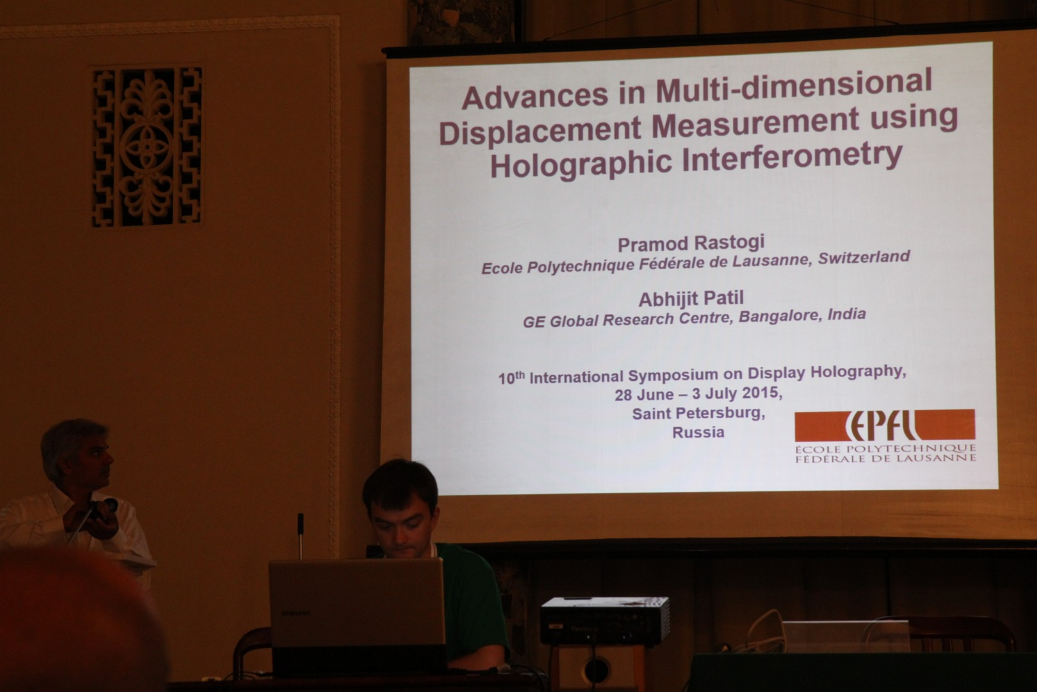
Advances in Multi-dimensional Displacement Measurement using Holographic Interferometry
Pramod Rastogi
Ecole Polytechnique Fédérale de Lausanne, Switzerland
Abhijit Patil
GE Global Research Centre, Bangalore, India
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Russia
EPFL
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Outline of Presentation

- Holographic interferometry
- Holographic moiré
- Novel approaches for multiple phase extraction
- Experimental verification
- Conclusion

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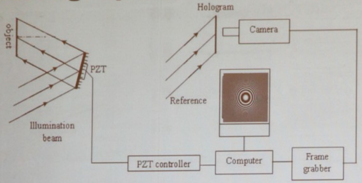
Holographic Interferometry



$$\varphi(n', j') = \frac{2\pi s_z \sin\theta}{\lambda}$$

$$I(n', j') = I_{dc}(n', j')\{1 + \gamma(n', j')\cos[\varphi(n', j')]\}$$

- Compares two different states of the same object
- Displacements, deformations, vibrations, shape change of rough surfaces such as concrete and steel

Important : Only one displacement component is obtained

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Why two displacements are important?

- Application in various fields such as fracture mechanics, bio-mechanics, model verification for large flawed structures, non-destructive evaluations, etc.
- Measurement of strain fields near stationary and growing cracks
- Crack tip opening displacement during crack growth
- Strain measurements near crack-tip at high temperatures
- Study of deformations in concrete during compressive loadings
- Sometimes out of plane motion can introduce significant errors in in-plane displacement

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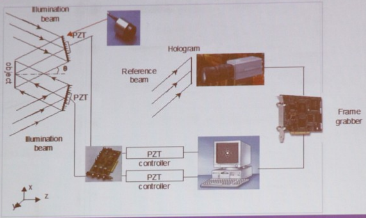
Holographic Moiré



$$I(n',j') = I_{dc}(n',j')\{1+\gamma_1(n',j')\cos[\varphi_1(n',j')]+\gamma_2(n',j')\cos[\varphi_2(n',j')]\}$$

$$\varphi_1(n',j')-\varphi_2(n',j') = \frac{2\pi s_x \sin\theta}{\lambda}$$

In-plane displacement

$$\varphi_1(n',j')+\varphi_2(n',j') = \frac{2\pi s_z(1+\cos\theta)}{\lambda}$$

out-of-plane displacement

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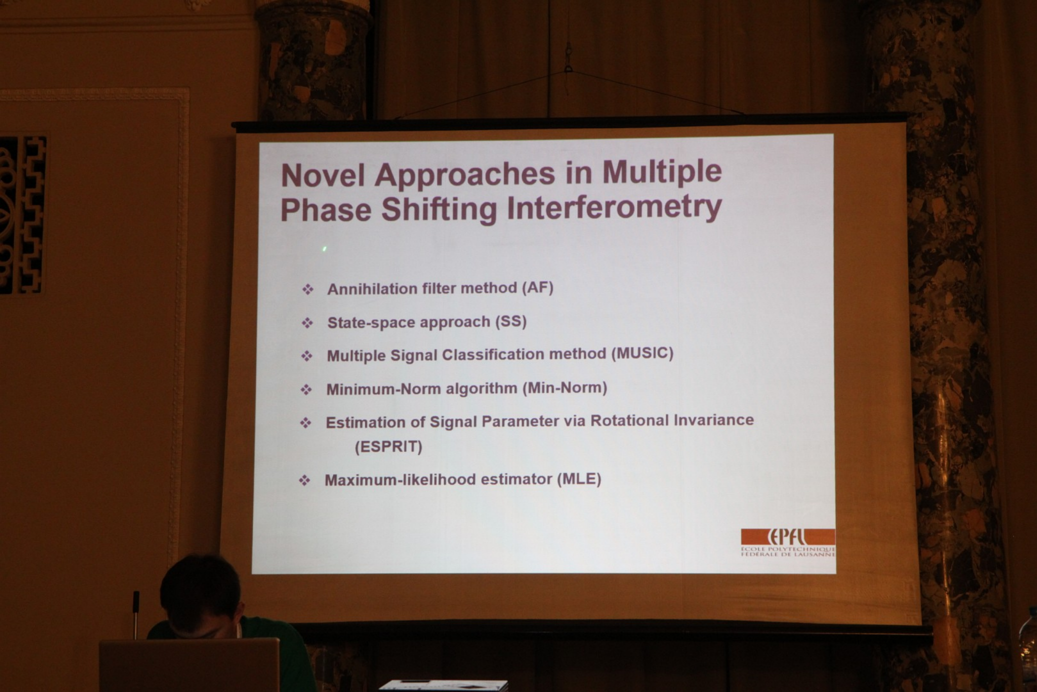
Novel Approaches in Multiple Phase Shifting Interferometry
Annihilation filter method (AF)
State-space approach (SS)
Multiple Signal Classification method (MUSIC)
Minimum-Norm algorithm (Min-Norm)
Estimation of Signal Parameter via Rotational Invariance (ESPRIT)
Maximum-likelihood estimator (MLE)
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High Resolution Approach – I

- Design of Annihilation Filter
  - The method draws parallelism between the frequencies present in the spectrum and the phase steps
  - Design an annihilation filter (a polynomial) which has zeros at the frequencies (phase steps)
- The algorithm extracts multiple phase steps in the presence of additive white Gaussian noise
- The denoising procedure is introduced to enhance the reliability in phase step extraction

Design of Annihilation Filter for Holographic Moiré

- **Interference equation for moiré fringes is given by**

$$I(n',j';n) = I_{dc} + \sum_{k=1}^{\kappa} a_{-k} \exp\left[-ik(\varphi_1 + n\alpha)\right] + \sum_{k=1}^{\kappa} a_k \exp\left[ik(\varphi_1 + n\alpha)\right] +$$

$$\sum_{k=1}^{\kappa} b_{-k} \exp\left[-ik(\varphi_2 + n\beta)\right] + \sum_{k=1}^{\kappa} b_k \exp\left[ik(\varphi_2 + n\beta)\right] + \eta \,; \quad \text{for } n = 0,1,2,\ldots\ldots,N-1$$

- **The equation is rewritten as**

$$I_n(n',j';n) = I_{dc} + \sum_{k=1}^{\kappa} \ell_k u_k^n + \sum_{k=1}^{\kappa} \ell_k^* \left(u_k^*\right)^n + \sum_{k=1}^{\kappa} \wp_k v_k^n + \sum_{k=1}^{\kappa} \wp_k^* \left(v_k^*\right)^n ,$$

$$\text{for } n = 0,1,2,\ldots\ldots,N-1$$

$$\ell_k = a_k \exp(ik\varphi_1),\; u_k = \exp(ik\alpha),\; \wp_k = b_k \exp(ik\varphi_2),\; v_k = \exp(ik\beta)$$

- **Frequencies present in intensity**

$0; \alpha, 2\alpha\ldots\ldots, \kappa\alpha; -\alpha, -2\alpha\ldots\ldots, -\kappa\alpha; \beta, 2\beta\ldots\ldots, \kappa\beta; -\beta, -2\beta\ldots\ldots, -\kappa\beta;$

Design of Annihilation Filter for Holographic Moiré

❖ **The Z-transform of intensity equation**

$$
\begin{aligned}
I(z) &= \sum_{n=0}^{N-1} I_n z^{-n} \\
&= \sum_{n=0}^{N-1} I_{dc} z^{-n} + \sum_{n=0}^{N-1}\sum_{k=1}^{\kappa} \ell_k u_k^{(n)} z^{-n} + \sum_{n=0}^{N-1}\sum_{k=1}^{\kappa} \ell_k^{*} \left(u_k^{*}\right)^{(n)} z^{-n} \\
&\quad + \sum_{n=0}^{N-1}\sum_{k=1}^{\kappa} \wp_k v_k^{(n)} z^{-n} + \sum_{n=0}^{N-1}\sum_{k=1}^{\kappa} \wp_k^{*} \left(v_k^{*}\right)^{(n)} z^{-n}
\end{aligned}
$$

❖ **The polynomial (annihilation filter)**

$$
\begin{aligned}
P(z) &= \left(1 - z^{-1}\right)\prod_{k=1}^{\kappa}\left(1 - e^{i\alpha_k} z^{-1}\right)\left(1 - e^{-i\alpha_k} z^{-1}\right)\left(1 - e^{i\beta_k} z^{-1}\right)\left(1 - e^{-i\beta_k} z^{-1}\right) \\
&= \sum_{k=0}^{4\kappa+1} p_k z^{-k}
\end{aligned}
$$

A. Patil, R. Langoju, and P. Rastogi, Optics Express 12, 4681-4697 (2004).
A. Patil, R. Langoju, and P. Rastogi, Optics Letters 30, 391-393 (2005).

Design of Annihilation Filter for Holographic Moiré

❖ The discrete convolution of intensity fringes and polynomial is given by

$$\sum_{k=0}^{4\kappa+1} I_{n-k}\, P_k = D_n \quad \text{for } \{n = 0,1,2,\ldots\ldots,N-1\}$$

$$\begin{bmatrix} I_0 & 0 & 0 & \cdot & \cdot & 0 \\ I_1 & I_0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hdashline I_{4\kappa+1} & I_{4\kappa} & \cdot & \cdot & \cdot & I_0 \\ I_{4\kappa+2} & I_{4\kappa+1} & \cdot & \cdot & \cdot & I_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ I_{N-2} & I_{N-3} & \cdot & \cdot & \cdot & I_{N-4\kappa-3} \\ I_{N-1} & I_{N-2} & \cdot & \cdot & \cdot & I_{N-4\kappa-2} \\ \hdashline 0 & I_{N-1} & I_{N-2} & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & I_{N-1} & I_{N-2} \\ 0 & 0 & 0 & 0 & 0 & I_{N-1} \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_{4\kappa+1} \end{bmatrix} = \begin{bmatrix} D_0 \\ D_1 \\ \cdot \\ D_{4\kappa} \\ \hdashline 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ \hdashline D_N \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ D_{N+4\kappa} \end{bmatrix}$$

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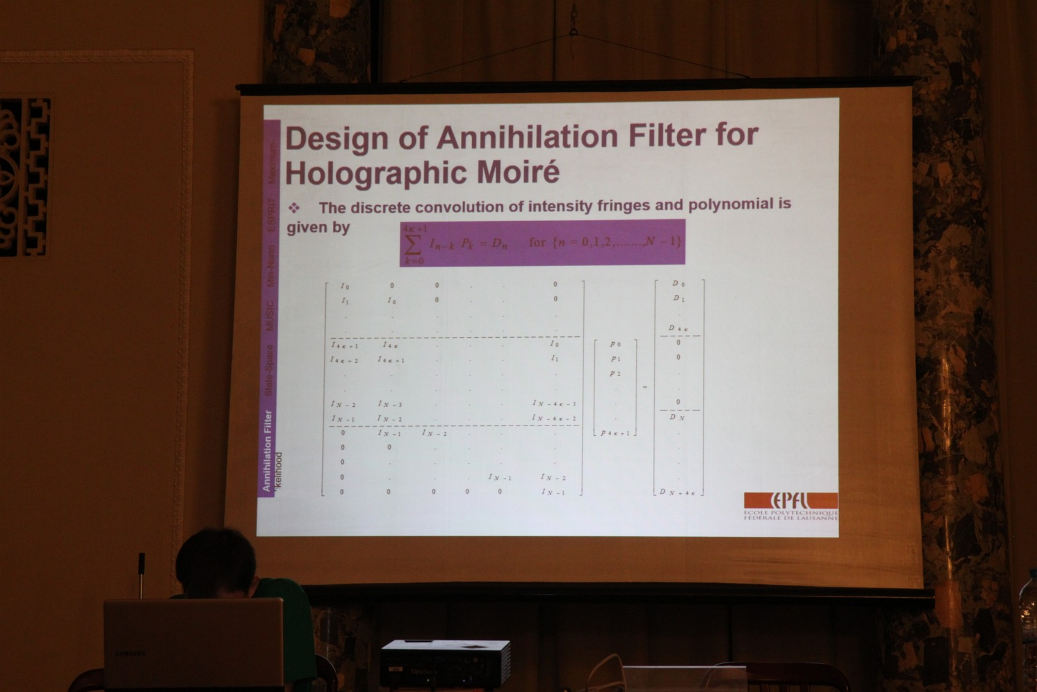
Design of Annihilation Filter for Holographic Moiré
The discrete convolution of intensity fringes and polynomial is given by
$\sum_{k=0}^{4\kappa+1} I_{n-k} P_k = D_n$ for $\{n = 0,1,2,\ldots\ldots,N-1\}$
Maximum-Likelihood
ESPRIT
Min-Norm
MUSIC
State-Space
Annihilation Filter
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Design of Annihilation Filter for Holographic Moiré

❖ Selecting only the central rows corresponding to zero on right hand side of matrix gives

$$\sum_{k=0}^{4\kappa+1} I_{n-k}\, P_k = 0 \qquad \text{for } \{n=4\kappa+1,\ 4\kappa+2,\ldots\ldots,N-1\}$$

$$\underbrace{\begin{bmatrix} I_{4\kappa+1} & I_{4\kappa} & \cdot & \cdot & \cdot & I_0 \\ I_{4\kappa+2} & I_{4\kappa+1} & \cdot & \cdot & \cdot & I_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ I_{N-2} & I_{N-4} & \cdot & \cdot & \cdot & I_{N-4\kappa-3} \\ I_{N-1} & I_{N-3} & \cdot & \cdot & \cdot & I_{N-4\kappa-2} \end{bmatrix}}_{\mathbf{R}} \underbrace{\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_{4\kappa+1} \end{bmatrix}}_{\mathbf{P}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

❖ The phase steps can be computed from roots of polynomial P(*z*)

$\alpha = \Re(\ln u_1 / i)$ $\beta = \Re(\ln v_1 / i)$

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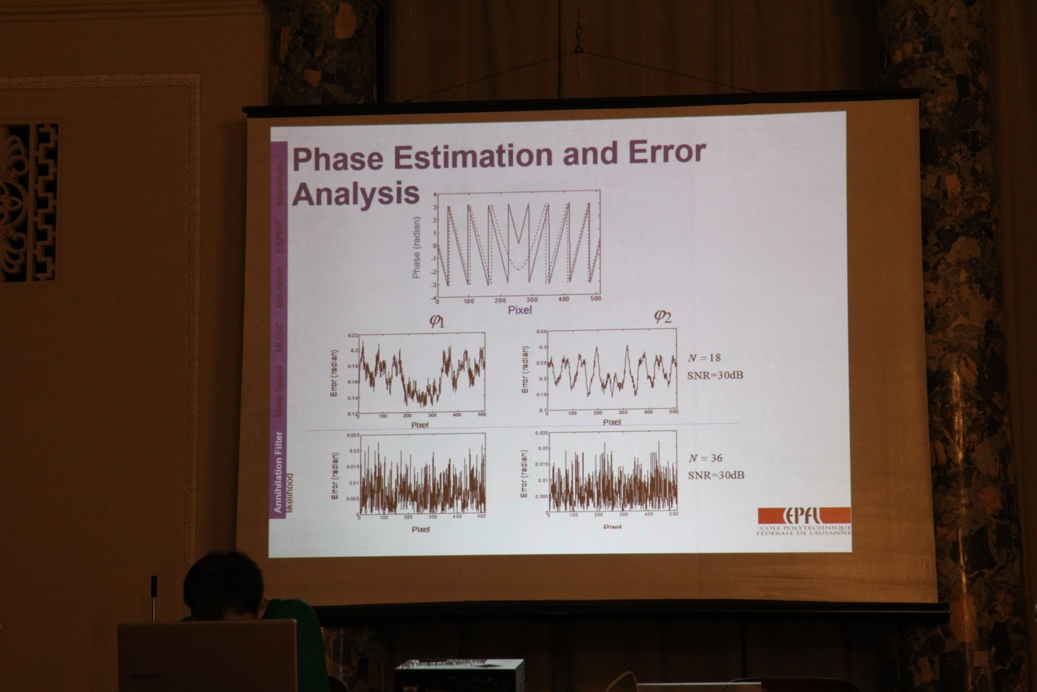

Phase Estimation and Error Analysis
Maximum-likelihood
ESPRIT
Min-Norm
MUSIC
State-Space
Annihilation Filter
Phase (radian)
Pixel
φ1
φ2
Error (radian)
Pixel
N = 18
SNR=30dB
N = 36
SNR=30dB
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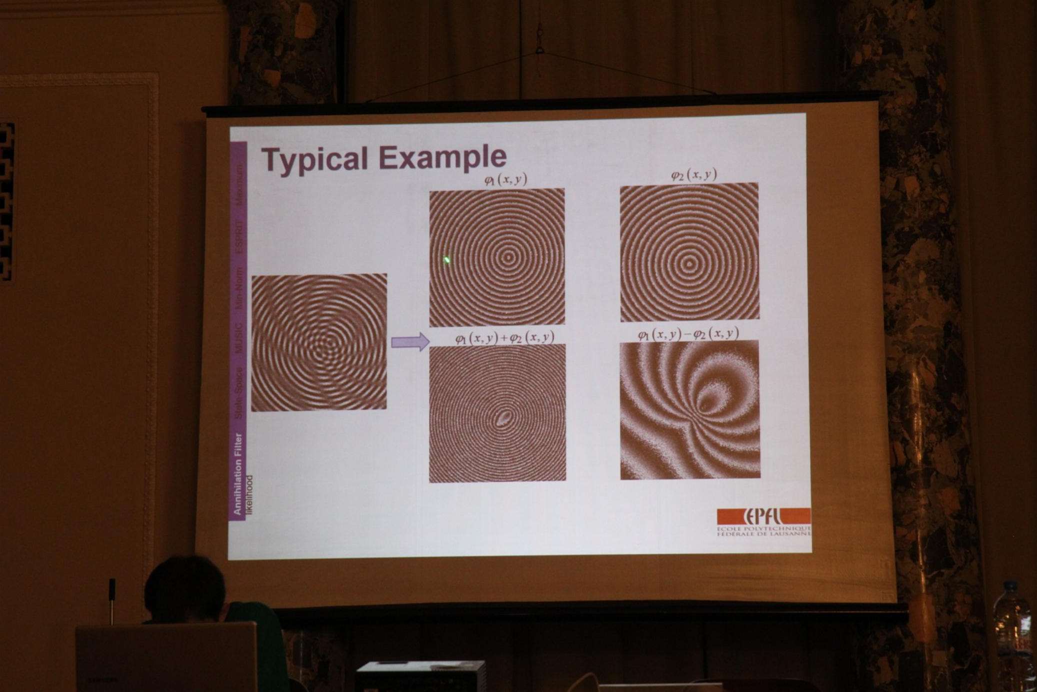

Typical Example
$\varphi_1(x,y)$
$\varphi_2(x,y)$
$\varphi_1(x,y)+\varphi_2(x,y)$
$\varphi_1(x,y)-\varphi_2(x,y)$
Annihilation Filter
State Space
MUSIC
Min-Norm
ESPRIT
Maximum likelihood
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Comments on Annihilation Filter Technique

Annihilation Filter | State-Space | MUSIC | Min-Norm | ESPRIT | Maximum-likelihood

Salient features

- Identifies multiple phases in the presence of noise
- Handles non-sinusoidal waveform
- Allows converging as well as diverging beams
- Arbitrary phase steps can be imparted
- Phase steps are estimated in real time
- Denoising procedure adds robustness in phase estimation

Concerns !

- Requires a denoising procedure
- Works directly on the signal data
- Needs considerable number of data frames

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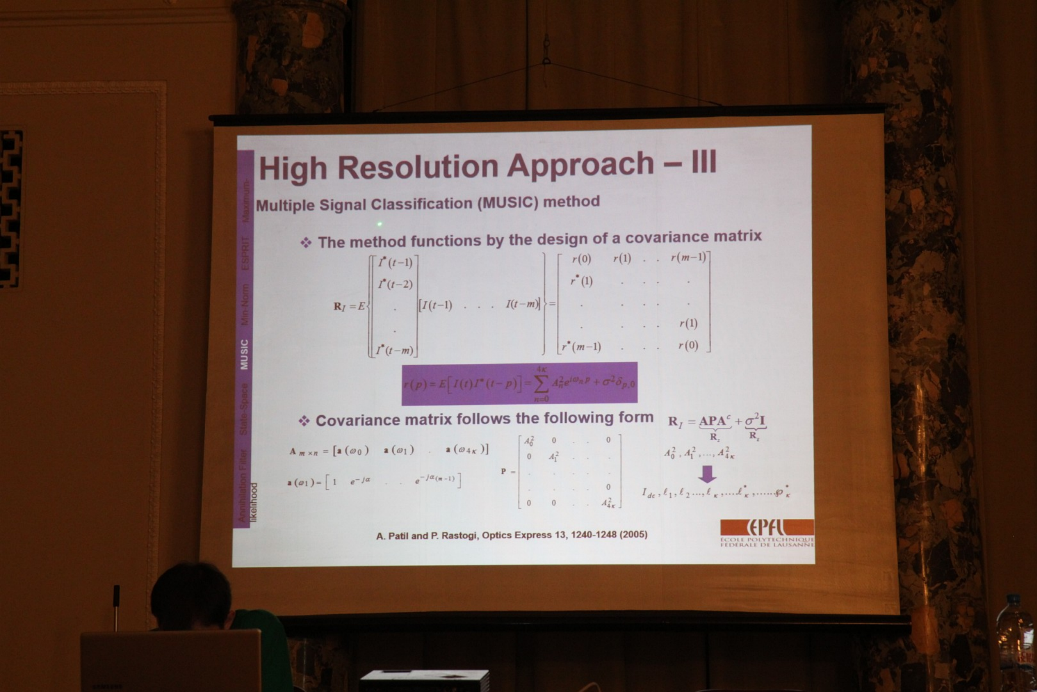

High Resolution Approach – III
Multiple Signal Classification (MUSIC) method
Maximum-likelihood
ESPRIT
Min-Norm
MUSIC
State-Space
Annihilation Filter
❖ The method functions by the design of a covariance matrix
$\mathbf{R}_I = E\left\{\begin{bmatrix} I^*(t-1) \\ I^*(t-2) \\ . \\ . \\ I^*(t-m) \end{bmatrix} \begin{bmatrix} I(t-1) & . & . & . & I(t-m) \end{bmatrix}\right\} = \begin{bmatrix} r(0) & r(1) & . & . & r(m-1) \\ r^*(1) & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & r(1) \\ r^*(m-1) & . & . & . & r(0) \end{bmatrix}$
$r(p) = E\left[I(t)I^*(t-p)\right] = \sum_{n=0}^{4\kappa} A_n^2 e^{i\omega_n p} + \sigma^2 \delta_{p,0}$
❖ Covariance matrix follows the following form
$\mathbf{R}_I = \underbrace{\mathbf{APA}^c}_{\mathbf{R}_s} + \underbrace{\sigma^2\mathbf{I}}_{\mathbf{R}_z}$
$\mathbf{A}_{m\times n} = [\mathbf{a}(\omega_0) \quad \mathbf{a}(\omega_1) \quad . \quad \mathbf{a}(\omega_{4\kappa})]$
$\mathbf{a}(\omega_1) = [1 \quad e^{-j\alpha} \quad . \quad . \quad e^{-j\alpha(m-1)}]$
$\mathbf{P} = \begin{bmatrix} A_0^2 & 0 & . & . & 0 \\ 0 & A_1^2 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 0 \\ 0 & 0 & . & . & A_{4\kappa}^2 \end{bmatrix}$
$A_0^2, A_1^2, \ldots, A_{4\kappa}^2$
$I_{dc}, \ell_1, \ell_2 \ldots, \ell_\kappa, \ldots \ell_\kappa^*, \ldots \ldots \wp_\kappa^*$
A. Patil and P. Rastogi, Optics Express 13, 1240-1248 (2005)
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High Resolution Approach – III

Multiple Signal Classification (MUSIC) method

- The signal and noise spaces are separated by performing the singular value decomposition of $\mathbf{R}_I$

$\lambda_1 \geq \lambda_2 \geq .....\lambda_{4K+1} \geq \sigma^2$	$\lambda_{4K+2} \geq \lambda_{4K+3} \geq .....\lambda_m \approx \sigma^2$

- The orthonormal eigen vectors associated with $\lambda_1 \geq \lambda_2 \geq .....\lambda_{4K+1}$ is $\mathbf{S}_{m\times n} = [\mathbf{s}_1, \mathbf{s}_2, ... \mathbf{s}_{4K+1}]$: Signal subspace

- The orthonormal eigen vectors associated with $\lambda_{4K+2} \geq \lambda_{4K+3} \geq .....\lambda_m$ is $\mathbf{G}_{m\times(m-n)} = [\mathbf{g}_1, \mathbf{g}_2, ... \mathbf{g}_{m-4K+1}]$: Noise subspace

- MUSIC uses the fact that the noise subspace is orthogonal to the $\{\mathbf{a}(\omega_k)\}_{k=0}^{n}$

$\sum_{k=m-n}^{m} \mathbf{a}^c(\omega)\,\mathbf{G}_k = 0$	$\mathbf{a}^T(\omega)\mathbf{G}\mathbf{G}^c\mathbf{a}(\omega) = \left\| \mathbf{G}^c\mathbf{a}(\omega) \right\|^2 = 0$ for $m > n$

Maximum-likelihood | ESPRIT | Min-Norm | MUSIC | State-Space | Annihilation Filter

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High Resolution Approach – III

Maximum-likelihood
ESPRIT
Min-Norm
MUSIC
State-Space
Annihilation Filter

Design of a sample covariance matrix

- **Forward approach**

$$\hat{\mathbf{R}}_I = \frac{1}{N}\sum_{t=m}^{N}\begin{bmatrix} I^*(t-1) \\ I^*(t-2) \\ \vdots \\ I^*(t-m)\end{bmatrix}\begin{bmatrix} I(t-1) & I(t-2) & \cdots & I(t-m)\end{bmatrix}$$

- **Forward backward approach**

$$\hat{\mathbf{R}}_I = \frac{1}{2N}\sum_{t=m}^{N}\left(\begin{bmatrix} I^*(t-1) \\ I^*(t-2) \\ \vdots \\ I^*(t-m)\end{bmatrix}\begin{bmatrix} I(t-1) & I(t-2) & \cdots & I(t-m)\end{bmatrix} + \begin{bmatrix} I^*(t-m) \\ \vdots \\ I^*(t-2) \\ I^*(t-1)\end{bmatrix}\begin{bmatrix} I(t-m) & \cdots & I(t-2) & I(t-1)\end{bmatrix}\right)$$

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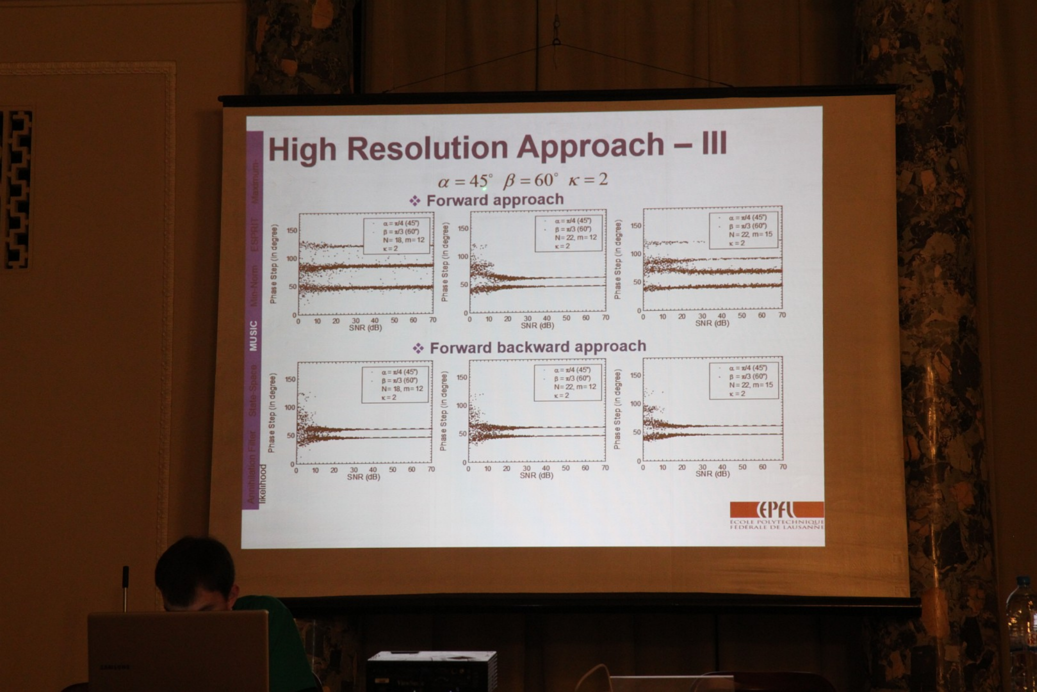

High Resolution Approach – III
$\alpha = 45^\circ \quad \beta = 60^\circ \quad \kappa = 2$
Forward approach
α = π/4 (45°)
β = π/3 (60°)
N= 18, m= 12
κ = 2
α = π/4 (45°)
β = π/3 (60°)
N= 22, m= 12
κ = 2
α = π/4 (45°)
β = π/3 (60°)
N= 22, m= 15
κ = 2
Phase Step (in degree)
SNR (dB)
Forward backward approach
α = π/4 (45°)
β = π/3 (60°)
N= 18, m= 12
κ = 2
α = π/4 (45°)
β = π/3 (60°)
N= 22, m= 12
κ = 2
α = π/4 (45°)
β = π/3 (60°)
N= 22, m= 15
κ = 2
Phase Step (in degree)
SNR (dB)
Maximum-likelihood
ESPRIT
Min-Norm
MUSIC
State-Space
Annihilation Filter
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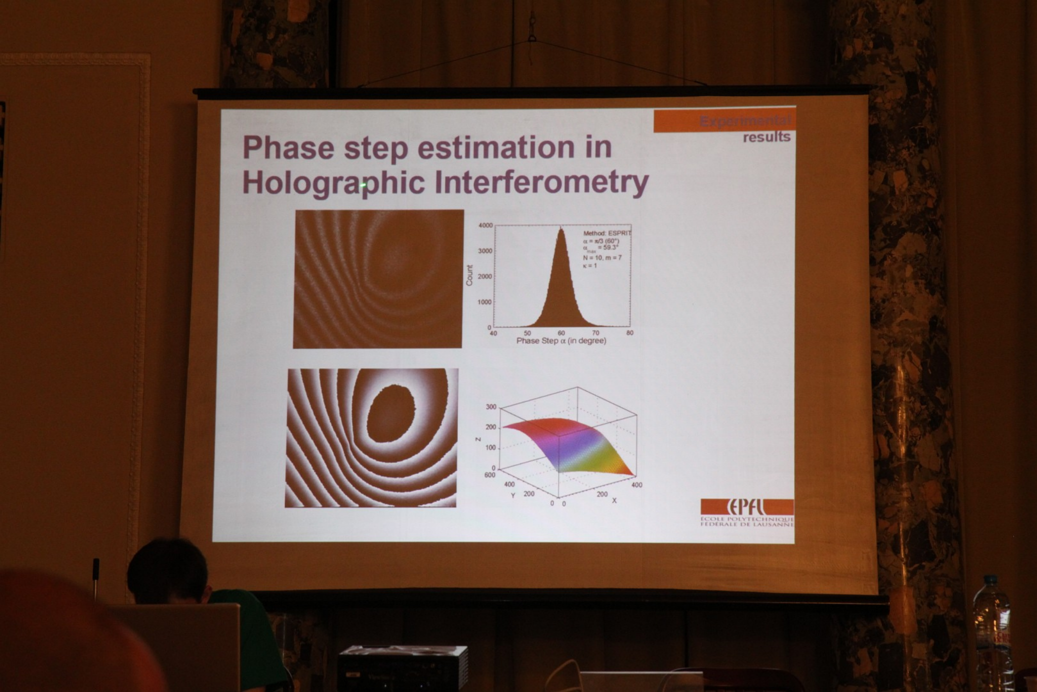
Experimental
results
Phase step estimation in
Holographic Interferometry
Method: ESPRIT
α = π/3 (60°)
α_max = 59.3°
N = 10, m = 7
κ = 1
Count
Phase Step α (in degree)
4000
3000
2000
1000
0
40
50
60
70
80
300
200
100
0
Z
600
400
200
0
Y
0
200
400
X
EPFL
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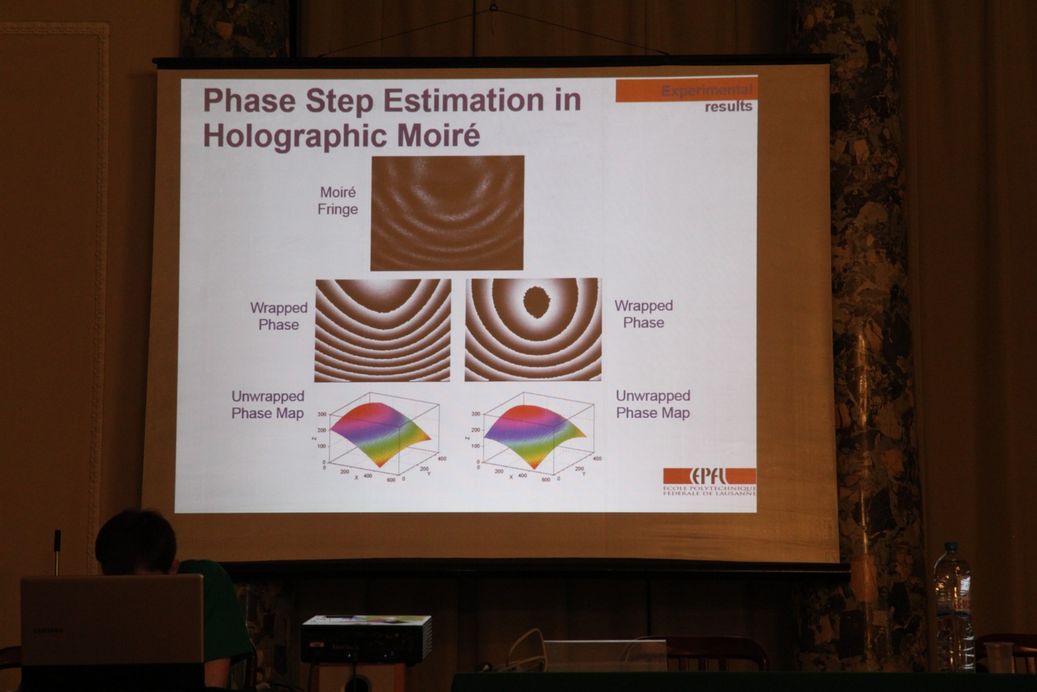

Phase Step Estimation in Holographic Moiré
Experimental results
Moiré Fringe
Wrapped Phase
Wrapped Phase
Unwrapped Phase Map
Unwrapped Phase Map
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Concluding Remarks

Salient features

- **High resolution approach is applied to optical metrology**
- **Multiple wholefield displacement components can be measured simultaneously using temporal techniques**
- **Multiple PZTs can be allowed in an optical configuration**
- **Practical implementation of the concept has been demonstrated**

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