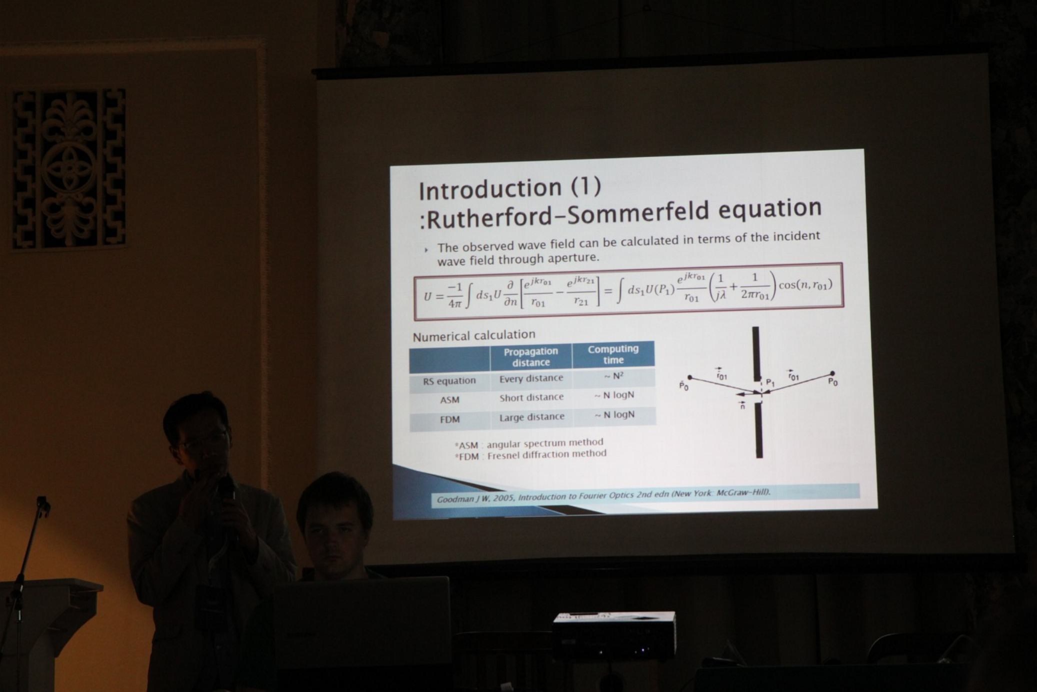
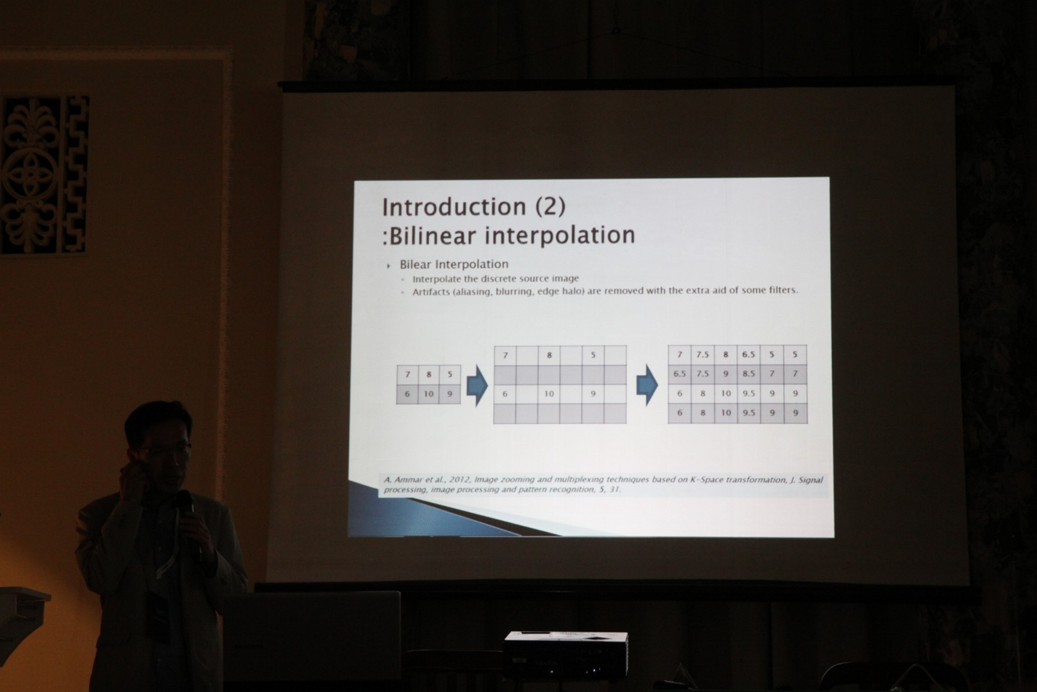
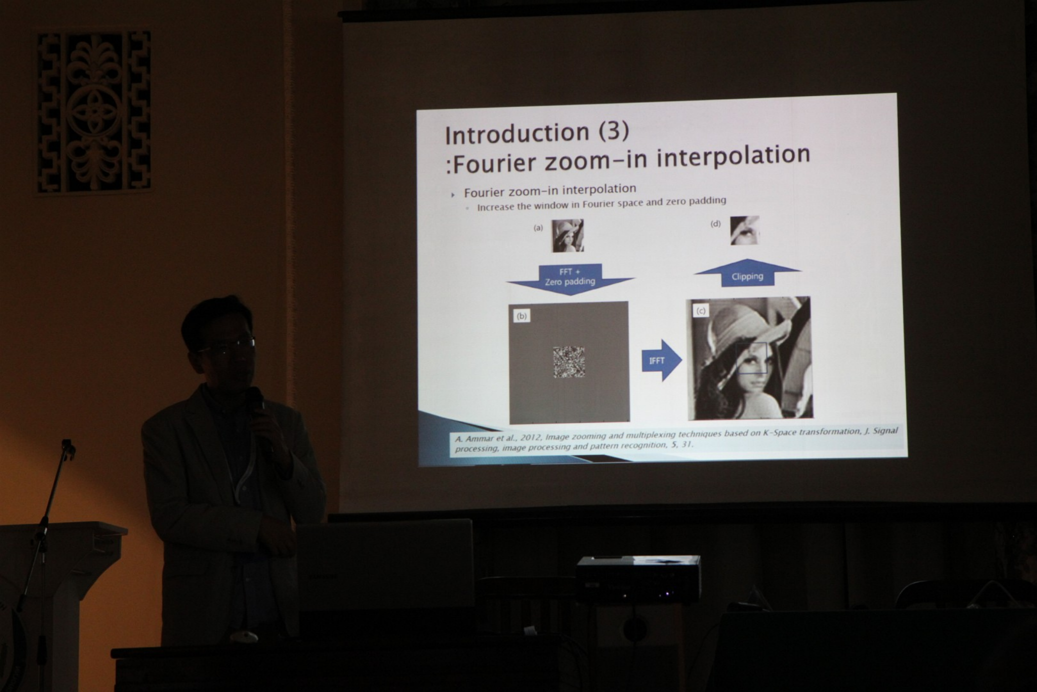
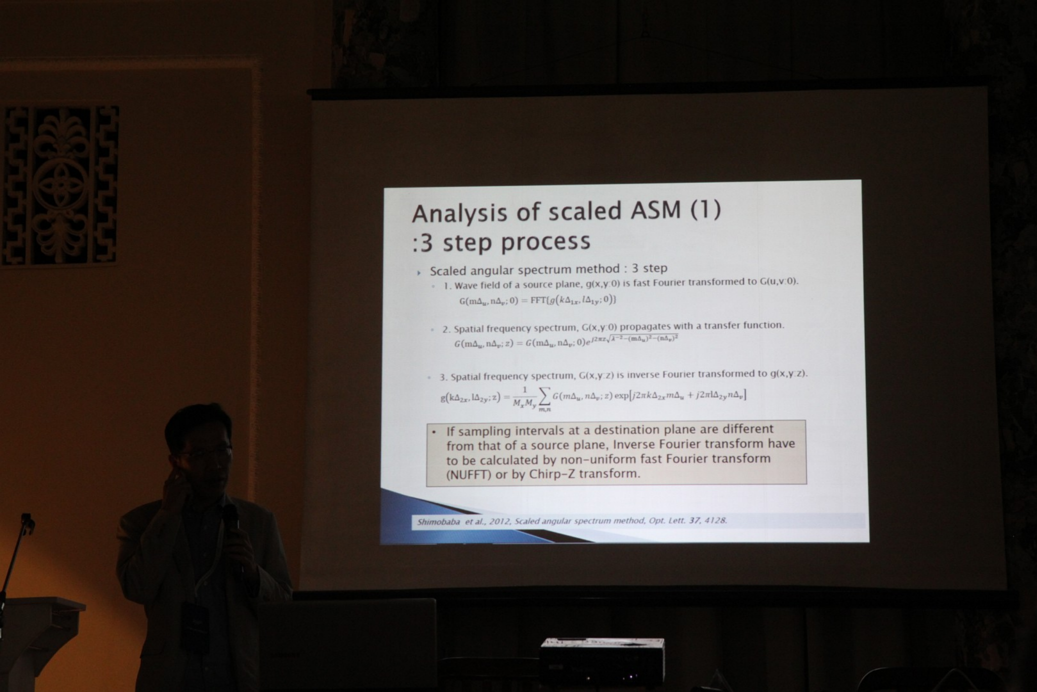
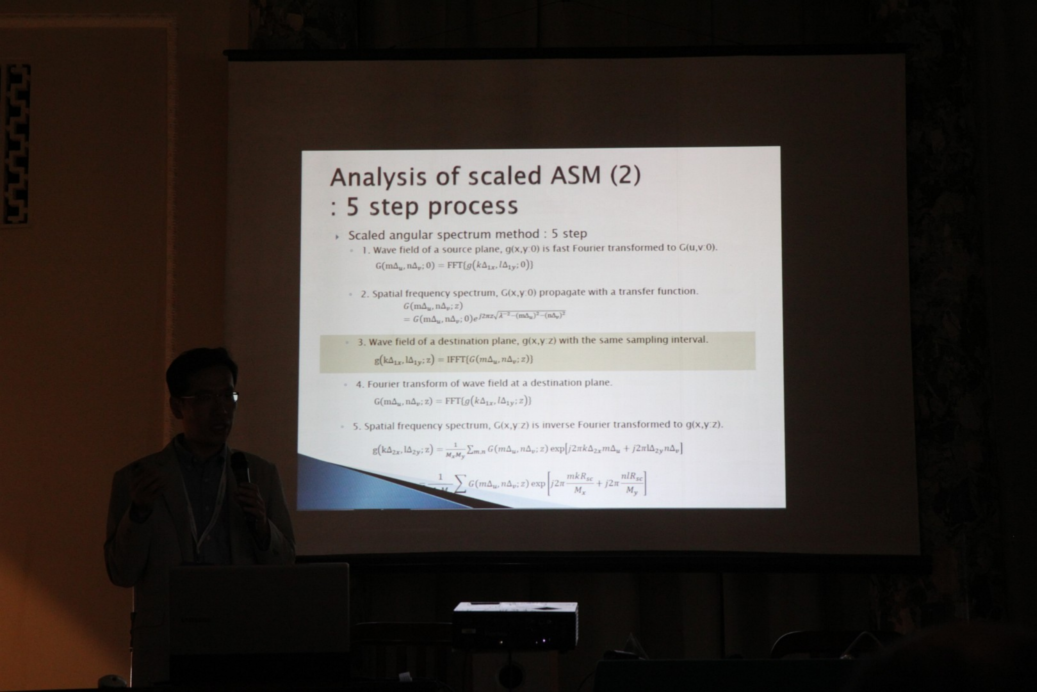

Introduction (1)
:Rutherford-Sommerfeld equation
The observed wave field can be calculated in terms of the incident wave field through aperture.
$U = \frac{-1}{4\pi}\int ds_1 U \frac{\partial}{\partial n}\left[\frac{e^{jkr_{01}}}{r_{01}} - \frac{e^{jkr_{21}}}{r_{21}}\right] = \int ds_1 U(P_1)\frac{e^{jkr_{01}}}{r_{01}}\left(\frac{1}{j\lambda} + \frac{1}{2\pi r_{01}}\right)\cos(n, r_{01})$
Numerical calculation
Propagation distance
Computing time
RS equation
Every distance
~ N^2
ASM
Short distance
~ N logN
FDM
Large distance
~ N logN
*ASM : angular spectrum method
*FDM : Fresnel diffraction method
$\vec{r}_{01}$
$\vec{r}_{01}$
$\bar{P}_0$
P_1
P_0
$\vec{n}$
Goodman J W, 2005, Introduction to Fourier Optics 2nd edn (New York: McGraw-Hill).

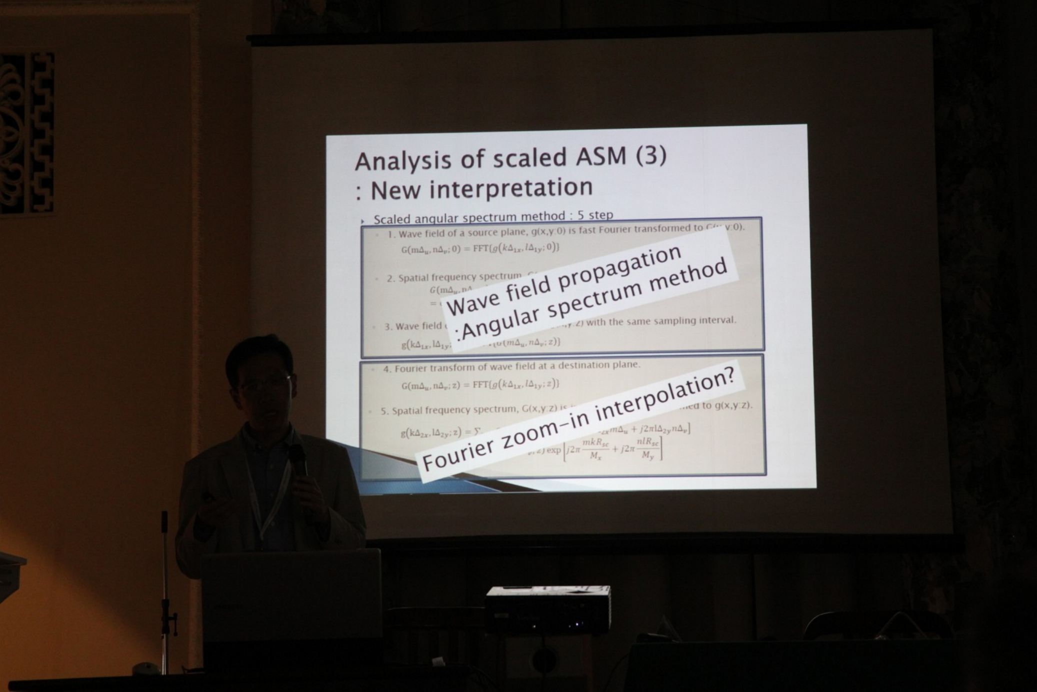

Introduction (2)
:Bilinear interpolation
Bilear Interpolation
Interpolate the discrete source image
Artifacts (aliasing, blurring, edge halo) are removed with the extra aid of some filters.
7 8 5
6 10 9
7 8 5
6 10 9
7 7.5 8 6.5 5 5
6.5 7.5 9 8.5 7 7
6 8 10 9.5 9 9
6 8 10 9.5 9 9
A. Ammar et al., 2012, Image zooming and multiplexing techniques based on K-Space transformation, J. Signal processing, image processing and pattern recognition, 5, 31.

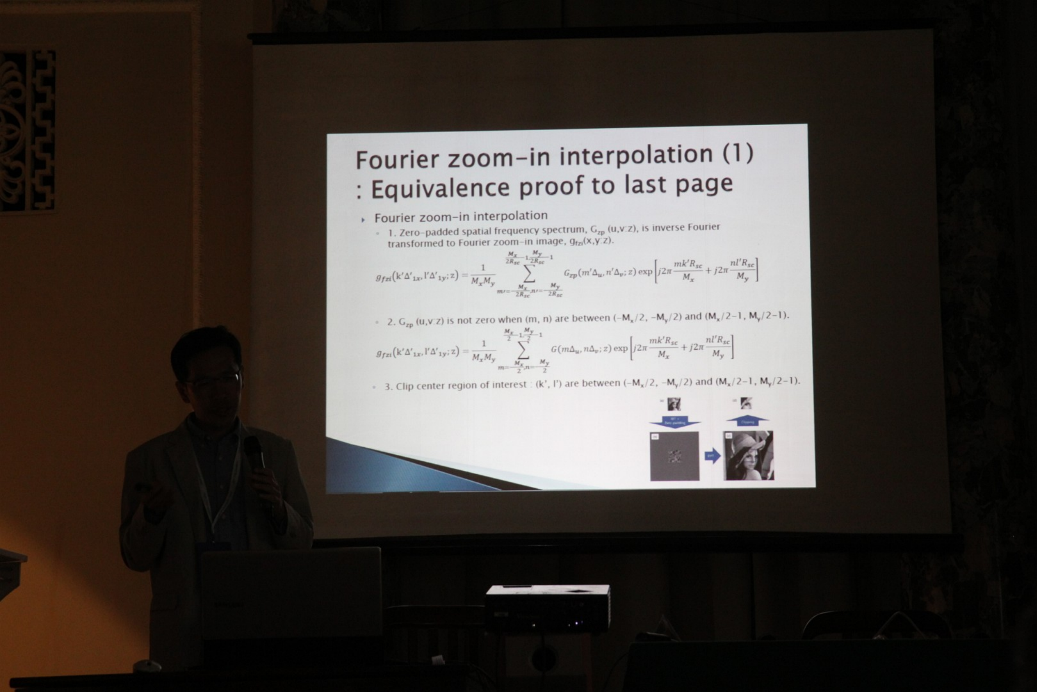

Introduction (3)
:Fourier zoom-in interpolation
Fourier zoom-in interpolation
Increase the window in Fourier space and zero padding
(a)
(d)
FFT + Zero padding
Clipping
(b)
IFFT
(c)
A. Ammar et al., 2012, Image zooming and multiplexing techniques based on K-Space transformation, J. Signal processing, image processing and pattern recognition, 5, 31.


Analysis of scaled ASM (1)
:3 step process
Scaled angular spectrum method : 3 step
1. Wave field of a source plane, g(x,y:0) is fast Fourier transformed to G(u,v:0).
$G(m\Delta_u, n\Delta_v; 0) = \mathrm{FFT}\{g(k\Delta_{1x}, l\Delta_{1y}; 0)\}$
2. Spatial frequency spectrum, G(x,y:0) propagates with a transfer function.
$G(m\Delta_u, n\Delta_v; z) = G(m\Delta_u, n\Delta_v; 0) e^{j2\pi z\sqrt{\lambda^{-2}-(m\Delta_u)^2-(n\Delta_v)^2}}$
3. Spatial frequency spectrum, G(x,y:z) is inverse Fourier transformed to g(x,y:z).
$g(k\Delta_{2x}, l\Delta_{2y}; z) = \frac{1}{M_x M_y}\sum_{m,n} G(m\Delta_u, n\Delta_v; z)\exp[j2\pi k\Delta_{2x} m\Delta_u + j2\pi l\Delta_{2y} n\Delta_v]$
If sampling intervals at a destination plane are different from that of a source plane, Inverse Fourier transform have to be calculated by non-uniform fast Fourier transform (NUFFT) or by Chirp-Z transform.
Shimobaba et al., 2012, Scaled angular spectrum method, Opt. Lett. 37, 4128.

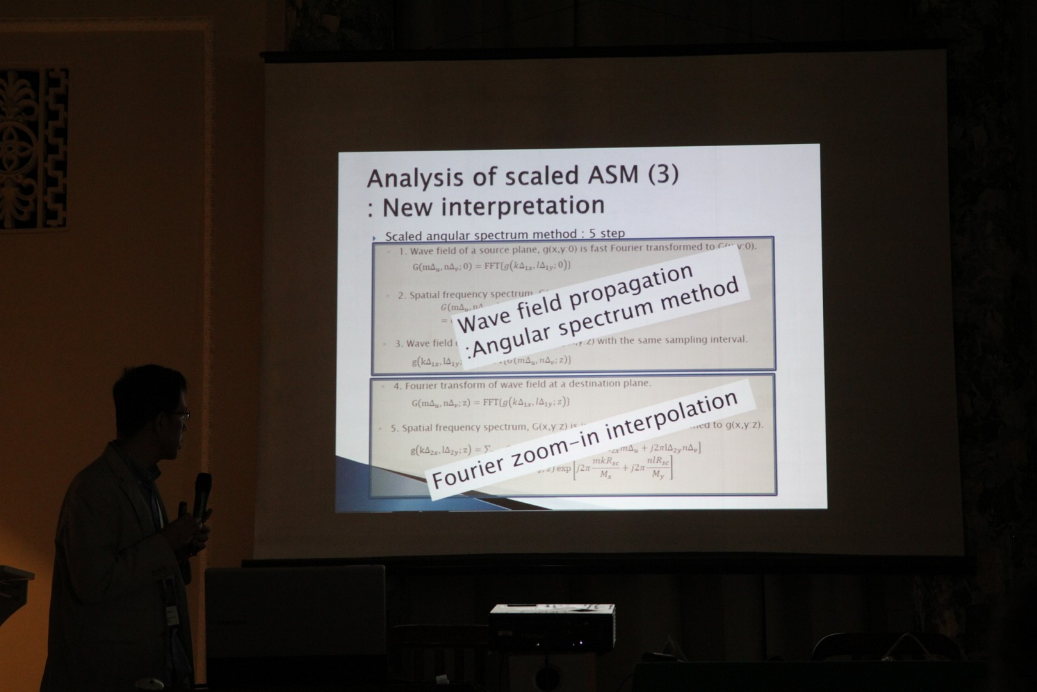


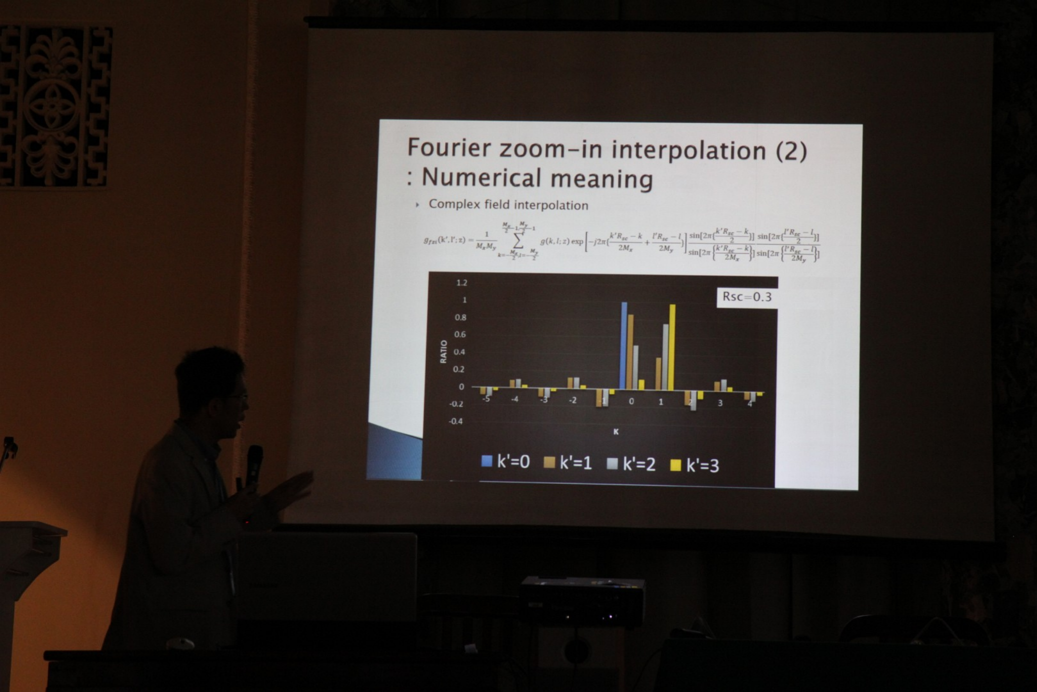
Analysis of scaled ASM (2)
: 5 step process
Scaled angular spectrum method : 5 step
1. Wave field of a source plane, g(x,y:0) is fast Fourier transformed to G(u,v:0).
$G(m\Delta_u, n\Delta_v; 0) = FFT\{g(k\Delta_{1x}, l\Delta_{1y}; 0)\}$
2. Spatial frequency spectrum, G(x,y:0) propagate with a transfer function.
$G(m\Delta_u, n\Delta_v; z) = G(m\Delta_u, n\Delta_v; 0) e^{j2\pi z\sqrt{\lambda^{-2}-(m\Delta_u)^2-(n\Delta_v)^2}}$
3. Wave field of a destination plane, g(x,y:z) with the same sampling interval.
$g(k\Delta_{1x}, l\Delta_{1y}; z) = IFFT\{G(m\Delta_u, n\Delta_v; z)\}$
4. Fourier transform of wave field at a destination plane.
$G(m\Delta_u, n\Delta_v; z) = FFT\{g(k\Delta_{1x}, l\Delta_{1y}; z)\}$
5. Spatial frequency spectrum, G(x,y:z) is inverse Fourier transformed to g(x,y:z).
$g(k\Delta_{2x}, l\Delta_{2y}; z) = \frac{1}{M_x M_y}\sum_{m,n} G(m\Delta_u, n\Delta_v; z)\exp[j2\pi k\Delta_{2x} m\Delta_u + j2\pi l\Delta_{2y} n\Delta_v]$

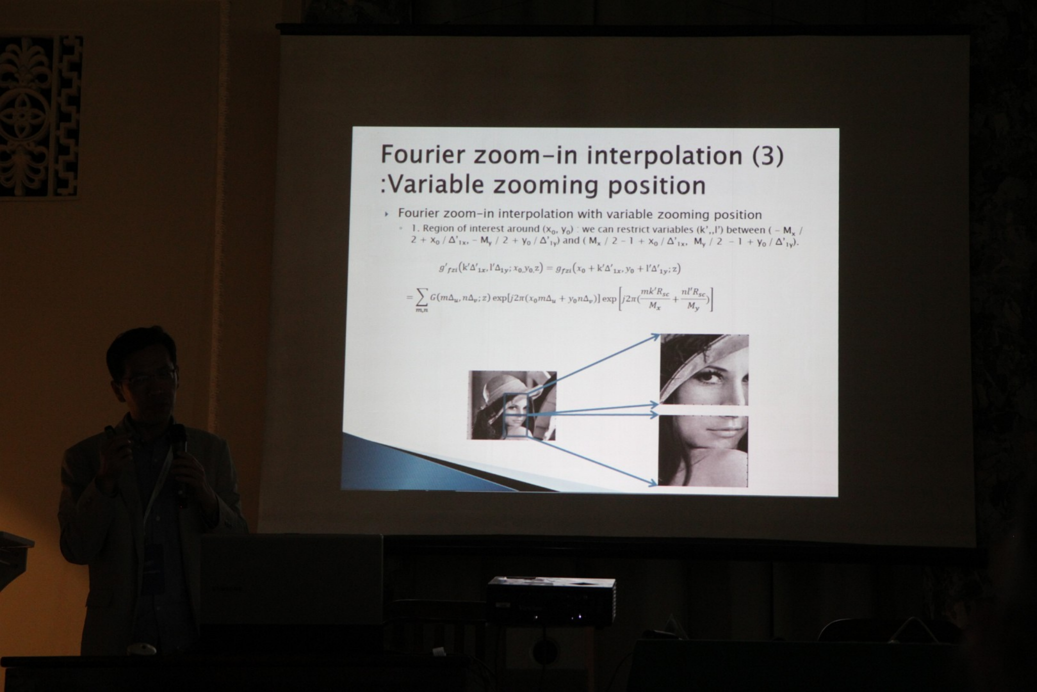

Analysis of scaled ASM (3)
: New interpretation
Scaled angular spectrum method : 5 step
1. Wave field of a source plane, g(x,y:0) is fast Fourier transformed to
G(mΔu, nΔv; 0) = FFT{g(kΔ1x, lΔ1y; 0)}
2. Spatial frequency spectrum
3. Wave field
with the same sampling interval.
4. Fourier transform of wave field at a destination plane.
G(mΔu, nΔv; z) = FFT{g(kΔ1x, lΔ1y; z)}
5. Spatial frequency spectrum, G(x,y:z) is
to g(x,y:z).
Wave field propagation
:Angular spectrum method
Fourier zoom-in interpolation?



Fourier zoom-in interpolation (1)
: Equivalence proof to last page
Fourier zoom-in interpolation
1. Zero-padded spatial frequency spectrum, G_{zp} (u,v:z), is inverse Fourier transformed to Fourier zoom-in image, g_{fzi}(x,y:z).
$g_{fzi}(k'\Delta'_{1x}, l'\Delta'_{1y}; z) = \frac{1}{M_x M_y} \sum_{m'=-\frac{M_x}{2R_{sc}}, n'=-\frac{M_y}{2R_{sc}}}^{\frac{M_x}{2R_{sc}}-1, \frac{M_y}{2R_{sc}}-1} G_{zp}(m'\Delta_u, n'\Delta_v; z) \exp\left[j2\pi \frac{mk'R_{sc}}{M_x} + j2\pi \frac{nl'R_{sc}}{M_y}\right]$
2. G_{zp} (u,v:z) is not zero when (m, n) are between ($-M_x/2$, $-M_y/2$) and ($M_x/2-1$, $M_y/2-1$).
$g_{fzi}(k'\Delta'_{1x}, l'\Delta'_{1y}; z) = \frac{1}{M_x M_y} \sum_{m=-\frac{M_x}{2}, n=-\frac{M_y}{2}}^{\frac{M_x}{2}-1, \frac{M_y}{2}-1} G(m\Delta_u, n\Delta_v; z) \exp\left[j2\pi \frac{mk'R_{sc}}{M_x} + j2\pi \frac{nl'R_{sc}}{M_y}\right]$
3. Clip center region of interest : (k', l') are between ($-M_x/2$, $-M_y/2$) and ($M_x/2-1$, $M_y/2-1$).

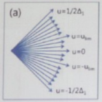

Analysis of scaled ASM (3)
: New interpretation
Scaled angular spectrum method : 5 step
1. Wave field of a source plane, g(x,y:0) is fast Fourier transformed to
2. Spatial frequency spectrum
3. Wave field
with the same sampling interval.
4. Fourier transform of wave field at a destination plane.
5. Spatial frequency spectrum, G(x,y:z) is
Wave field propagation
:Angular spectrum method
Fourier zoom-in interpolation


Fourier zoom-in interpolation (2)
: Numerical meaning
Complex field interpolation
Rsc=0.3
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
RATIO
-5
-4
-3
-2
-1
0
1
2
3
4
K
k'=0
k'=1
k'=2
k'=3


Fourier zoom-in interpolation (3)
:Variable zooming position
Fourier zoom-in interpolation with variable zooming position
1. Region of interest around (x_0, y_0) : we can restrict variables (k',,l') between (– M_x / 2 + x_0 / Δ'_{1x}, – M_y / 2 + y_0 / Δ'_{1y}) and (M_x / 2 – 1 + x_0 / Δ'_{1x}, M_y / 2 – 1 + y_0 / Δ'_{1y}).
$g'_{fzi}(k'\Delta'_{1x}, l'\Delta_{1y}; x_0, y_0, z) = g_{fzi}(x_0 + k'\Delta'_{1x}, y_0 + l'\Delta'_{1y}; z)$
$= \sum_{m,n} G(m\Delta_u, n\Delta_v; z) \exp[j2\pi(x_0 m\Delta_u + y_0 n\Delta_v)] \exp\left[j2\pi(\frac{mk'R_{sc}}{M_x} + \frac{nl'R_{sc}}{M_y})\right]$

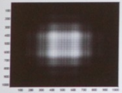
Wide range ASM (1)
:Band-limited ASM

- Nyquist theorem : local frequency of a function have to be below a half of a sampling frequency to avoid an aliasing error.

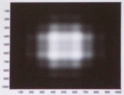


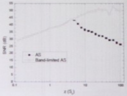
$$\frac{1}{2\pi}\frac{\partial w}{\partial u} < \frac{1}{2\Delta_u}, \frac{1}{2\pi}\frac{\partial w}{\partial v} < \frac{1}{2\Delta_v}$$

- Angular spectrum method

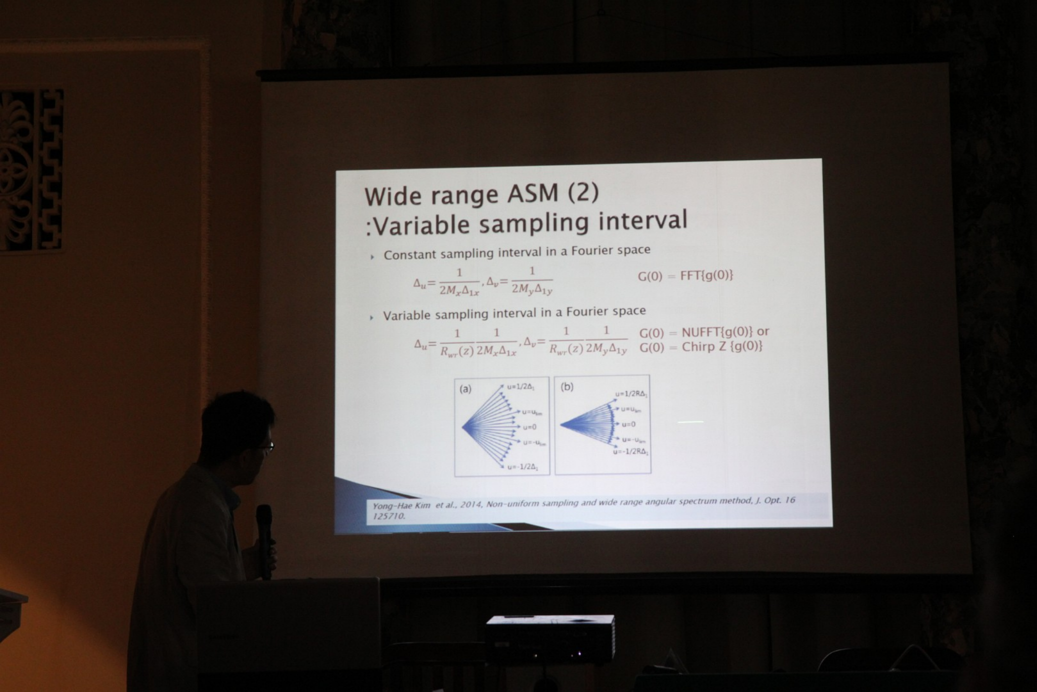


- Band-limited ASM

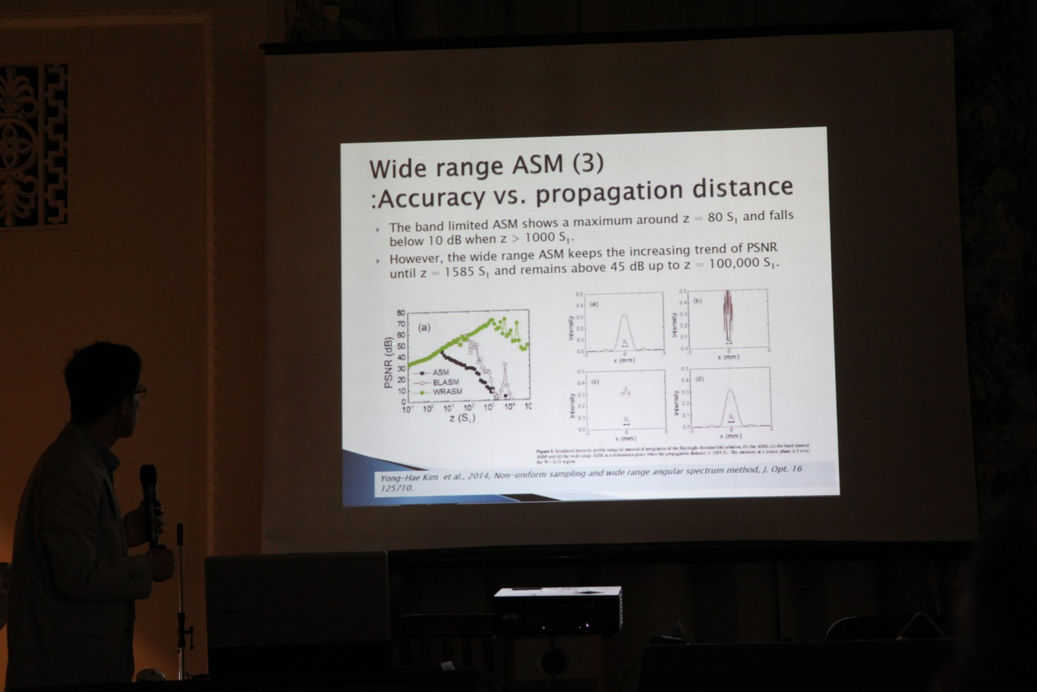


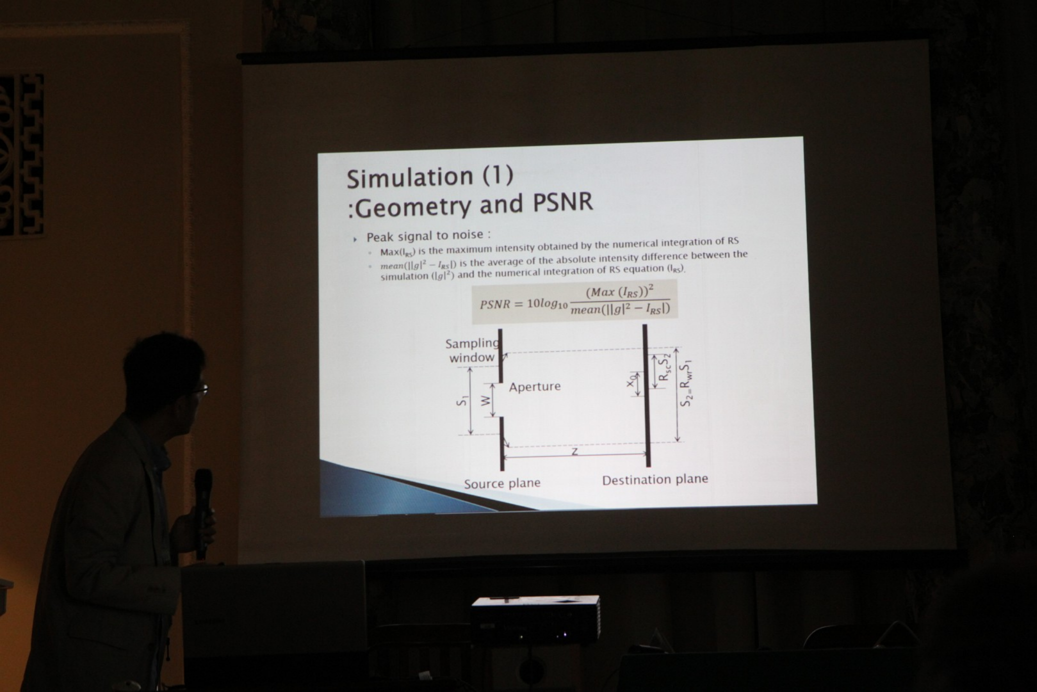


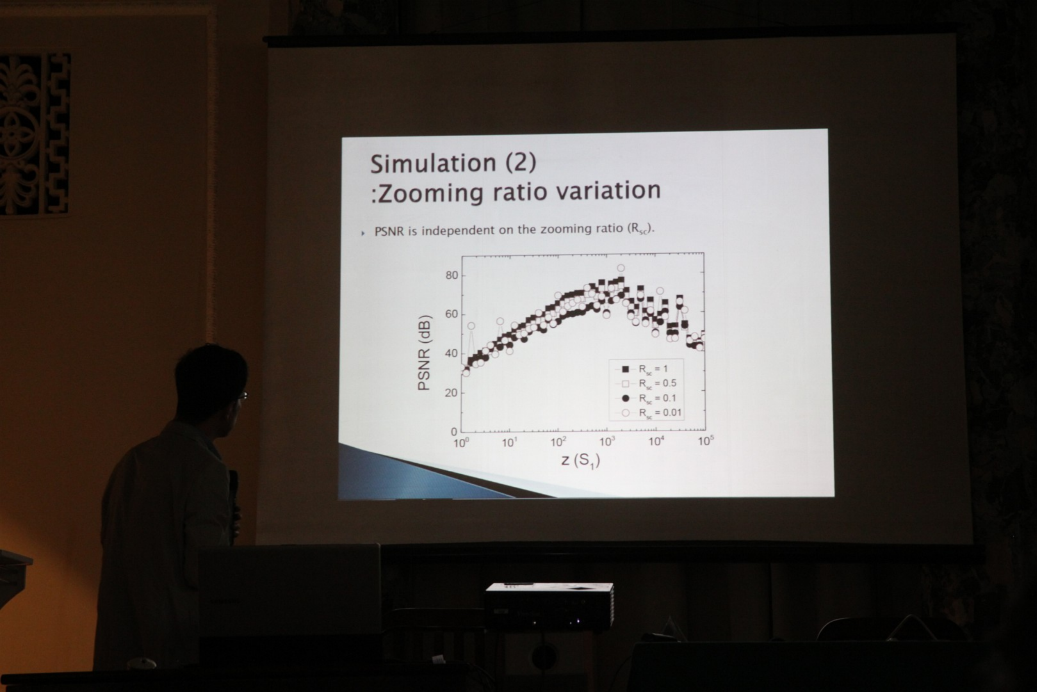
Matsushima K and Shimobaba T, 2009, Band-limited angular spectrum method for numerical simulation of free-space propagation in far and near fields, Opt. Express 17 19662



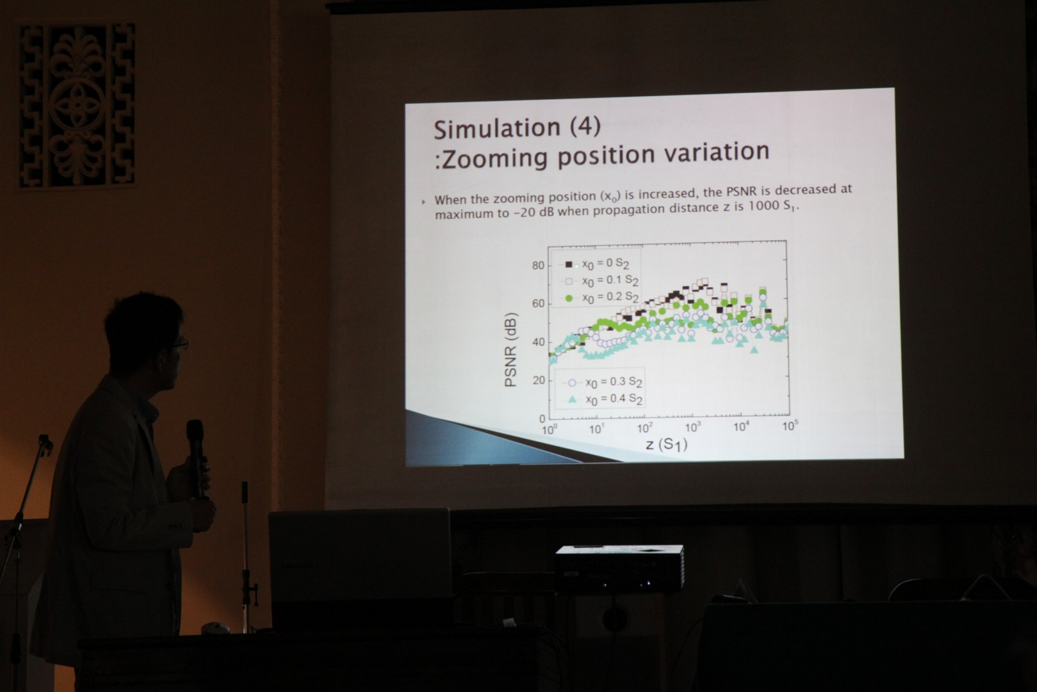
Wide range ASM (2)
:Variable sampling interval
Constant sampling interval in a Fourier space
$\Delta_u = \frac{1}{2M_x\Delta_{1x}}, \Delta_v = \frac{1}{2M_y\Delta_{1y}}$
G(0) = FFT{g(0)}
Variable sampling interval in a Fourier space
$\Delta_u = \frac{1}{R_{wr}(z)}\frac{1}{2M_x\Delta_{1x}}, \Delta_v = \frac{1}{R_{wr}(z)}\frac{1}{2M_y\Delta_{1y}}$
G(0) = NUFFT{g(0)} or
G(0) = Chirp Z {g(0)}
(a)
u=1/2Δ1
u=ulim
u=0
u=-ulim
u=-1/2Δ1
(b)
u=1/2RΔ1
u=ulim
u=0
u=-ulim
u=-1/2RΔ1
Yong-Hae Kim et al., 2014, Non-uniform sampling and wide range angular spectrum method, J. Opt. 16 125710.

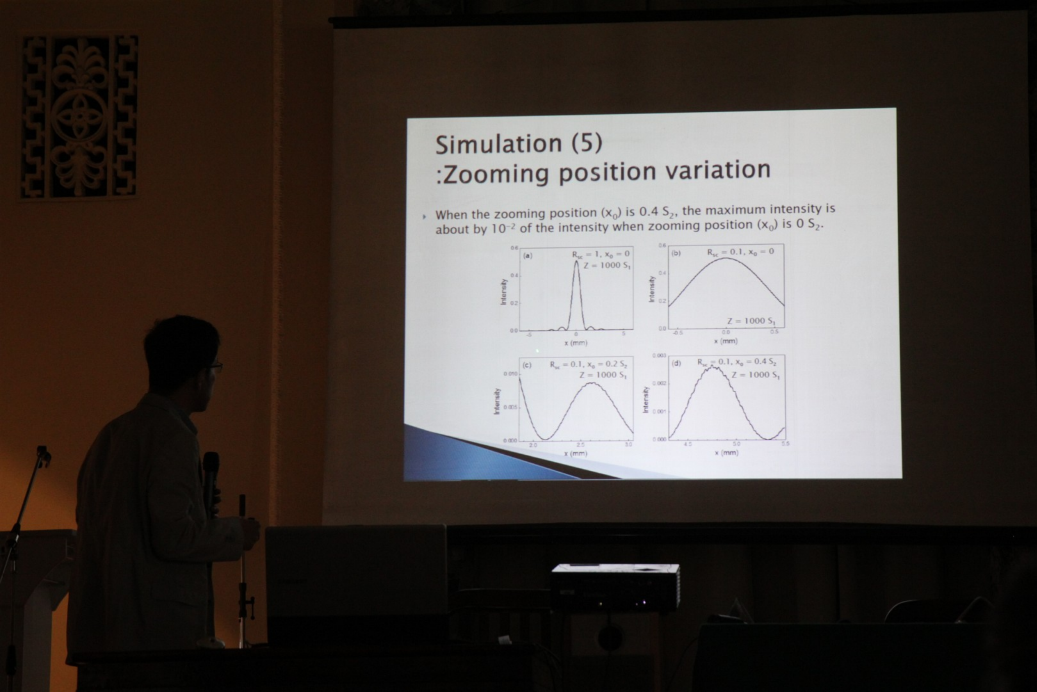

Wide range ASM (3)
:Accuracy vs. propagation distance
The band limited ASM shows a maximum around z = 80 S1 and falls below 10 dB when z > 1000 S1.
However, the wide range ASM keeps the increasing trend of PSNR until z = 1585 S1 and remains above 45 dB up to z = 100,000 S1.
(a)
PSNR (dB)
ASM
BLASM
WRASM
z (S1)
(a)
(b)
(c)
(d)
Intensity
x (mm)
Yong-Hae Kim et al., 2014, Non-uniform sampling and wide range angular spectrum method, J. Opt. 16 125710.

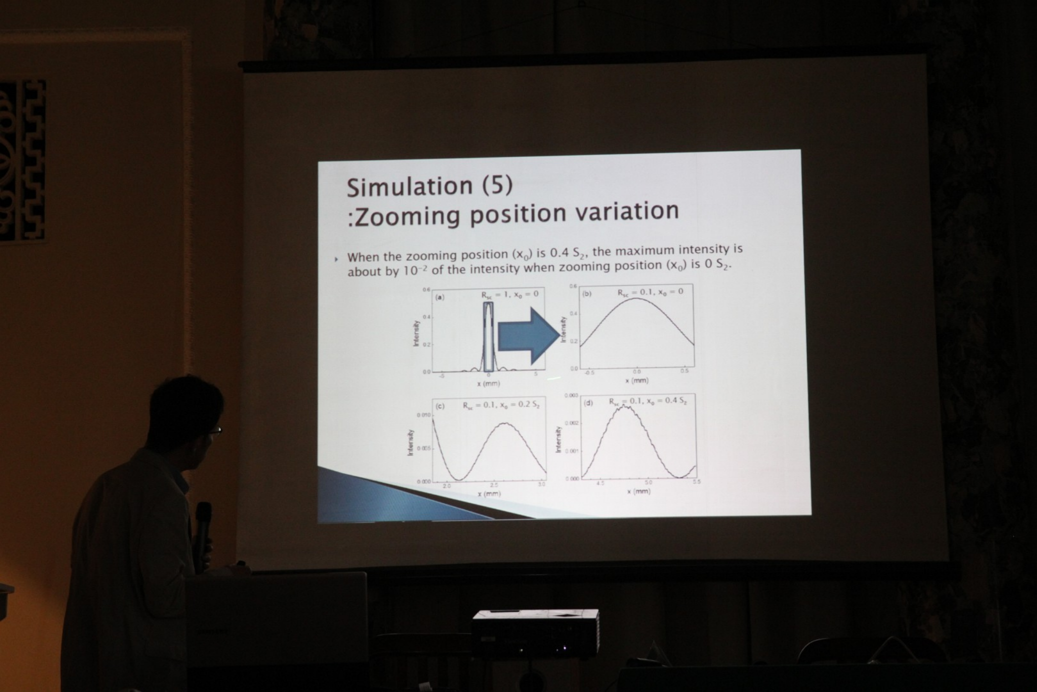

Simulation (1)
:Geometry and PSNR
Peak signal to noise :
$Max(I_{RS})$ is the maximum intensity obtained by the numerical integration of RS
$mean(||g|^2 - I_{RS}|)$ is the average of the absolute intensity difference between the simulation ($|g|^2$) and the numerical integration of RS equation (I_{RS}).
$PSNR = 10log_{10}\frac{(Max\ (I_{RS}))^2}{mean(||g|^2 - I_{RS}|)}$
Sampling window
Aperture
S_1
W
x_0
$R_{sc}S_2$
$S_2 = R_{wr}S_1$
z
Source plane
Destination plane

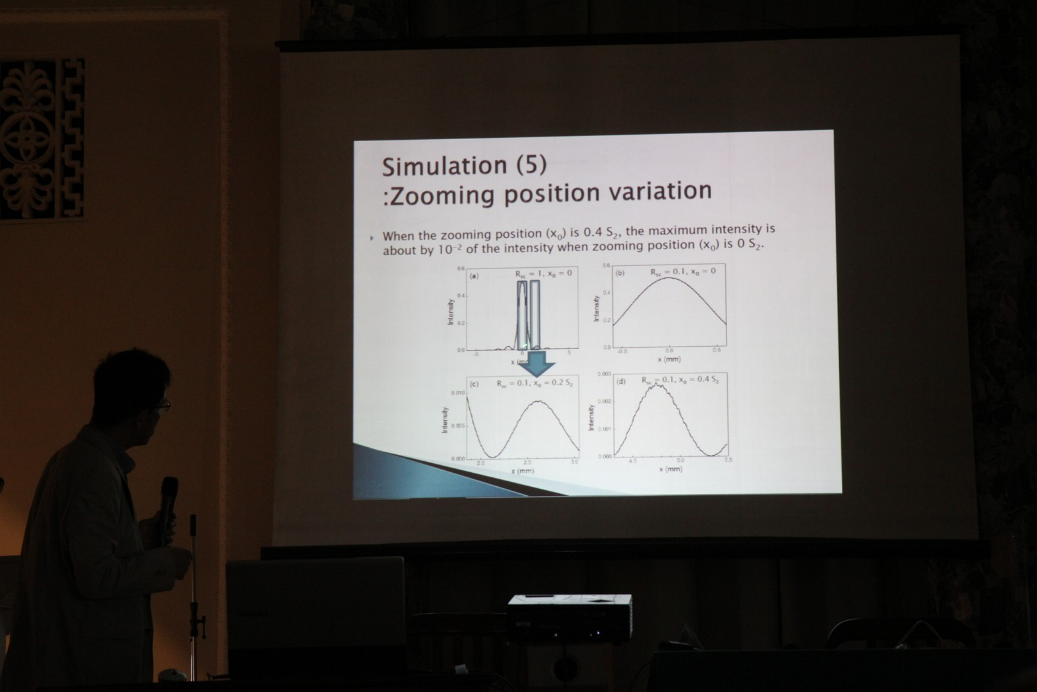

Simulation (2)
:Zooming ratio variation
PSNR is independent on the zooming ratio (R_{sc}).
80
60
40
20
0
PSNR (dB)
R_{sc} = 1
R_{sc} = 0.5
R_{sc} = 0.1
R_{sc} = 0.01
10^0
10^1
10^2
10^3
10^4
10^5
z (S_1)

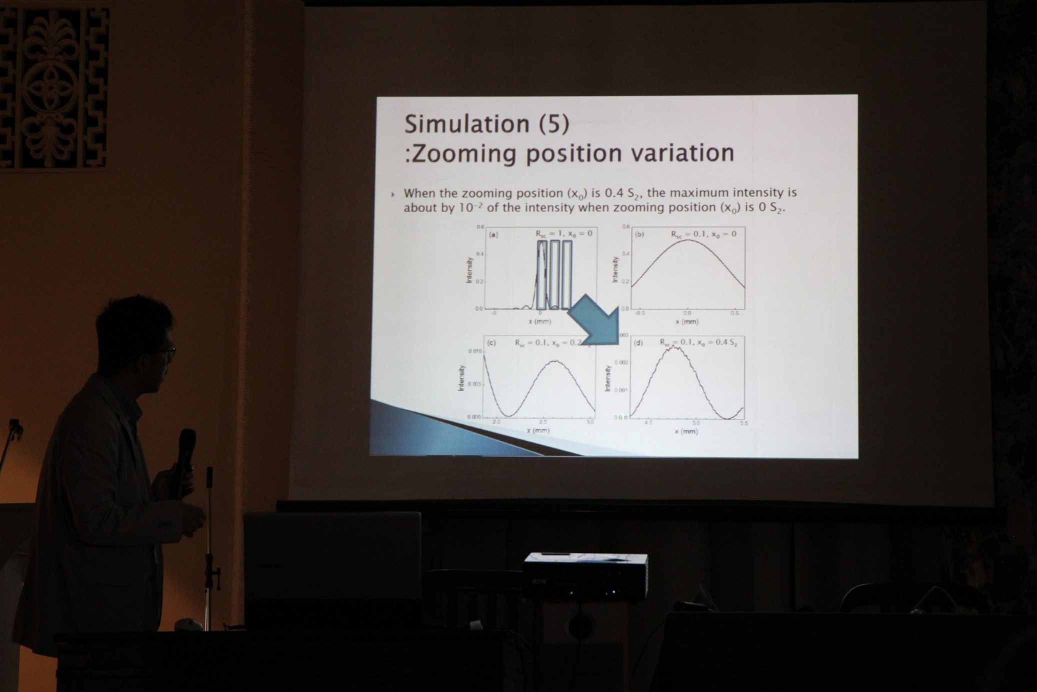

Simulation (3)
:Zooming ratio variation
Maximum intensity is nearly independent to zooming ratio (R_{sc}).
RS integral
$Z = 10\ S_1, R_{sc} = 1$
$Z = 10\ S_1, R_{sc} = 0.1$
Wide Range ASM
$Z = 10\ S_1, R_{sc} = 1$
$Z = 10\ S_1, R_{sc} = 0.1$


Simulation (4)
:Zooming position variation
When the zooming position (x_0) is increased, the PSNR is decreased at maximum to ~20 dB when propagation distance z is 1000 S_1.
x_0 = 0 S_2
x_0 = 0.1 S_2
x_0 = 0.2 S_2
x_0 = 0.3 S_2
x_0 = 0.4 S_2
PSNR (dB)
z (S_1)

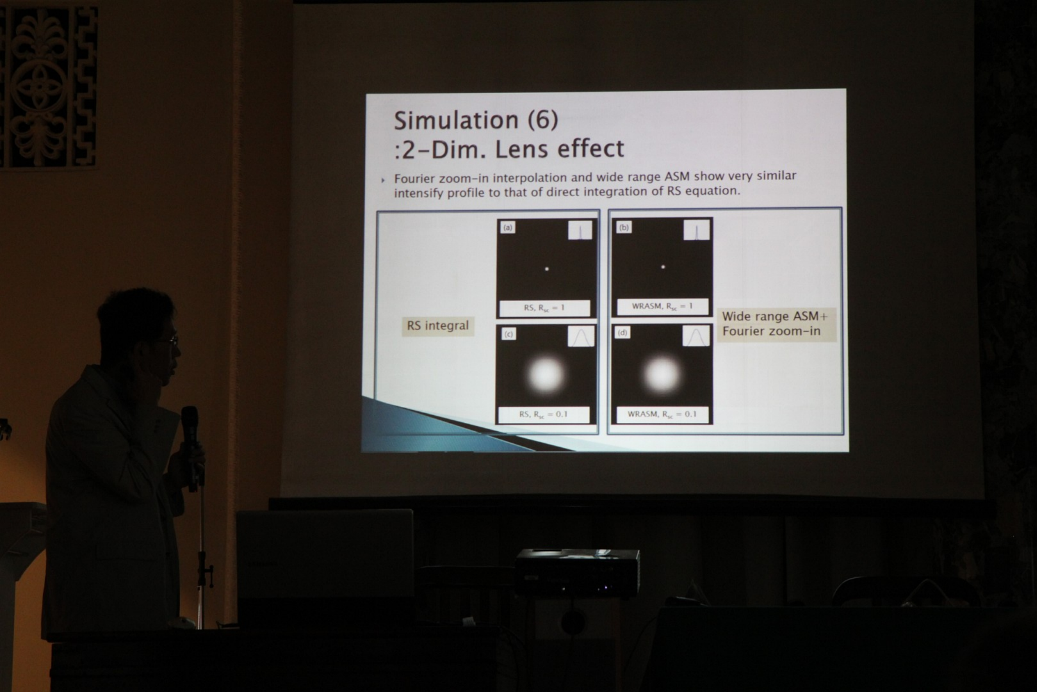

Simulation (5)
:Zooming position variation
When the zooming position (x_0) is 0.4 S_2, the maximum intensity is about by 10^{-2} of the intensity when zooming position (x_0) is 0 S_2.
(a) $R_{sc} = 1$, $x_0 = 0$
$Z = 1000\ S_1$
(b) $R_{sc} = 0.1$, $x_0 = 0$
$Z = 1000\ S_1$
(c) $R_{sc} = 0.1$, $x_0 = 0.2\ S_2$
$Z = 1000\ S_1$
(d) $R_{sc} = 0.1$, $x_0 = 0.4\ S_2$
$Z = 1000\ S_1$
Intensity
x (mm)

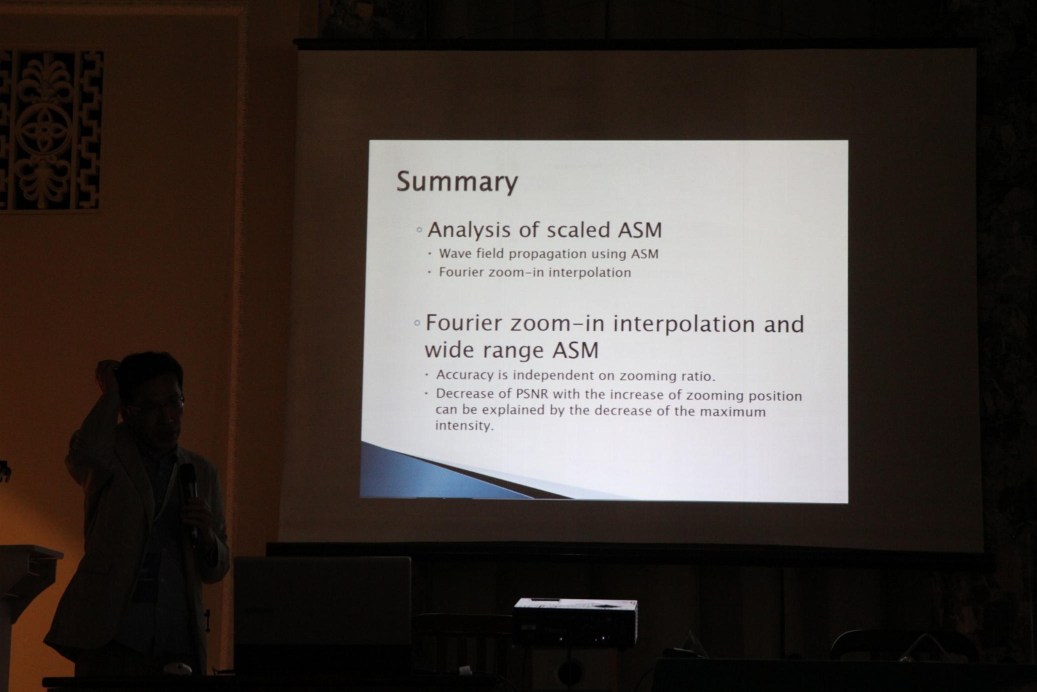

Simulation (5)
:Zooming position variation
When the zooming position (x_0) is 0.4 S_2, the maximum intensity is about by 10^{-2} of the intensity when zooming position (x_0) is 0 S_2.
(a) $R_{sc} = 1, x_0 = 0$
(b) $R_{sc} = 0.1, x_0 = 0$
(c) $R_{sc} = 0.1, x_0 = 0.2 S_2$
(d) $R_{sc} = 0.1, x_0 = 0.4 S_2$
Intensity
x (mm)


Simulation (5)
:Zooming position variation
When the zooming position (x_0) is 0.4 S_2, the maximum intensity is about by 10^{-2} of the intensity when zooming position (x_0) is 0 S_2.
(a) $R_{sc} = 1, x_0 = 0$
(b) $R_{sc} = 0.1, x_0 = 0$
(c) $R_{sc} = 0.1, x_0 = 0.2 S_2$
(d) $R_{sc} = 0.1, x_0 = 0.4 S_2$
Intensity
x (mm)



Simulation (5)
:Zooming position variation
When the zooming position (x_0) is 0.4 S_2, the maximum intensity is about by 10^{-2} of the intensity when zooming position (x_0) is 0 S_2.
(a) R_{sc} = 1, x_0 = 0
(b) R_{sc} = 0.1, x_0 = 0
(c) R_{sc} = 0.1, x_0 = 0.2
(d) R_{sc} = 0.1, x_0 = 0.4 S_2
Intensity
x (mm)


Simulation (6)
:2-Dim. Lens effect
Fourier zoom-in interpolation and wide range ASM show very similar intensify profile to that of direct integration of RS equation.
(a)
RS, R_{sc} = 1
(b)
WRASM, R_{sc} = 1
RS integral
(c)
(d)
Wide range ASM+ Fourier zoom-in
RS, R_{sc} = 0.1
WRASM, R_{sc} = 0.1


Summary
Analysis of scaled ASM
Wave field propagation using ASM
Fourier zoom-in interpolation
Fourier zoom-in interpolation and wide range ASM
Accuracy is independent on zooming ratio.
Decrease of PSNR with the increase of zooming position can be explained by the decrease of the maximum intensity.


Thank you!