FACULTY
OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

DEPARTMENT OF
COMPUTER SCIENCE
AND ENGINEERING

CENTRE
OF COMPUTER GRAPHICS
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Double look-up table method for fast light propagation calculations

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The problem

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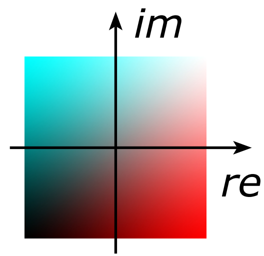
Radially symmetric functions in holography

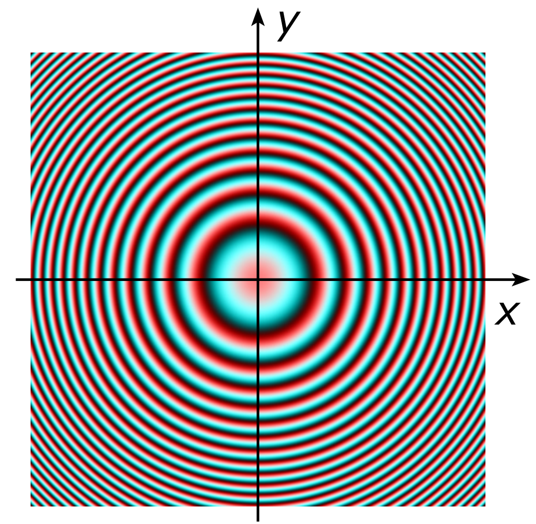
- Rayleigh-Sommerfeld convolution kernel

$$K_{\mathrm{RS}}(x, y; z_0) = -\frac{1}{2\pi}\left(\mathrm{j}\frac{2\pi}{\lambda} - \frac{1}{r}\right)\frac{\exp(\mathrm{j}2\pi r/\lambda)}{r}\frac{z_0}{r}$$

$$r = \sqrt{x^2 + y^2 + {z_0}^2}$$

- free space transfer function (angular spectrum kernel)
- lens phase
  shift function
- …





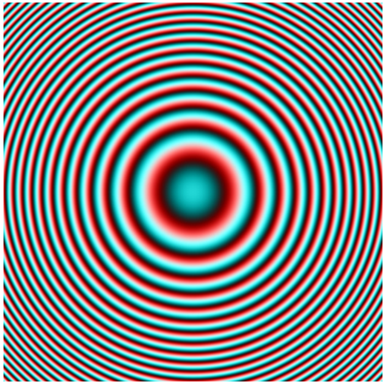
The problem

Radially symmetric functions in holography

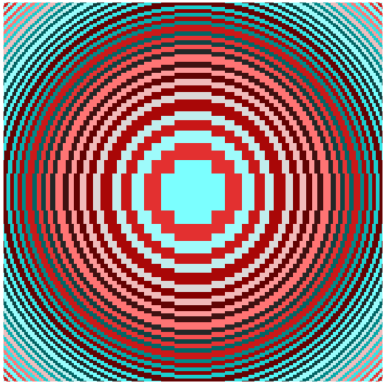
- require complicated elementary functions evaluation (sin, cos, sqrt, …)
- sometimes hard to calculate (e.g. filtered kernels)
- highly oscillatory

  ⇒ require high precision arithmetic

Example:
Rayleigh-Sommerfeld kernel
λ = 500 nm
z_0 = 2.1 m



correct calculation



single precision

The wish list

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Function evaluation should be

- fast and precise, easy to parallelize
- easy to implement on GPU or in hardware

GPU features

Many simple cores optimized for

- single precision arithmetic
- independent per-pixel operations
- linear code execution (no branching)
- computer graphics calculations (texture look-up)
- fast cache memory access

D-LUT principle

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Two-step calculation

Example: Rayleigh-Sommerfeld convolution kernel

1. $$\rho(x, y) = \sqrt{x^2 + y^2}$$

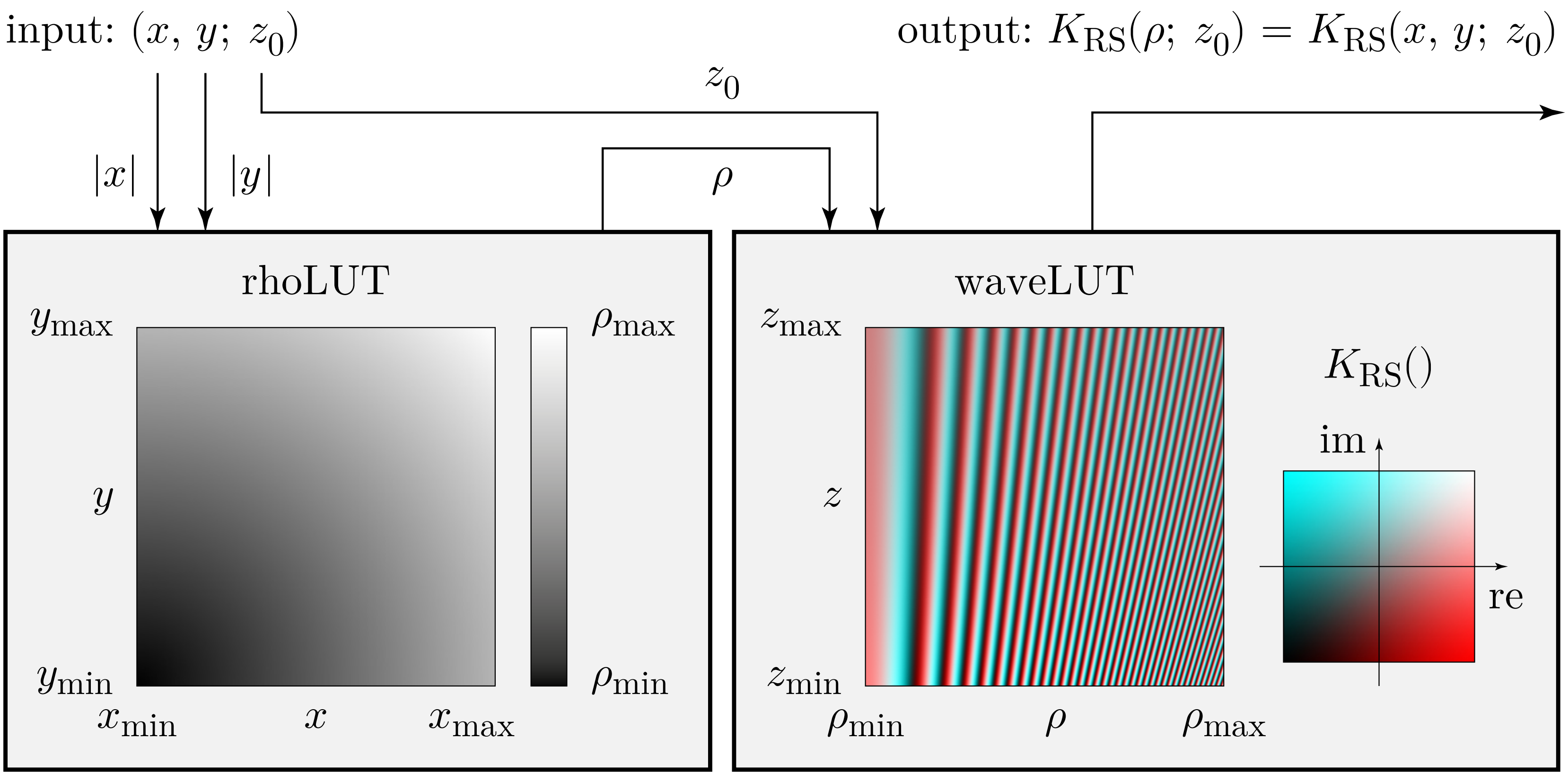
2. $$K_{\mathrm{RS}}(\rho; z_0) = -\frac{1}{2\pi}\left(\mathrm{j}\frac{2\pi}{\lambda} - \frac{1}{r}\right)\frac{\exp(\mathrm{j}2\pi r/\lambda)}{r}\frac{z_0}{r}$$

$$r = \sqrt{\rho^2 + {z_0}^2}$$

Proposed method

Precalculate $\rho(x, y)$ and $K_{\mathrm{RS}}(\rho; z_0)$ to look-up tables **rhoLUT** and **waveLUT** (double look-up table method)

D-LUT principle



rhoLUT properties

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Definition and usage of the rhoLUT

- $\text{rhoLUT}[m, n] = \sqrt{(m\Delta_\rho)^2 + (n\Delta_\rho)^2}$

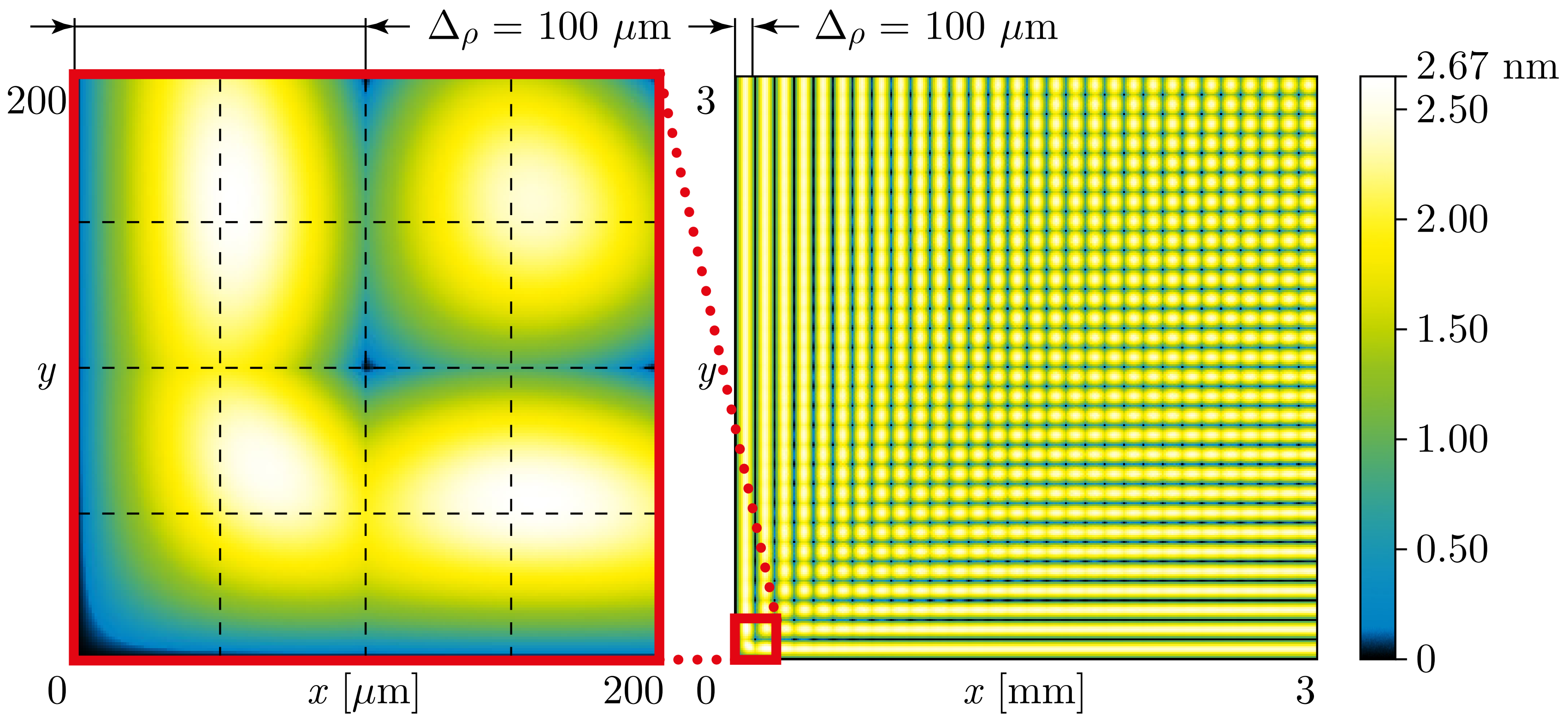
$$\rho(x, y) \approx \text{rhoLUT}[x/\Delta_\rho, y/\Delta_\rho]$$

  – use nearest-neighbour or bilinear interpolation for noninteger x/Δ_ρ, y/Δ_ρ

- sampling distance Δ_ρ small
  $\Rightarrow$ higher precision, larger table
- nearest-neighbour interpolation
  $\Rightarrow$ faster, less precise $\Rightarrow$ requires smaller Δ_ρ

rhoLUT properties

- error of $r = \sqrt{\rho^2 + {z_0}^2}$ should be small
- example: error of r for bilinear interp. in the rhoLUT



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Selection of Δ_ρ

- estimate of the maximum error

$$\mathrm{maxRError}(\Delta_\rho; z_0) = 1.2\left\{\left[\rho_{\mathrm{Bilinear}}\left(\frac{\Delta_\rho}{2}, \frac{\Delta_\rho}{2}\right)\right]^2 + {z_0}^2\right\}^{1/2} - 1.2\left\{\left[\rho_{\mathrm{Exact}}\left(\frac{\Delta_\rho}{2}, \frac{\Delta_\rho}{2}\right)\right]^2 + {z_0}^2\right\}^{1/2}$$

(factor 1.2 results from deeper error analysis)

rhoLUT properties

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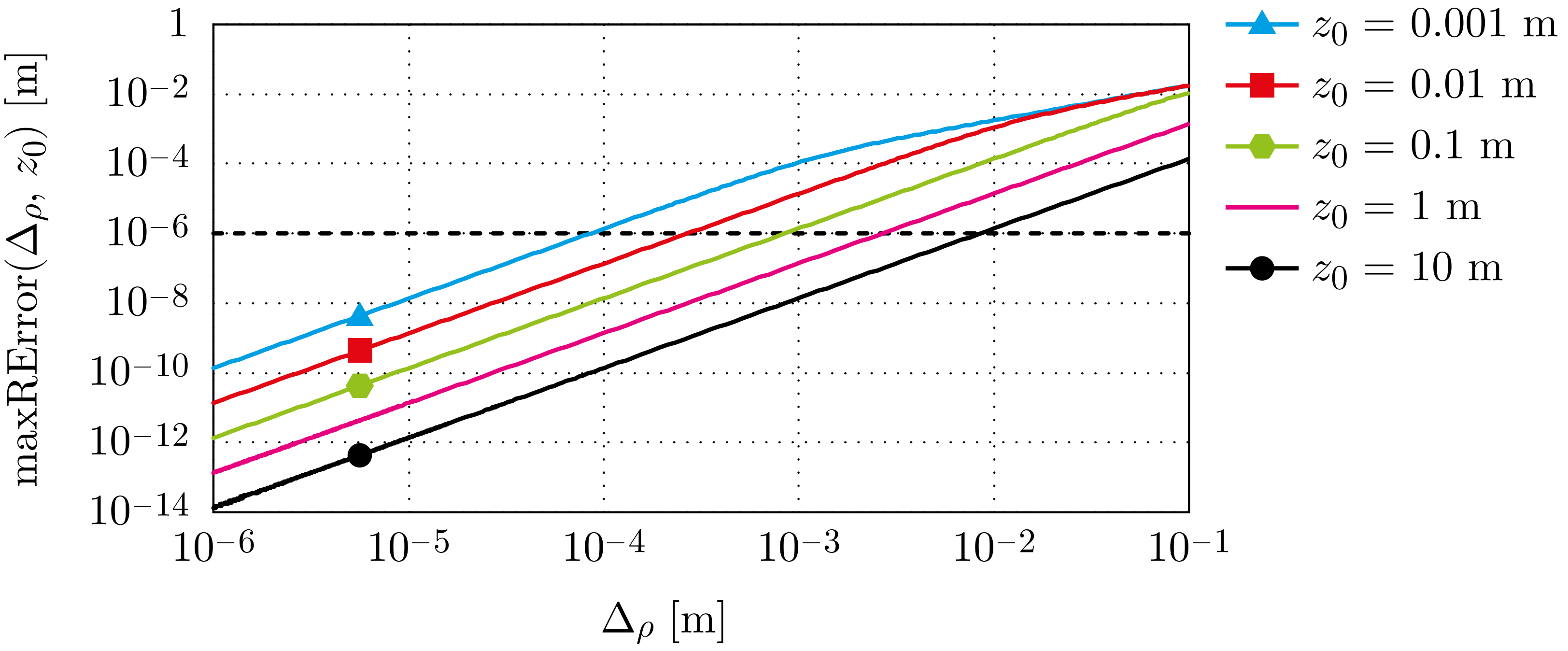
Selection of Δ_ρ

- maxRError looks like a line in a log-log graph
  for wide range of Δ_ρ, z_0 and acceptable error values



rhoLUT properties

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Selection of Δ_ρ

- linear approximation in the log-log graph leads to

$$\Delta_\rho \approx \exp\left\{\frac{1}{\kappa}\log\left[\frac{\text{maxRError}}{(z_0)^{\xi_1}\exp(\xi_0)}\right]\right\}$$

(where $\kappa \approx 1.9998$, $\xi_0 \approx -1.9886$, $\xi_1 \approx -0.9999$)

Example

propagation area 50×50 mm^2, $z_0 = 1$ m

- maxRError = $\lambda / 100 \Rightarrow$ rhoLUT size 264×264
- maxRError = $\lambda / 10 \Rightarrow$ rhoLUT size 84×84

waveLUT properties

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Definition and usage of the waveLUT

- $\text{waveLUT}[q; z_0] = K_{\text{RS}}(q\Delta_w; z_0)$

$$K_{\text{RS}}(\rho; z_0) \approx \text{waveLUT}[\rho/\Delta_w; z_0]$$

  - use nearest-neighbour or linear interpolation for noninteger ρ/Δ_w

- Δ_w should be small enough to capture oscillations of $K_{\text{RS}}(\rho; z_0)$
- calculate maximum local frequency of $K_{\text{RS}}(\rho; z_0)$
- set $\Delta_w \leq 1 / (8 \times \text{maximum local frequency})$

Results

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Accuracy

- look-up tables precalculated in double precision
- measured error according to theoretical analysis
- no problems with single precision calculations

Look-up tables sizes

- example values for typical scenario in computer generated holography (hologram size ≈ 5 × 5 cm, resolution 1 μm ⇒ hologram size 2.5 GB)
- rhoLUT size ≈ 500 × 500 samples (≈ 1 MB)
- waveLUT size ≈ 250 000 × Z samples (≈ 2Z MB)

Results

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CPU implemenation speed

- 10× to 200× faster than direct calculation for complicated filtered propagation kernels in realistic scenarios (just with waveLUT, ρ calculated directly)
- 1.7× faster than direct calculation for simple propagation kernel (just with waveLUT, ρ calculated directly)
- careful rhoLUT implementation 1.2–1.4× faster than calculating ρ directly

Results

GPU implemenation

- look-up tables precalculated on CPU in double precision
- incorporation of texture look-up and built-in texture interpolation
- no problems with GPU accuracy
- execution speed not yet fully tested

Work in progress (almost done)

- careful testing of GPU performance
- angular spectrum propagation method

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Thank you for your attention!

Feel free to ask me for
the manuscript of the full text and
scripts for detailed error analysis.

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