FACULTY
OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

DEPARTMENT OF
COMPUTER SCIENCE
AND ENGINEERING

CENTRE
OF COMPUTER GRAPHICS
AND VISUALIZATION

PLZEŇ
CZECH REPUBLIC

http://graphics.zcu.cz

COMPUTER GENERATED HOLOGRAPHY

3D VISION AND BEYOND

Petr Lobaz

University of West Bohemia

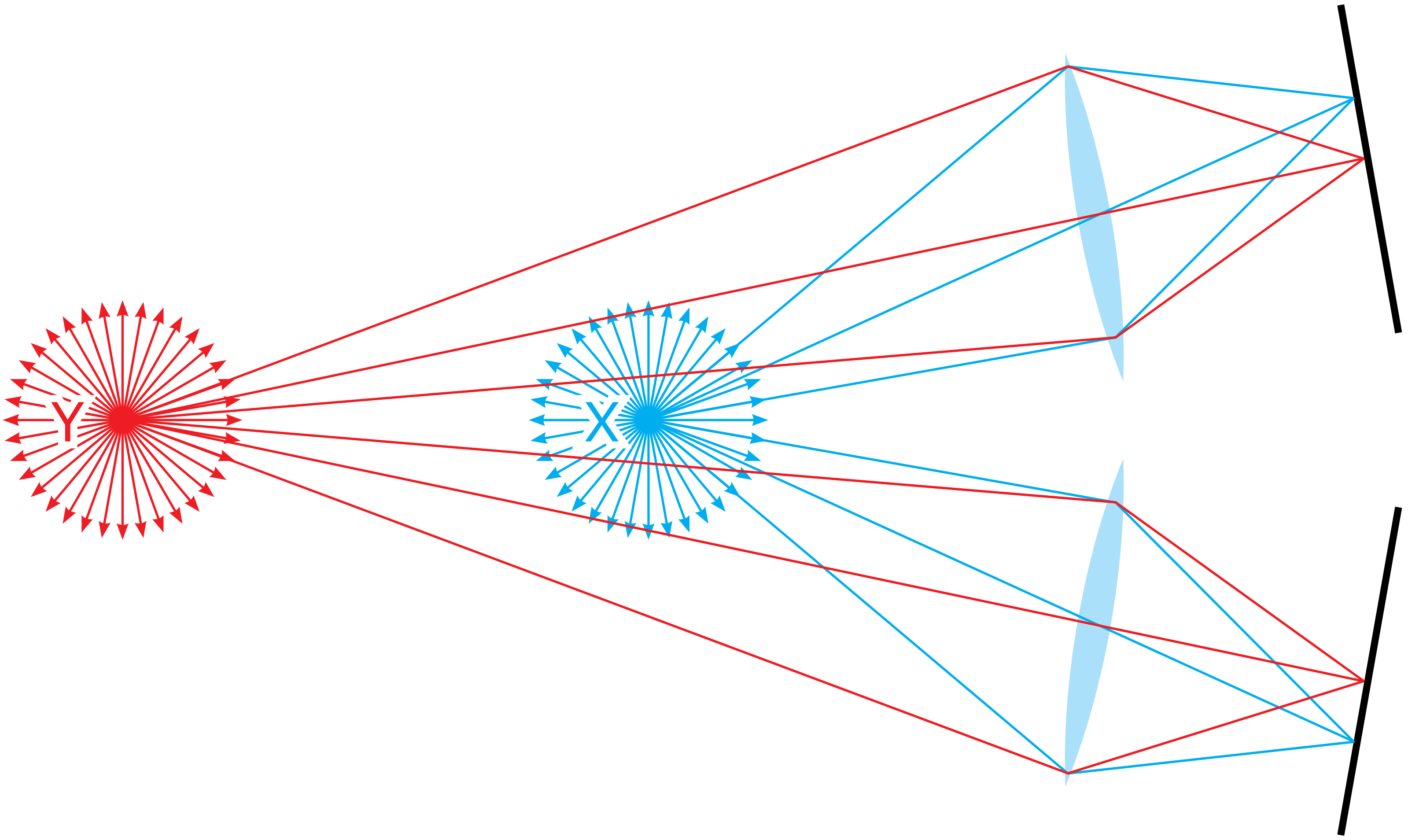
Pilsen, Czech republic

lobaz@kiv.zcu.cz

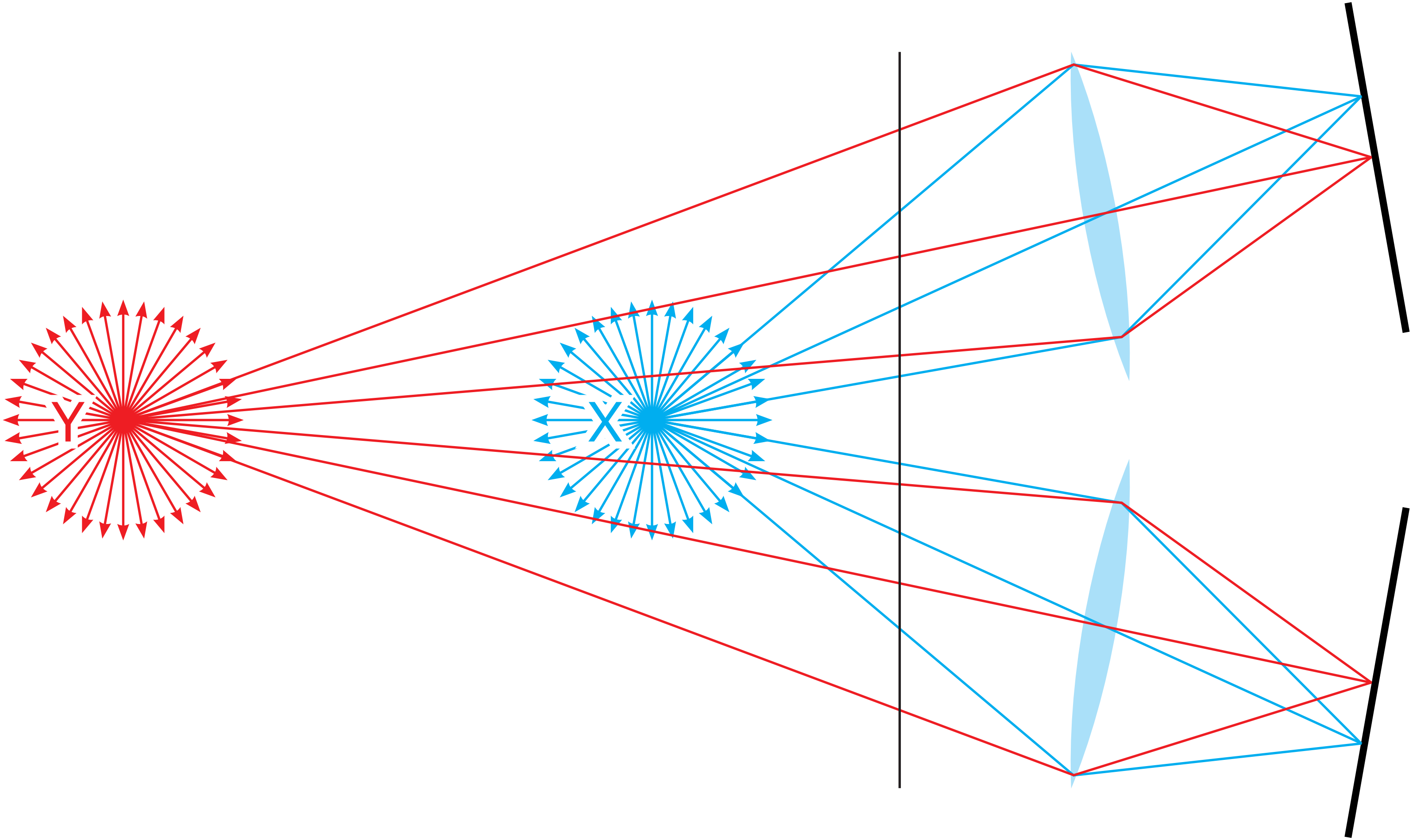
Plzeň
September 2013

Contents

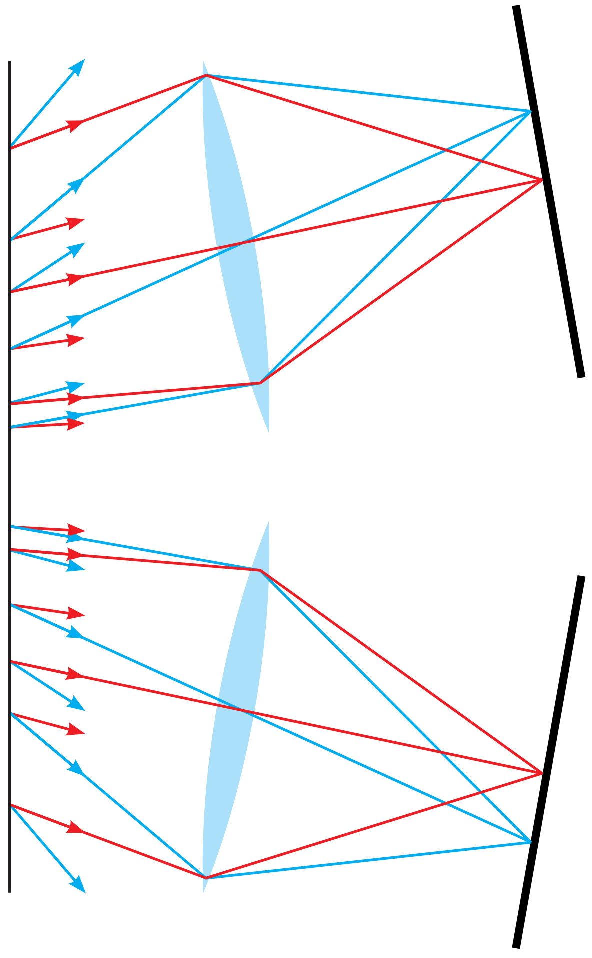
3D image



3D image

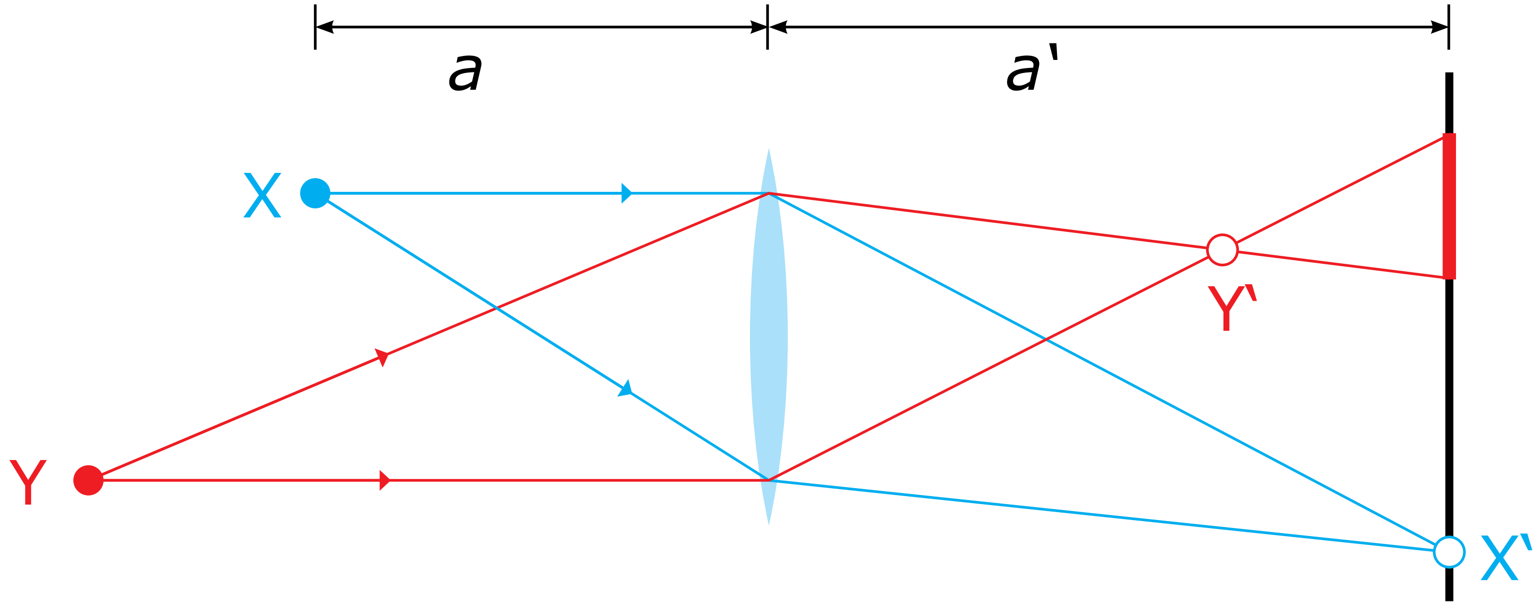


3D image



Thin lens formula

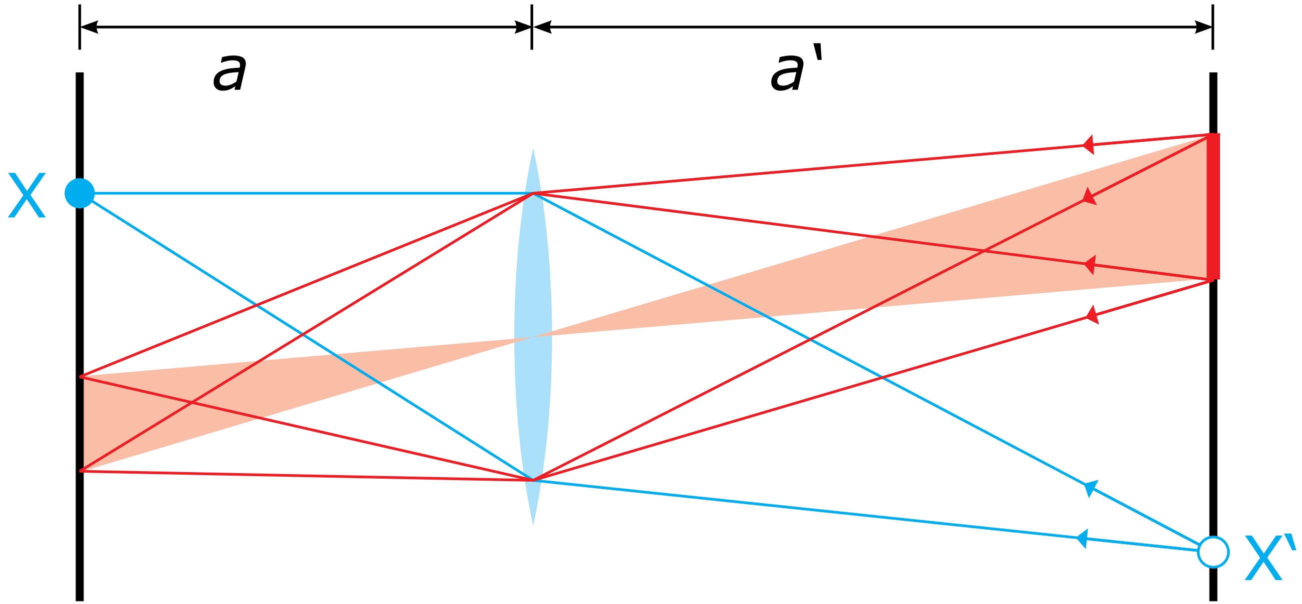
$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}$$



Photography

Original points reconstruction

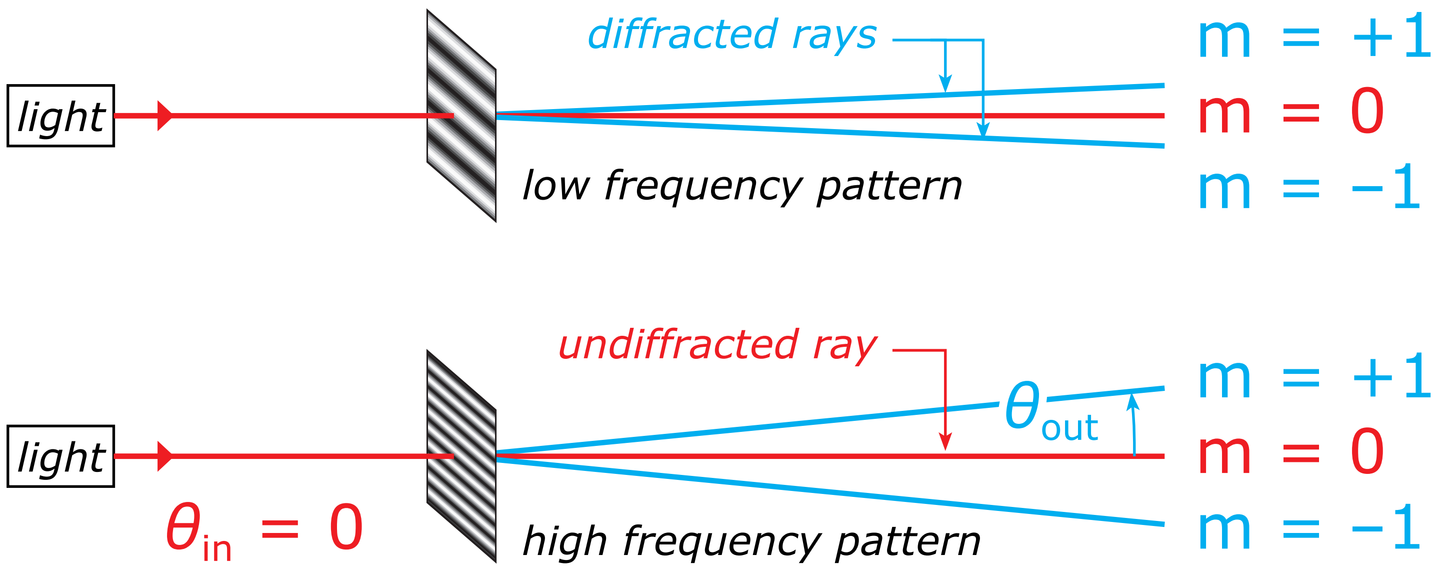
- perfect for points in focus only
- loss of information



Hologram principle

Light diffraction

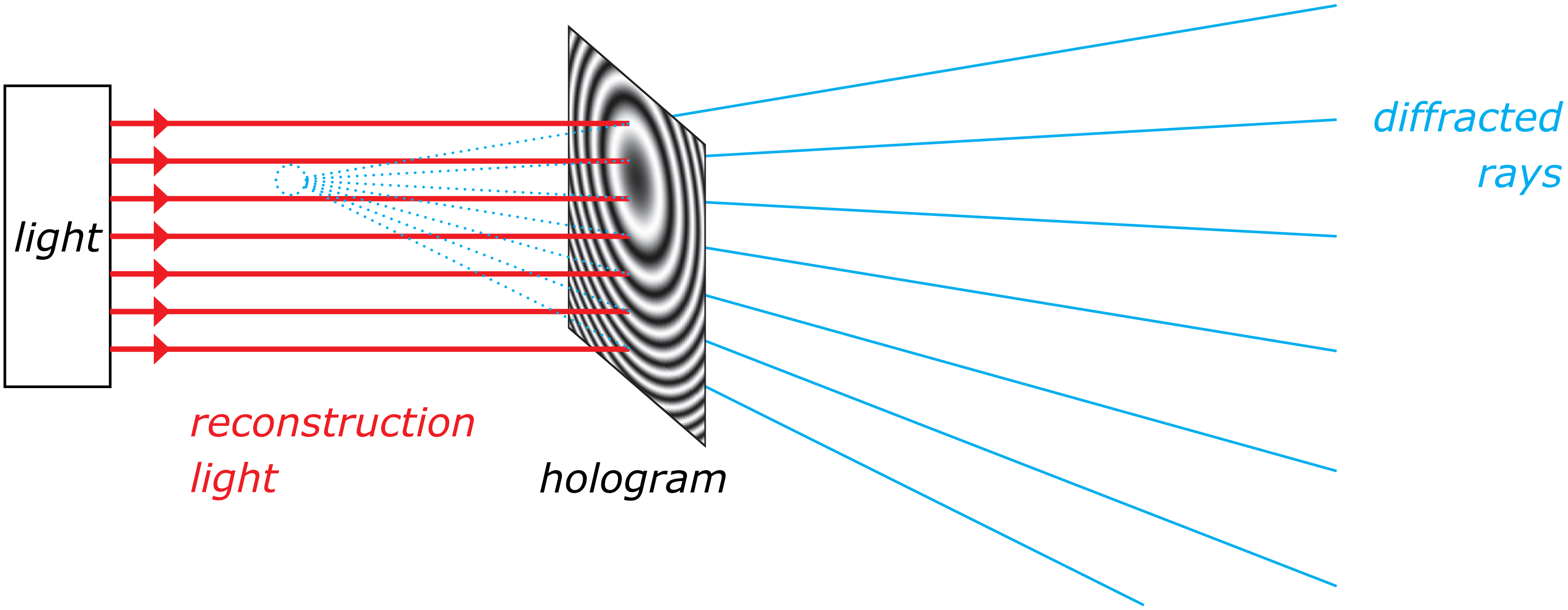
- depends mainly on frequency f of the pattern
  output angle of the rays: $\sin\theta_{out} = m\lambda f + \sin\theta_{in}$



Hologram principle

Hologram watching

- illuminate hologram with a light source
- light beams start to diffract on the pattern as if the original object was still present



Nature of the light

- force interaction between (oscillating) point charges
- point source of a light:
  movement up and down $\sim A\cos(\omega t - \varphi)$
- force (field) in a distance r:
  $$u(t, r) = \frac{A}{r}\cos(\omega[t - \frac{r}{c}] - \varphi) = A' \cos(\omega t - \varphi'(r))$$
- photographic emulsion reacts on intensity: $(A')^2$

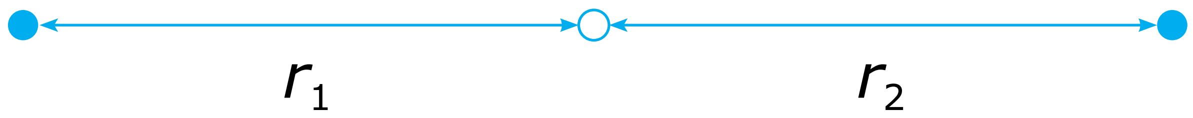
⇒ cannot tell close “darker light” from distant “brighter light”

Interference

$$\frac{A}{r_1}\cos(\omega[t - \frac{r_1}{c}]) \qquad + \qquad \frac{A}{r_2}\cos(\omega[t - \frac{r_2}{c}])$$

- T period of oscillation $1.7 \cdot 10^{-15}$ s
- $\omega = 2\pi/T$ angular frequency
- $f = 1/T$ frequency
- c speed of the light
- $\lambda = cT$ wavelength $0.5 \cdot 10^{-6}$ m
- $k = 2\pi/\lambda$ wavenumber $1.2 \cdot 10^{7}\ \mathrm{m}^{-1}$

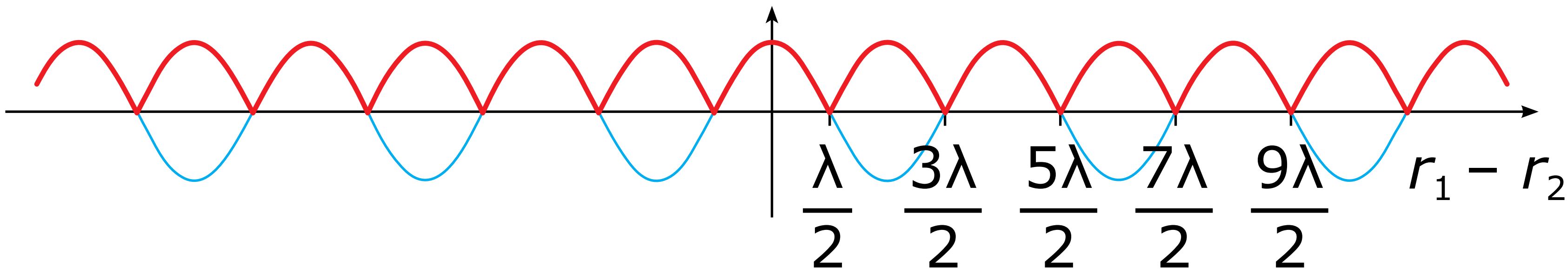
Interference



$$r_1 \approx r_2$$

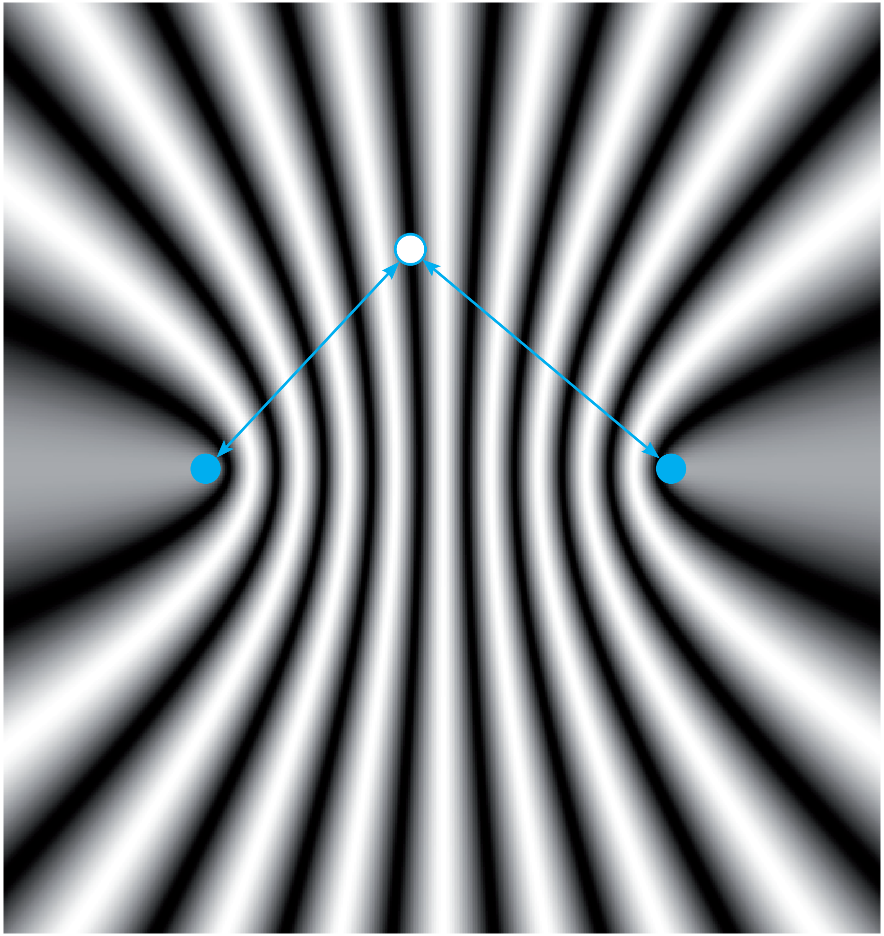
$$\frac{A}{r_1}\cos(\omega t - kr_1) \quad + \quad \frac{A}{r_2}\cos(\omega t - kr_2)$$

$$\approx 2\,\frac{A}{r_1}\cos\left(\frac{k(r_1 - r_2)}{2}\right)\cos\left(\frac{2\omega t - k(r_1 + r_2)}{2}\right)$$



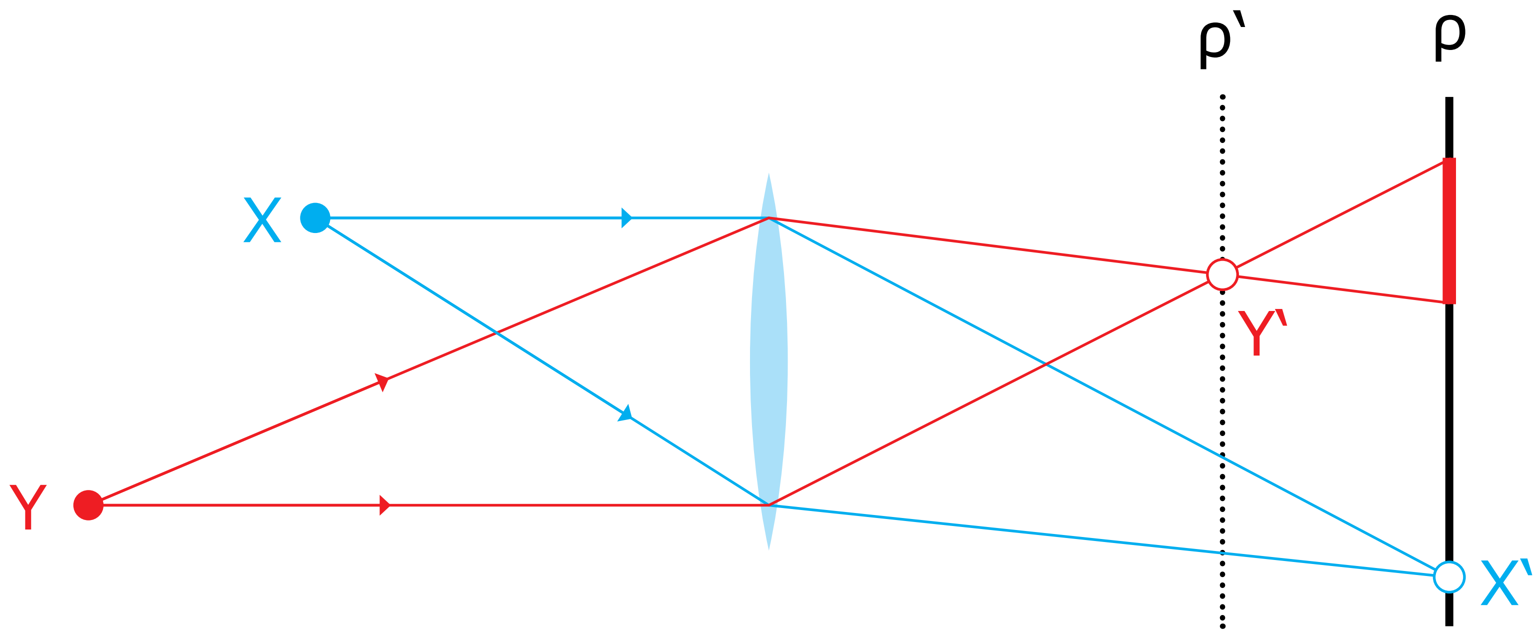
Interference

- constructive
  ×
  destructive
  interference



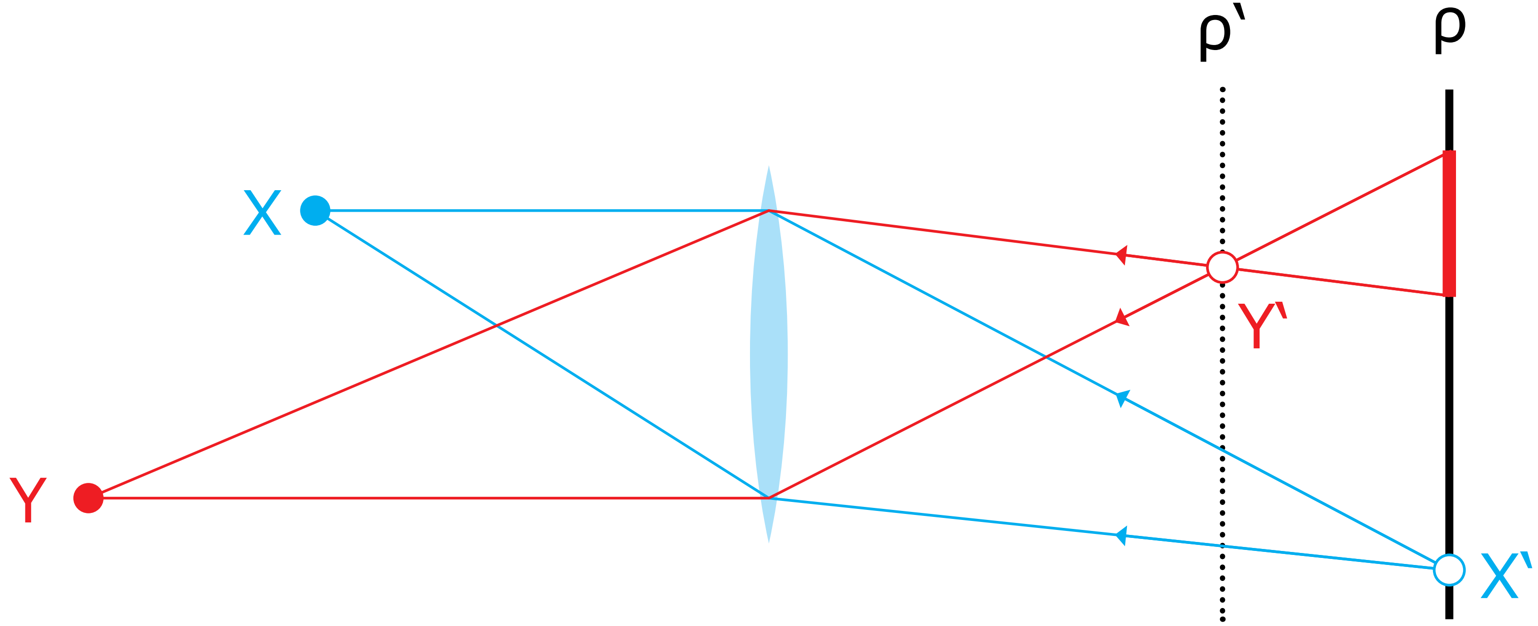
Perfect picture

- image of X in ρ: amplitude 0 except of X‵ (→∞)
- image of Y in ρ‵: amplitude 0 except of Y‵ (→∞)
- image of Y in ρ: amplitude and phase from Y‵



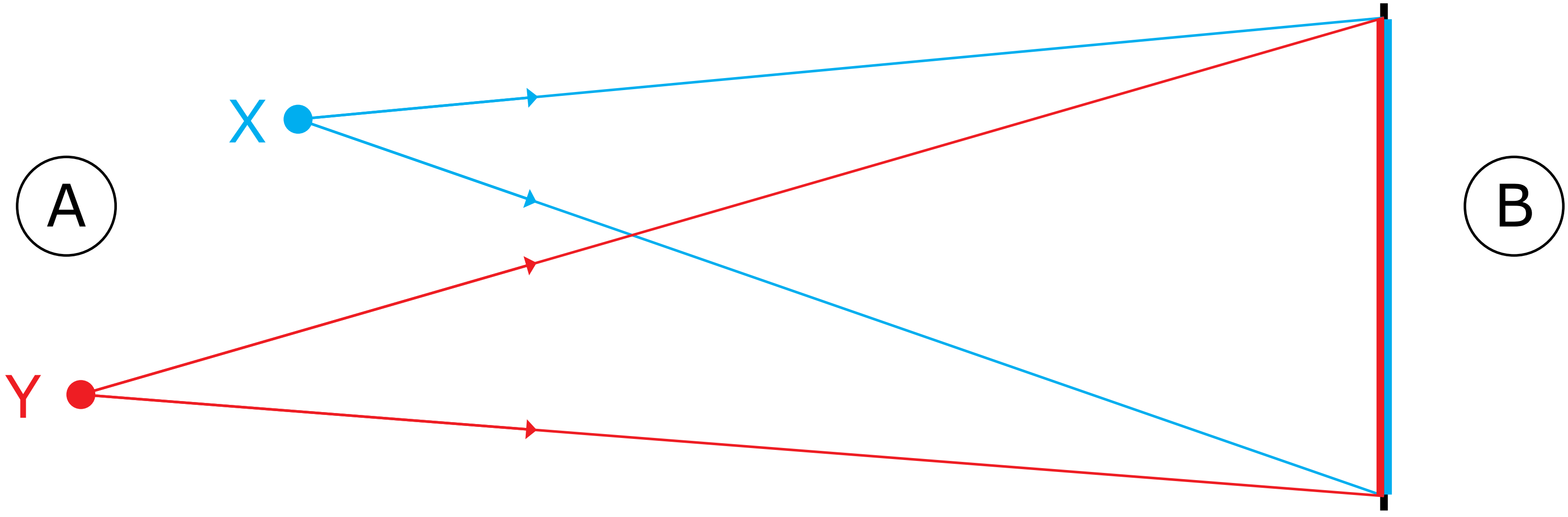
Perfect picture

- reconstruction of X‘: point X
- reconstruction of “blurry” Y‘: constructive interference in Y‘
  ⇒ reconstruction of Y



Perfect picture

- phase is critical for 3D image – how to capture it?
- no need for a lens anymore
- observation from A: pseudoscopic image
- observation from B: orthoscopic image



Complex notation

- $j^2 = -1$
- $e^{jx} = \cos x + j \sin x$
- $A \cos(\omega t - \varphi) = \mathrm{Re}\{A\, e^{j(\omega t - \varphi)}\}$

- $e^{jx} + e^{jy} = 2 \cos\left(\frac{x - y}{2}\right) \exp\left(j \frac{x + y}{2}\right)$
- $e^{jx} + e^{-jx} = 2 \cos x$

- intensity of $U = A\, e^{j(\omega t - \varphi)}$
  $|U|^2 = U U^* = A\, e^{j(\omega t - \varphi)} A\, e^{-j(\omega t - \varphi)} = A^2$

Complex notation

Advantage of phasor arithmetic

- optical fied – time dependent function:
  $u(t, r) = A \cos(\omega t - \varphi(r))$
- its phasor (complex amplitude):
  $U(r) = A \exp(-\mathrm{j}\varphi(r))$
- sum of optical fields:
  $A_1 \cos(\omega t - \varphi_1(r)) + A_2 \cos(\omega t - \varphi_2(r)) + \cdots = ?$
- in phasor arithmetic:
  $\mathrm{A}_1 \exp(-\mathrm{j}\varphi_1(r)) + \mathrm{A}_2 \exp(-\mathrm{j}\varphi_2(r)) + \cdots = U_{\mathrm{sum}}(r)$
- optical field (if needed):
  $\mathrm{u}_{\mathrm{sum}}(t, r) = \mathrm{Re}\{U_{\mathrm{sum}}(r)\, \mathrm{e}^{\mathrm{j}\omega t}\}$

Basic wavefront shapes

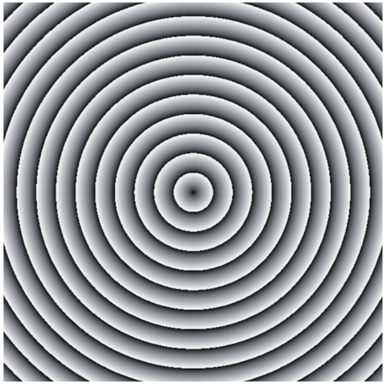
Spherical wavefront

- $u(t, r) = \frac{A}{r} \exp(\mathrm{j}[\omega t - kr - \varphi])$

  $= \frac{A}{r} \exp(\mathrm{j}\omega t) \exp(-\mathrm{j}[kr + \varphi])$
- complex amplitude:

  $U(r) = \frac{A}{r} \exp(-\mathrm{j}[kr + \varphi])$
- resembles a plane
  in a big distance

amplitude



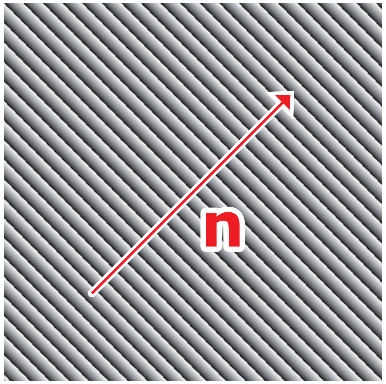
phase

Basic wavefront shapes

Plane wavefront

- wavefront normal $\mathbf{n}$, $|\mathbf{n}| = 1$
- wavefronts period λ
- wave vector $\mathbf{k} = k\,\mathbf{n} = 2\pi/\lambda\,\mathbf{n}$
- point in space $\mathbf{x} = (x, y, z)$
- wavefront plane equation
  $\mathbf{k} \cdot \mathbf{x} = \mathit{const.}$
- $U(\mathbf{x}) = A \exp(-\mathrm{j}[\mathbf{k} \cdot \mathbf{x} + \varphi])$
- rays: "directions perpendicular to wavefronts"

amplitude

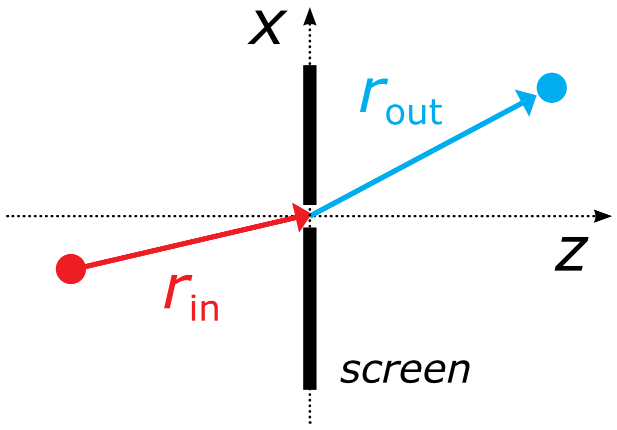


phase

Single slit diffraction

- complex amplitude at the slit of almost zero width

  $$U(r_{\text{in}}) = \frac{A}{r_{\text{in}}} \exp(-\text{j}[kr_{\text{in}} + \varphi])$$



- after "normalization"

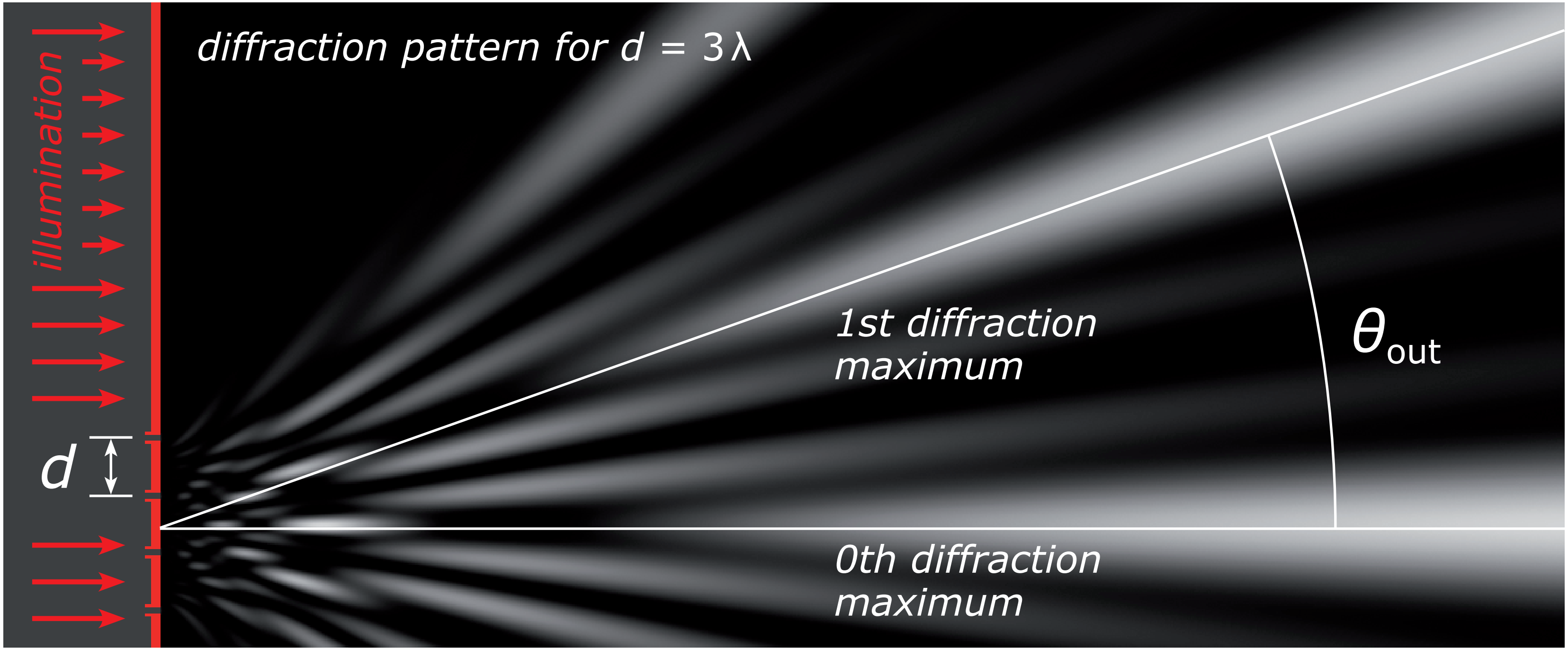
  $$U(\mathbf{0}) = A'$$

- complex amplitude behind the opaque screen

  $$U(r_{\text{out}}) = \frac{A'}{r_{\text{out}}} \exp(-\text{j}[kr_{\text{out}}])$$

Multiple slit diffraction

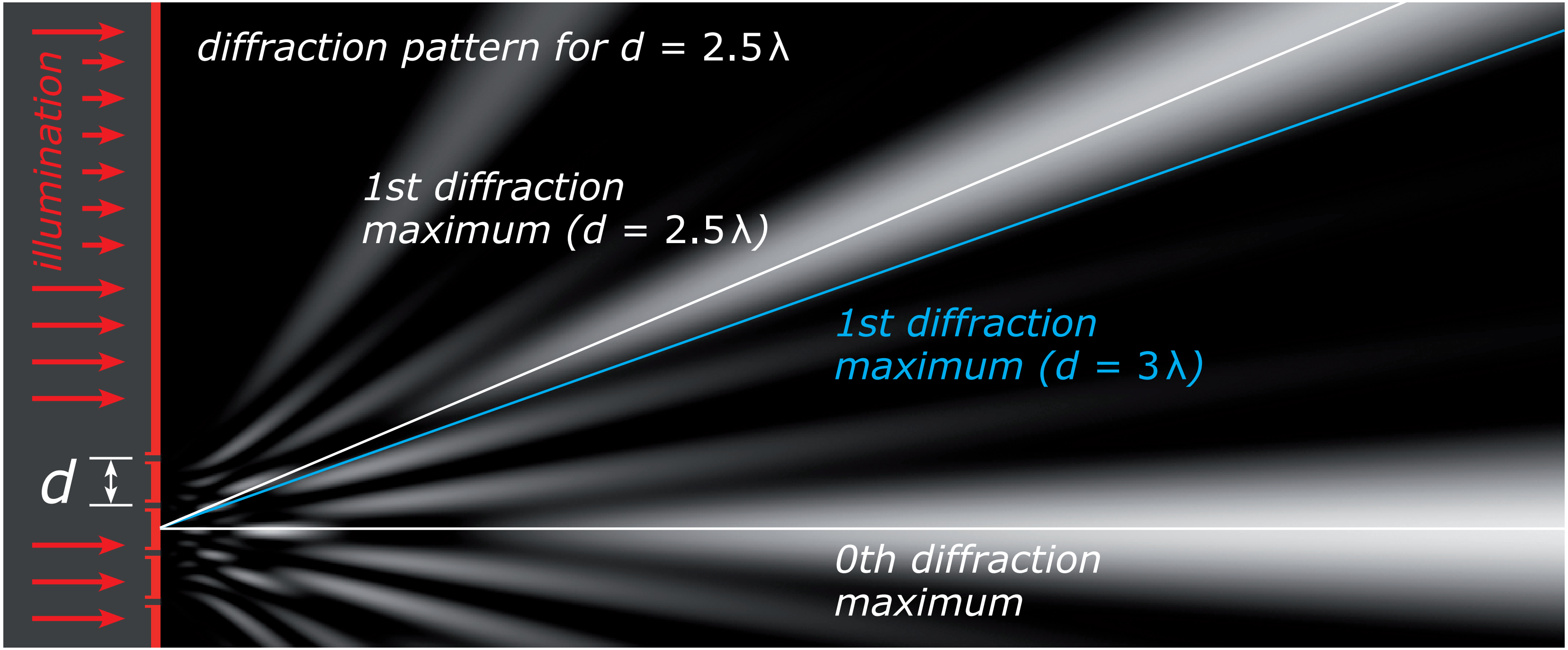
- *m*-th diffraction maximum:
  $\sin\theta_{out} = m \cdot \lambda/d$



Multiple slit diffraction

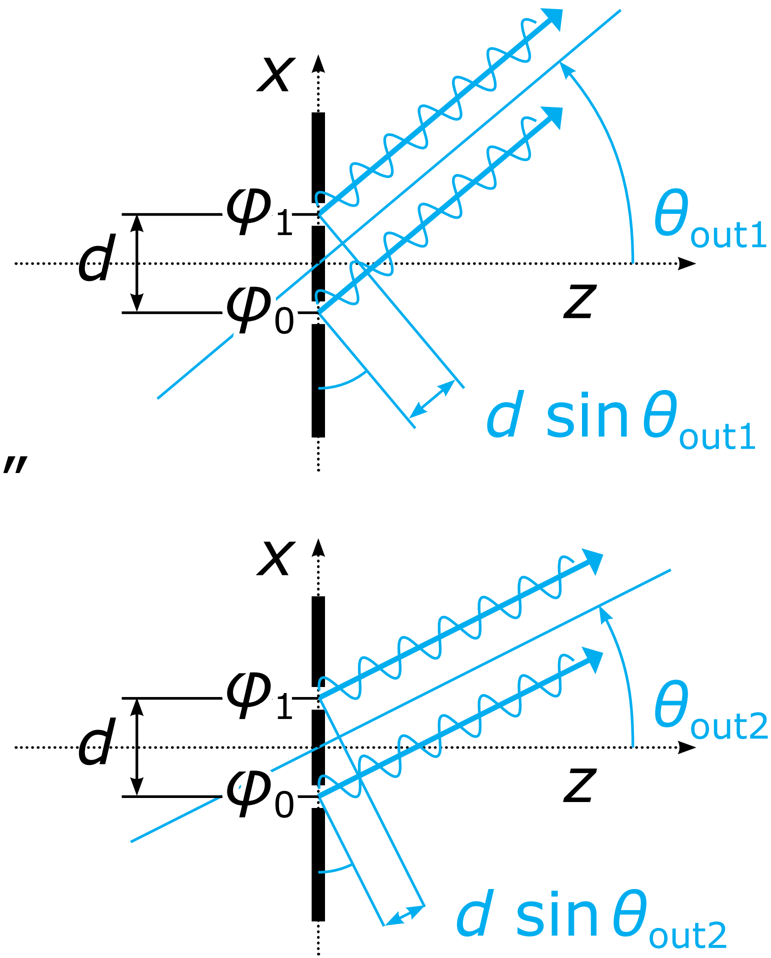
- change of period ⇒ change of diffraction angles
- change of illumination angle *(not shown)*

  ⇒ $\sin\theta_{out} = m \cdot \lambda/d + \sin\theta_{in}$ *(grating equation)*



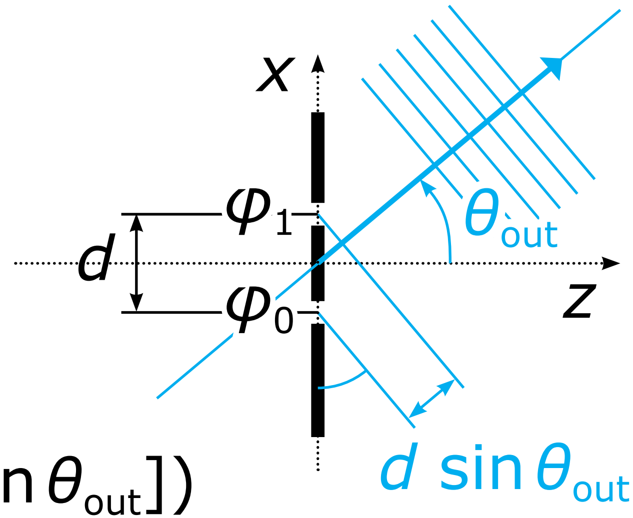
Double slit diffraction

- screen in a plane z = 0
- two slits in a distance d
- angle of observation θ_{out} in a distance $r_{out} \gg d$
- change in θ_{out}
  - ⇒ change in mutual "shift" of rays
  - ⇒ change of interference



Double slit diffraction

- in fact:
  two point sources
  common amplitude A′,
  phases φ_1, φ_2:
  $A'/r_{\text{out}} \exp(-\mathrm{j}[kr_{\text{out}} + \varphi_1])$
  $A'/r_{\text{out}} \exp(-\mathrm{j}[kr_{\text{out}} + \varphi_0 + kd\sin\theta_{\text{out}}])$



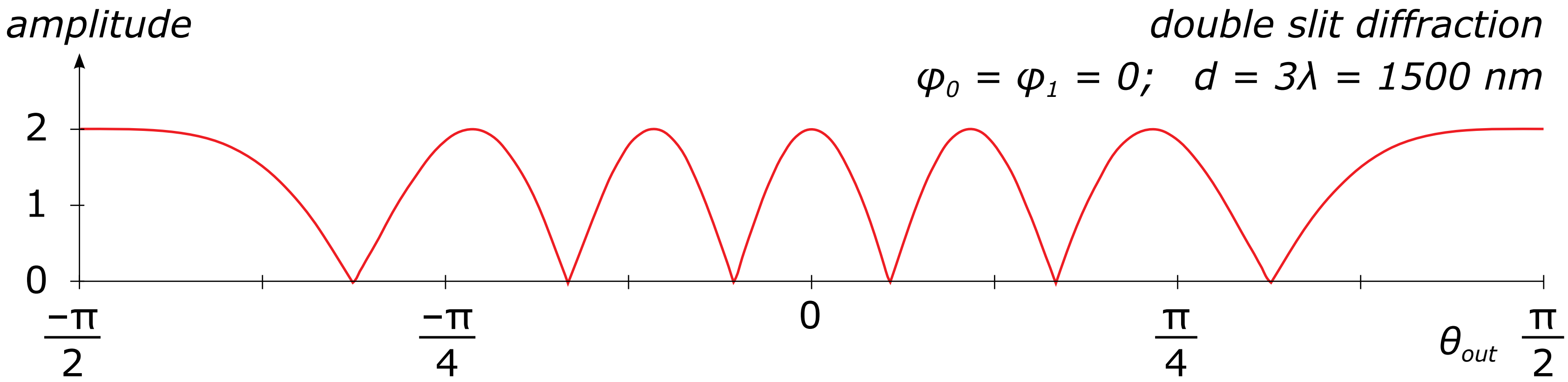
- their sum:

$$2A'/r_{\text{out}} \cos\frac{\varphi_0 - \varphi_1 + kd\sin\theta_{\text{out}}}{2} \times$$

$$\times \exp(-\mathrm{j}[kr_{\text{out}} + \frac{\varphi_0 + \varphi_1 + kd\sin\theta_{\text{out}}}{2}])$$

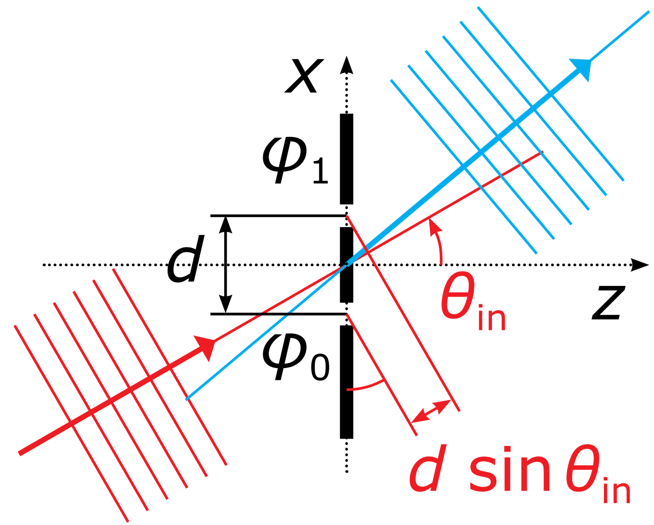
Double slit diffraction

- ±1st diffraction maximum:
  $$\frac{\varphi_0 - \varphi_1 + kd \sin \theta_{\text{out}}}{2} = \pm\pi$$
- *m*-th diffraction maximum:
  $$\varphi_0 - \varphi_1 + kd \sin \theta_{\text{out}} = m \cdot 2\pi$$



Double slit diffraction

- screen lighting by a plane wave at an angle θ_{in}
- $\varphi_1 = \varphi_0 + kd\sin\theta_{in}$



- m-th diffraction maximum:

  $\varphi_0 - \varphi_1 + kd\sin\theta_{out} = m \cdot 2\pi$

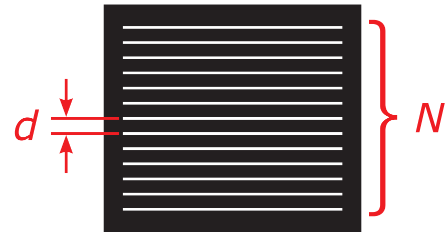
- after substitution:

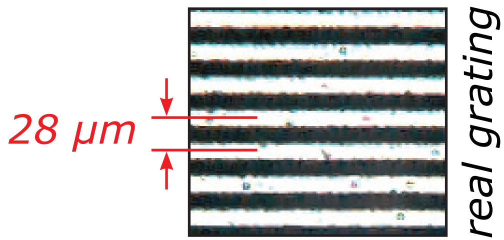
  $-kd\sin\theta_{in} + kd\sin\theta_{out} = m \cdot 2\pi$

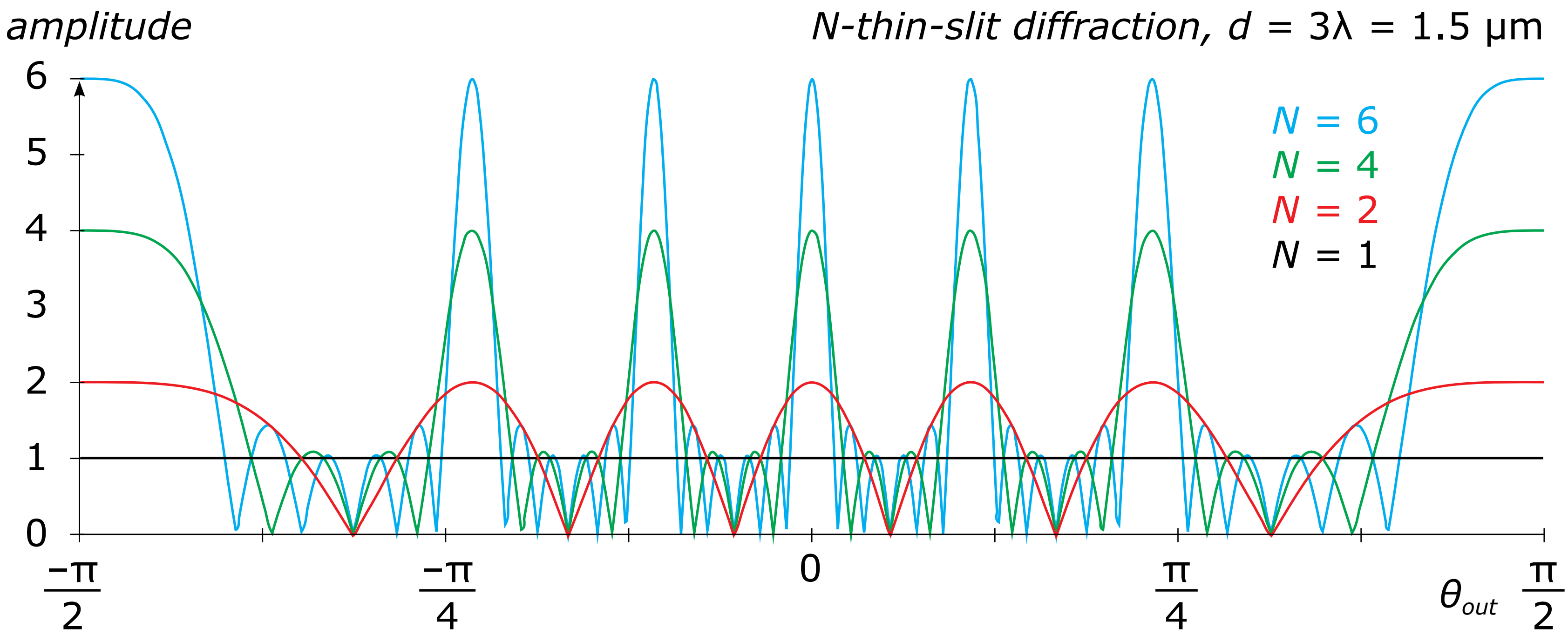
  $\sin\theta_{out} = m\lambda/d + \sin\theta_{in}$ **(grating equation)**

Amplitude diffraction grating

- opaque screen with thin N slits, period d



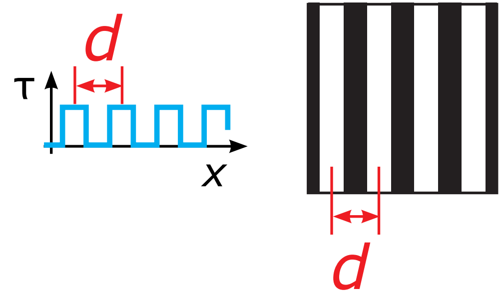


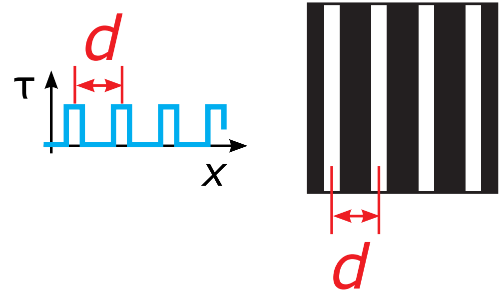


Amplitude diffraction grating

- other transmittance profiles:
  - different slit width
  - different transmittance shape

  ⇒ different brightness of maxima

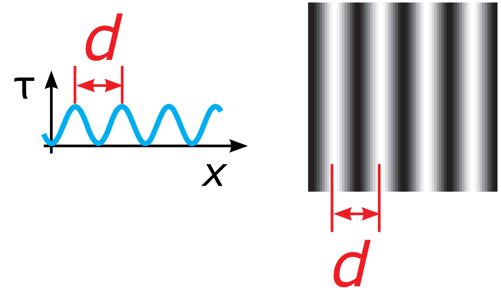




- transmittance profile

  $\tau(x) = (1 + \cos(2\pi x/d))/2$
  - the only important maxima:

    $m \in \{0, +1, -1\}$



Cosine pattern diffraction

- plane wave $U(\mathbf{x}) = \exp(-j[\mathbf{k}\cdot\mathbf{x}])$ passing through a pattern with cosine transmittance profile:

$$U(\mathbf{x})|_{z=0} = [1 + \cos(2\pi x/d)]/2 \exp(-j[\mathbf{k}\cdot\mathbf{x}])$$

$$= \frac{1}{2}\exp(-j[\mathbf{k}\cdot\mathbf{x}]) + \frac{1}{2}\cos(2\pi x/d)\exp(-j[\mathbf{k}\cdot\mathbf{x}])$$

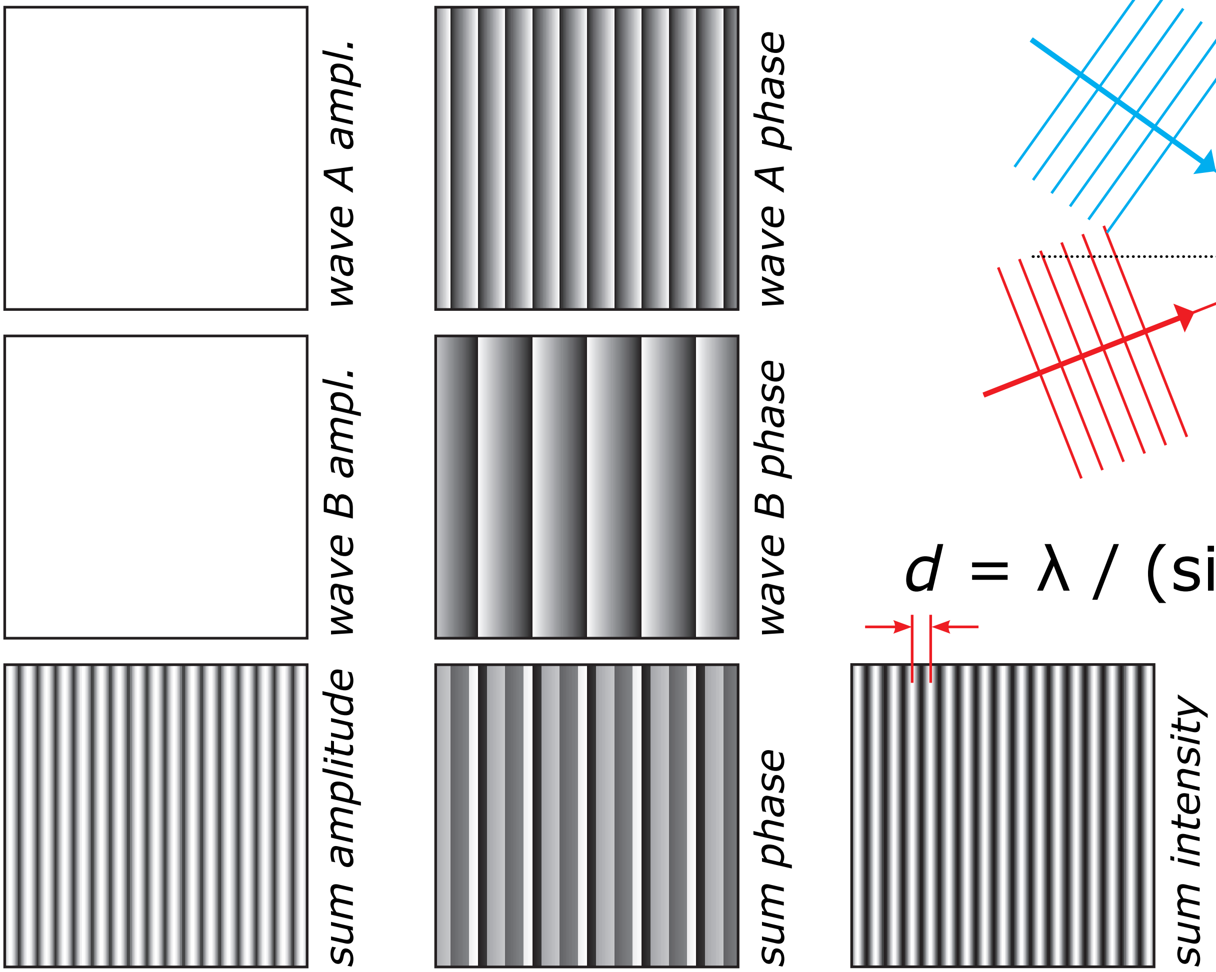
$$= \frac{1}{2}\exp(-j[\mathbf{k}\cdot\mathbf{x}])$$

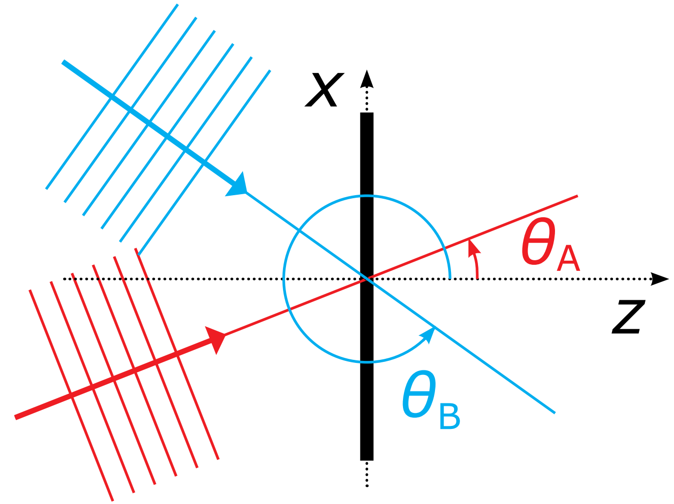
$$+ \frac{1}{4}[\exp(-j2\pi x/d) + \exp(j2\pi x/d)]\exp(-j[\mathbf{k}\cdot\mathbf{x}])$$

$$= \frac{1}{2}\exp(-j[\mathbf{k}\cdot\mathbf{x}])$$

$$+ \frac{1}{4}\exp(-j[\mathbf{k}_{+1}\cdot\mathbf{x}]) + \frac{1}{4}\exp(-j[\mathbf{k}_{-1}\cdot\mathbf{x}])$$

Cosine profile recording





$$d = \lambda / (\sin\theta_A - \sin\theta_B)$$

$$f = 1 / d$$

Cosine profile recording

- plane wave complex amplitude: $A \exp(-\mathrm{j}[\mathbf{k} \cdot \mathbf{x}])$
  inclination θ_A: $\boldsymbol{k} = k(\sin\theta_A, 0, \cos\theta_A)$
  in the plane $z = 0$: $\boldsymbol{x} = (x, y, 0)$

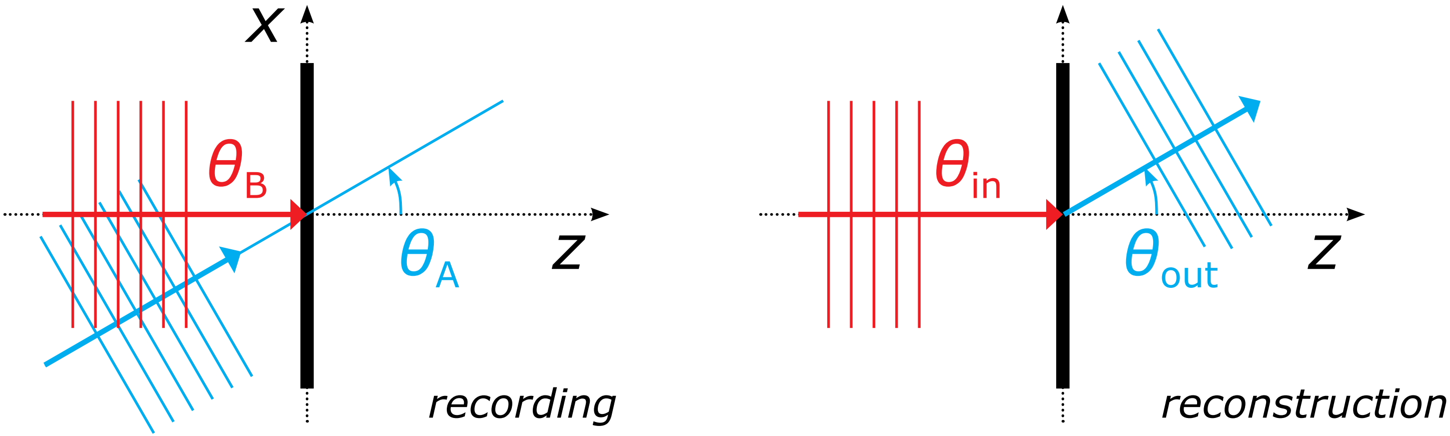
$\Rightarrow \boldsymbol{k} \cdot \boldsymbol{x} = kx \sin\theta_A$

- intensity of sum of plane waves from angles θ_A, θ_B:
  $|A \exp(-\mathrm{j}\boldsymbol{k}_A \cdot \boldsymbol{x}) + A \exp(-\mathrm{j}\boldsymbol{k}_B \cdot \boldsymbol{x})|^2 =$
  $= (A \exp(-\mathrm{j}\boldsymbol{k}_A \cdot \boldsymbol{x}) + A \exp(-\mathrm{j}\boldsymbol{k}_B \cdot \boldsymbol{x})) \times$
  $(A \exp(\mathrm{j}\boldsymbol{k}_A \cdot \boldsymbol{x}) + A \exp(\mathrm{j}\boldsymbol{k}_B \cdot \boldsymbol{x})) =$
  $= 2A + 2A \cos(\boldsymbol{k}_A \cdot \boldsymbol{x} - \boldsymbol{k}_B \cdot \boldsymbol{x}) =$
  $= 2A \{1 + \cos(k[\sin\theta_A - \sin\theta_B]x)\}$
- frequency of the pattern $f = (\sin\theta_A - \sin\theta_B)/\lambda$

Sin θ equation

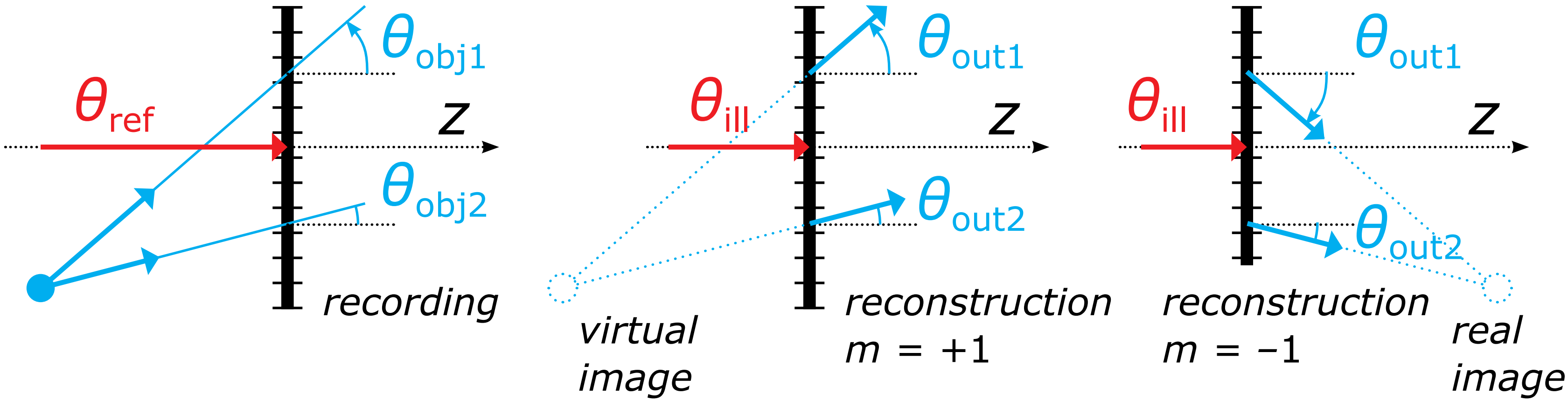
- grating equation:
  $\sin\theta_{out} = m\lambda/d + \sin\theta_{in} = m\lambda f + \sin\theta_{in}$
- after manipulation:
  $\sin\theta_{out} = m(\sin\theta_A - \sin\theta_B) + \sin\theta_{in}$
- example: m = +1, $\sin\theta_B = \sin\theta_{in}$

$\Rightarrow \sin\theta_{out} = \sin\theta_A$

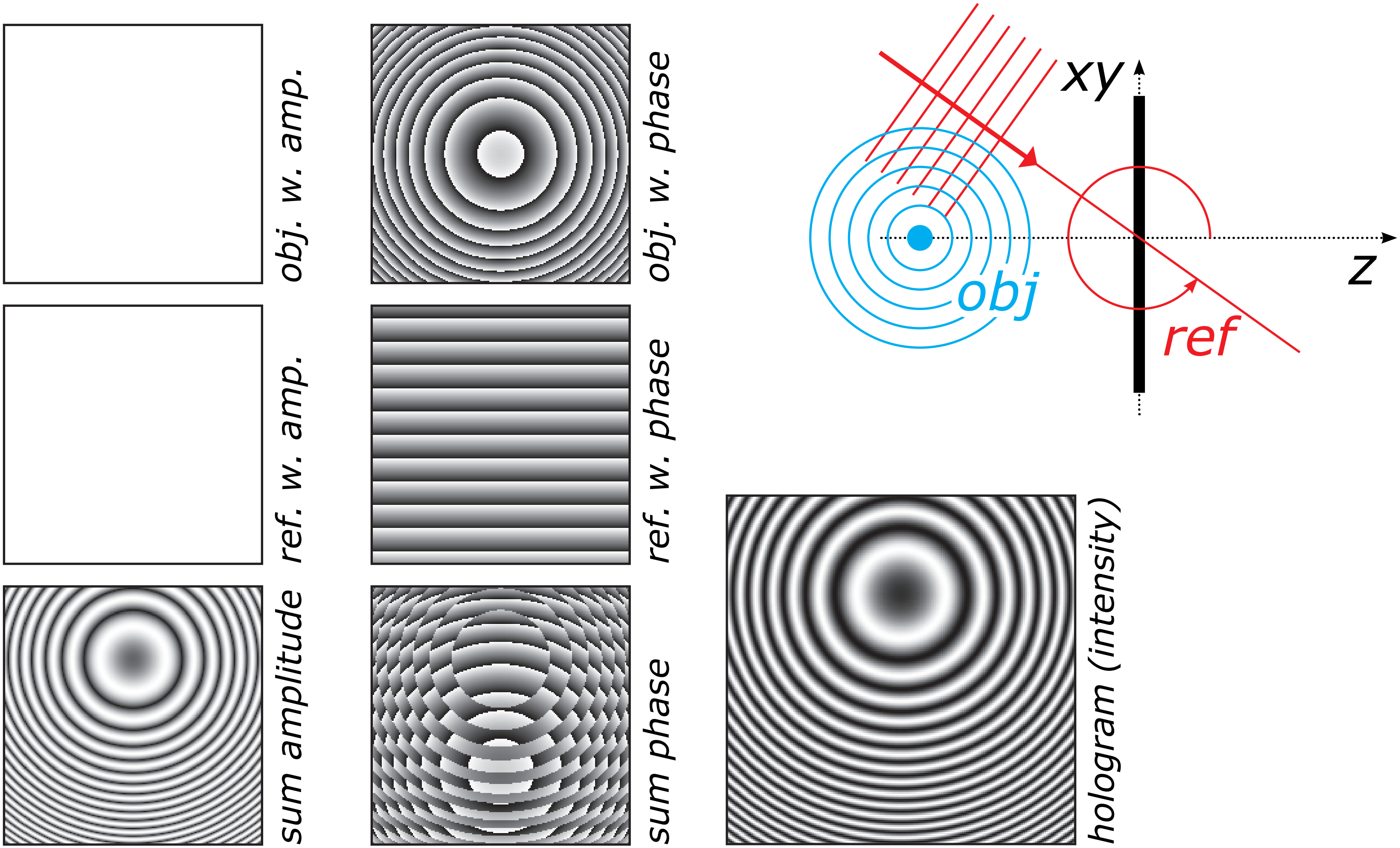


Hologram

- object wave: θ_{obj} (= θ_A), $\lambda = \lambda_{ref}$
- reference wave: θ_{ref} (= θ_B), $\lambda = \lambda_{ref}$
- illumination wave: θ_{ill} (= θ_{in}), $\lambda = \lambda_{ill}$
- $\sin\theta_{out} = m\frac{\lambda_{ill}}{\lambda_{ref}}(\sin\theta_{obj} - \sin\theta_{ref}) + \sin\theta_{ill}$
- example: $\lambda_{ill} = \lambda_{ref}$, $\theta_{ill} = \theta_{ref} = 0$



Hologram recording

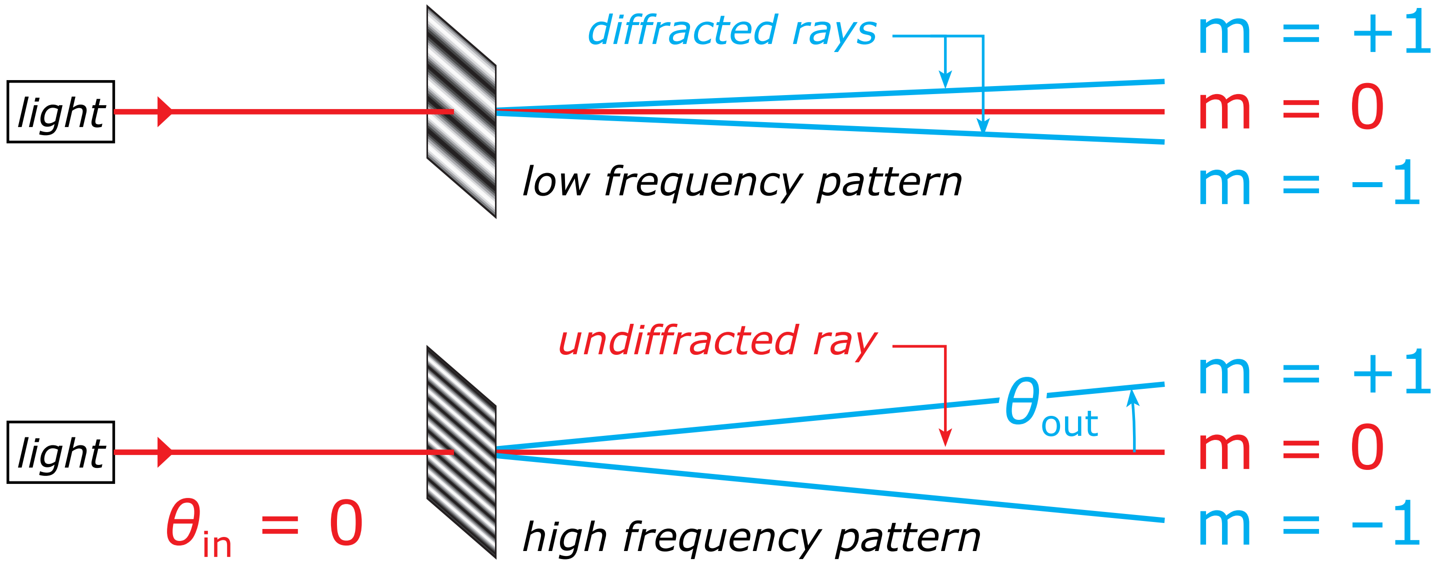


Hologram watching

Light diffraction

- depends mainly on frequency f of the pattern

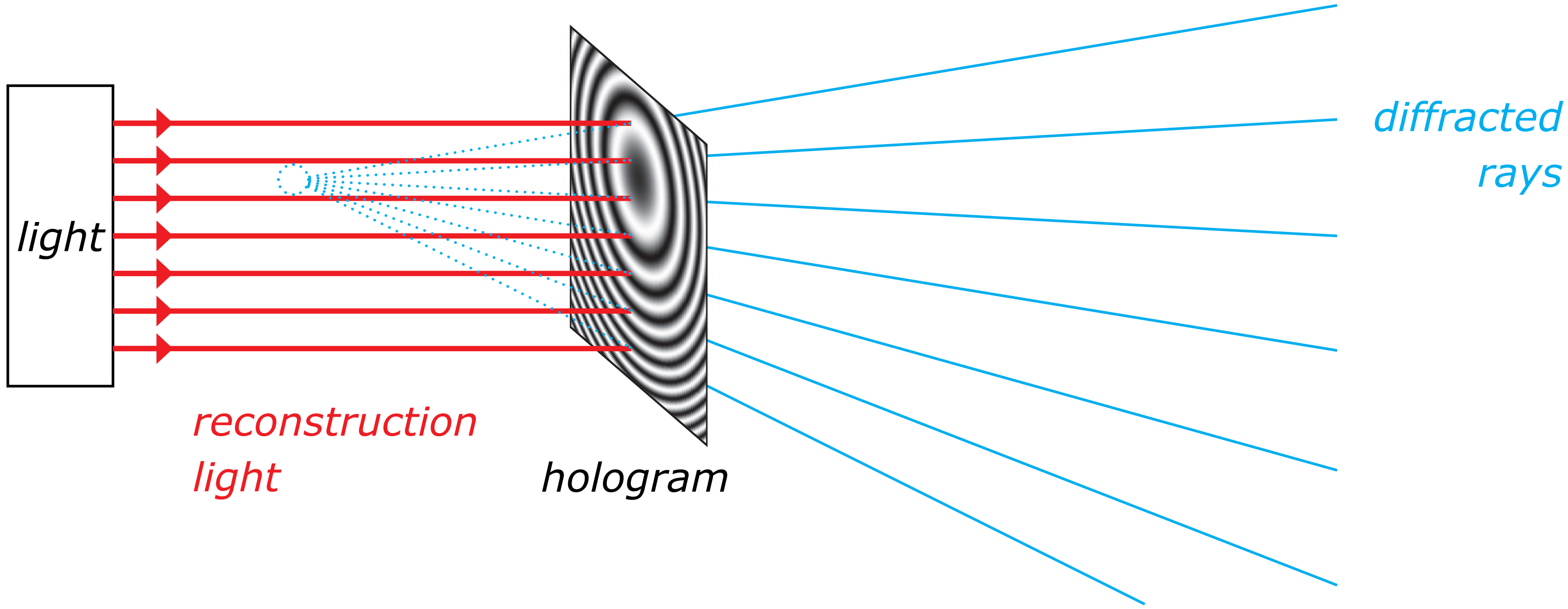
  output angle of the rays: $\sin\theta_{out} = m\lambda f + \sin\theta_{in}$



Hologram watching

Virtual image creation

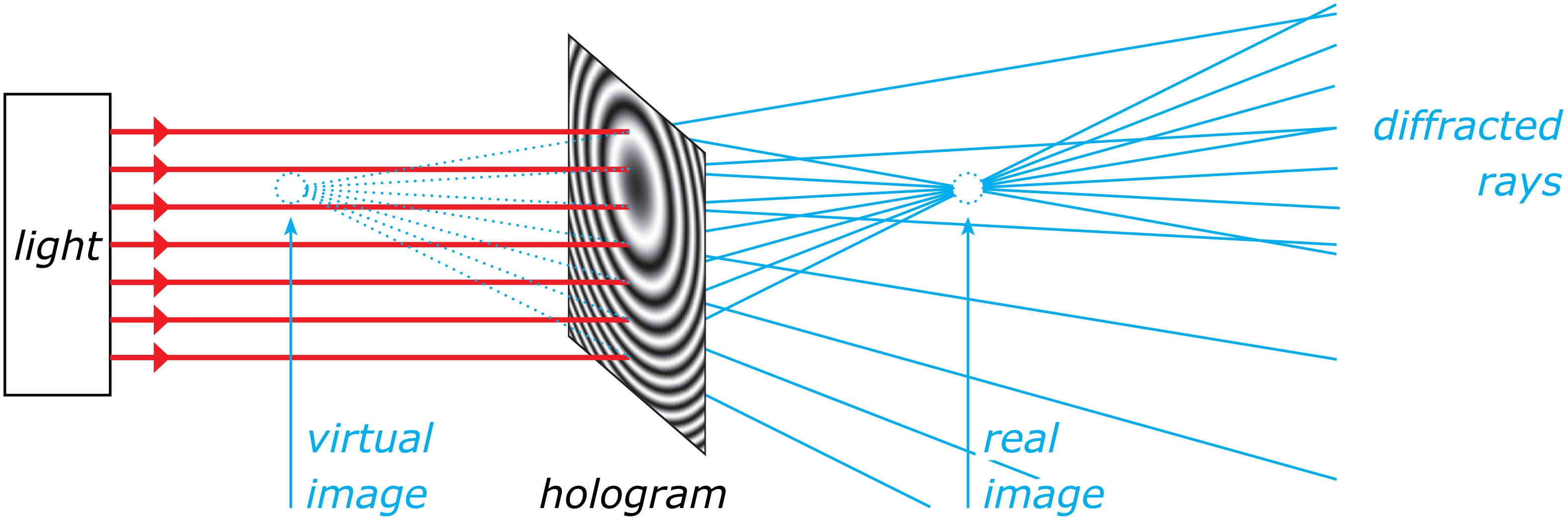
- illuminate hologram with a light source
- light beams start to diffract on the interference pattern as if the original object was still present



Hologram watching

Real image creation

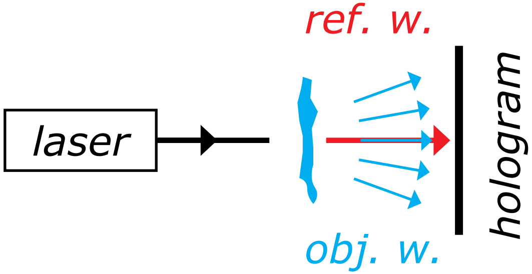
- output angle of the rays: $\sin\theta_{\text{out}} = m\lambda f + \sin\theta_{\text{in}}$
- for m = −1, rays create real image of the scene
- both rays for m = +1 and −1 appear at once
  ⇒ no need to distinguish between them



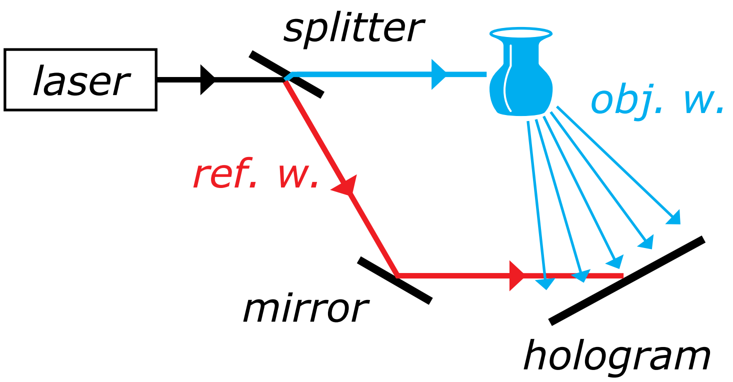
Hologram recording

Basic setups

- in-line (Gabor) hologram
  - for transparent objects
  - image damaged by 0th order
  - low spatial frequencies



- off-axis (Leith-Upatnieks) hologram
  - for opaque objects
  - clear image
  - high spatial frequencies (over 1000 lines/mm)
  - aberrations



Hologram principle proof

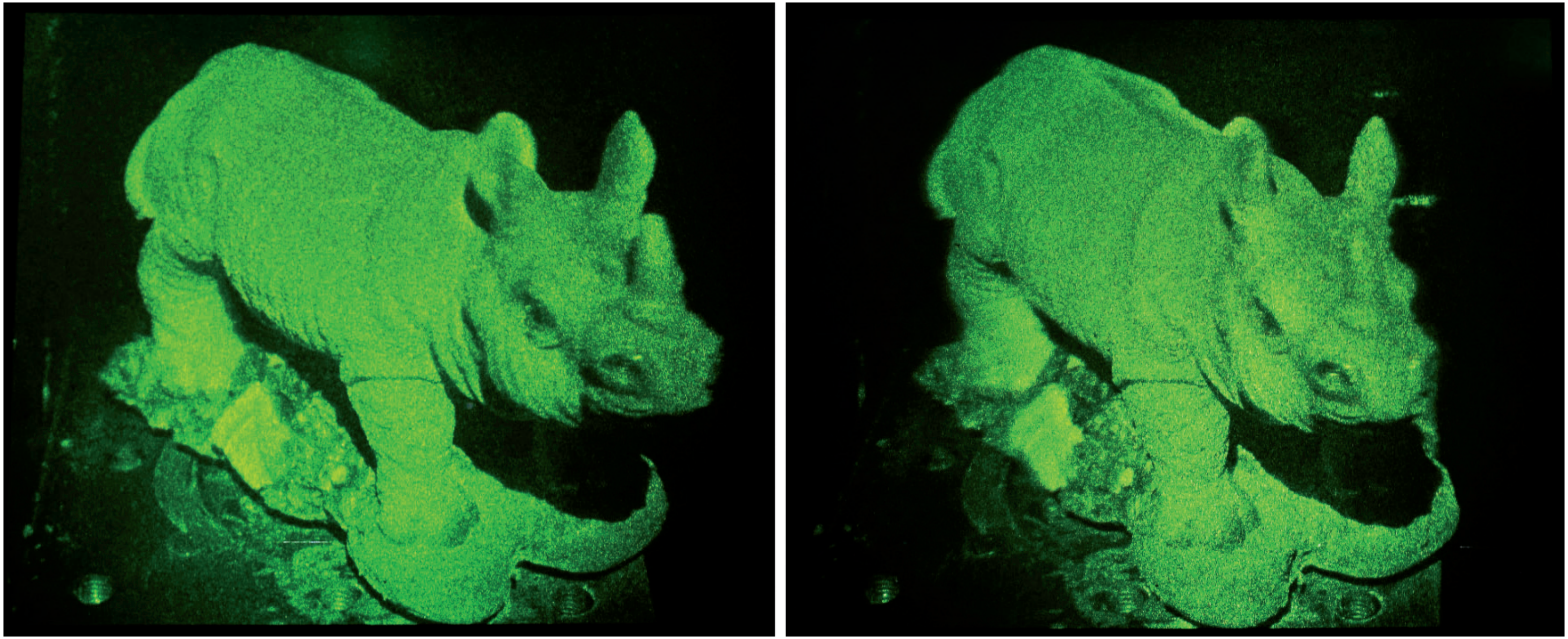
- hologram: recording of the interference of the object wave O and the reference wave R:
  $I = (O + R)(O + R)^* = OO^* + RR^* + OR^* + O^*R$
- after illumination by the copy of the reference wave:
  $U = IR$

$= (OO^*)R$	diffracted illumination wave
$+ (RR^*)R$	attenuated illumination wave
$+ O(RR^*)$	**copy of the object wave**
$+ O^*(RR)$	conjugate image

3D display holography

- reconstruction wave (hologram illumination) the same as reference wave (in recording process)

⇒ observation of the original object

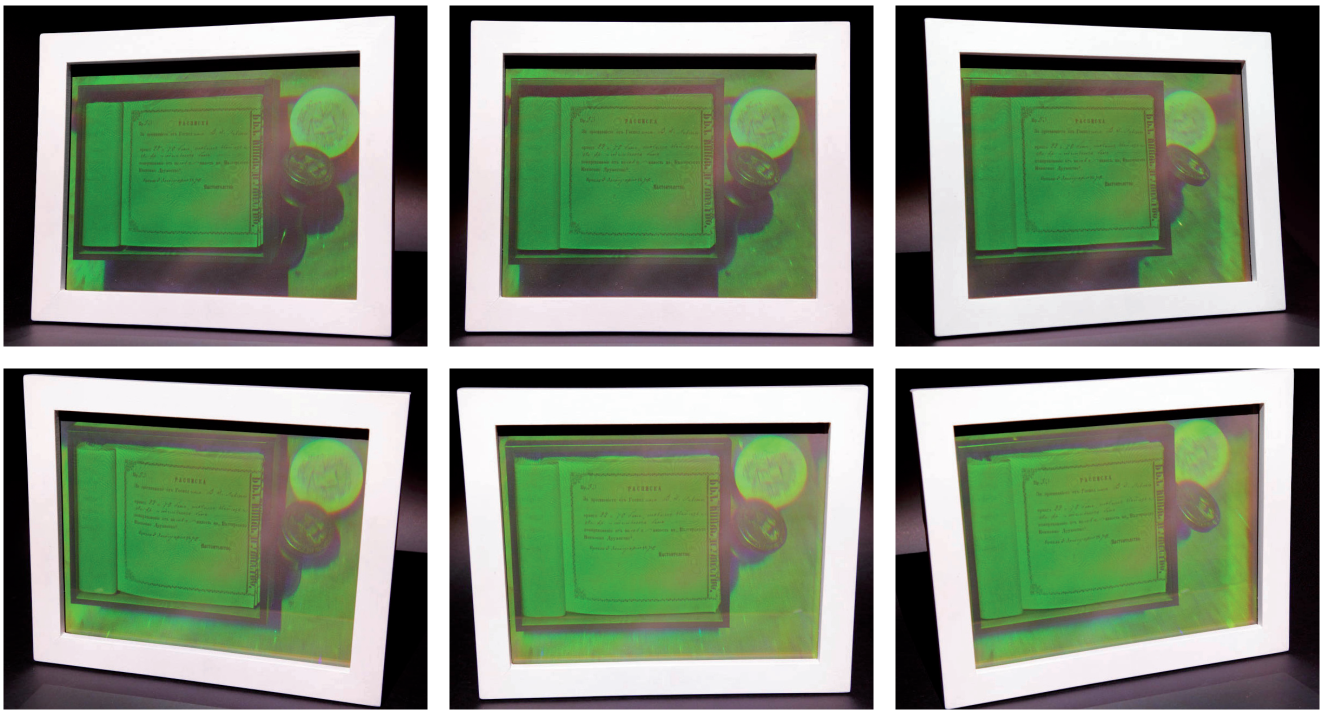


Hologram created by Šárka Němcová, ČVUT Praha

3D display holography

- reconstruction wave changes the angle

⇒ observation of the (deformed) original object from a varying viewpoint



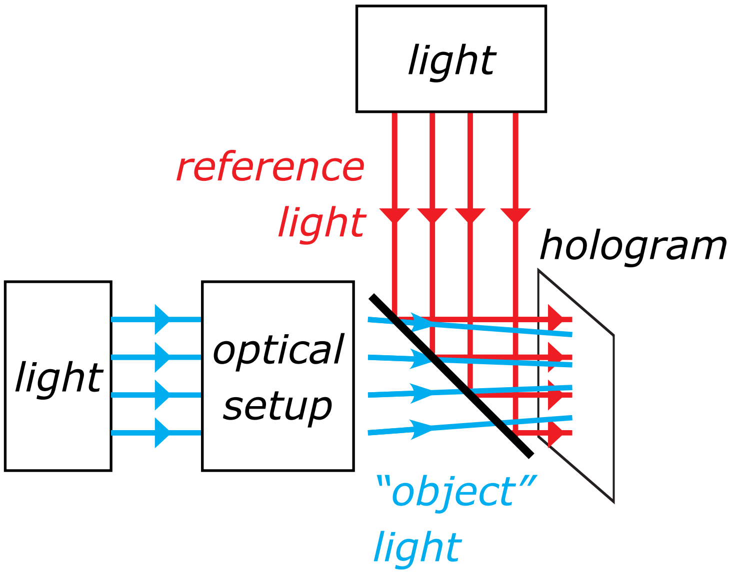
Hologram created in The Central Laboratory of Optical Storage and Processing of Information, Bulgarian Academy of Sciences

Holography applications

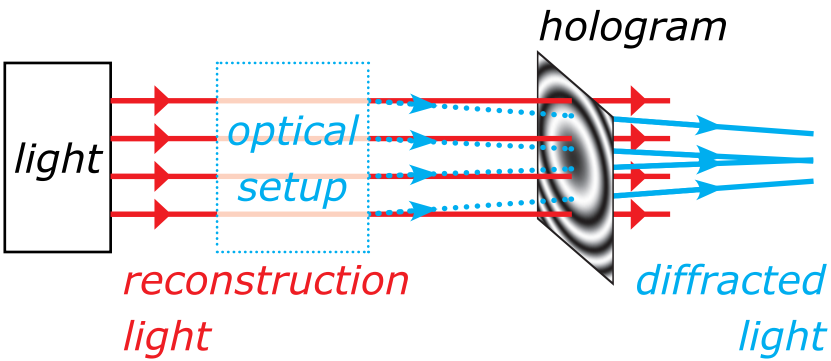
- microscopy, optical metrology
  - perfect light recording (biological sample, bubble chamber, …)
  - hologram examination (unlimited time of observation, safe environment, …)
- enhancing electron microscopy
  - original Gabor idea behind holography
  - hologram recording with electron beam (λ is 100 000× smaller than for visible light)
  - hologram enlargement, visible light illumination $\Rightarrow$ image 100 000× bigger

Holography applications

- diffractive (holographic) optical elements
  - mimicking any optical element
  - cheaper, easier abberation correction, …



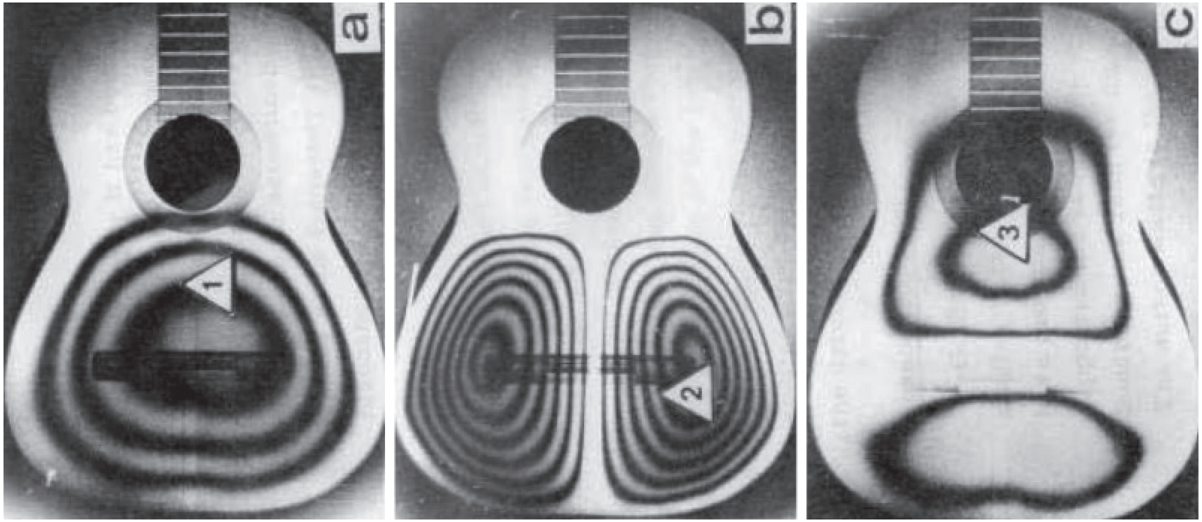
diffractive optical element recording



diffractive optical element usage

Holography applications

- non-destructive testing
  - double object recording on one hologram
  - microshifts cause interference strips
  - vibration causes loss of interference strips



Molin and Stetson, Institute of Optical Research, Stockholm (1971)

Digital holography

Hologram creation mathematically

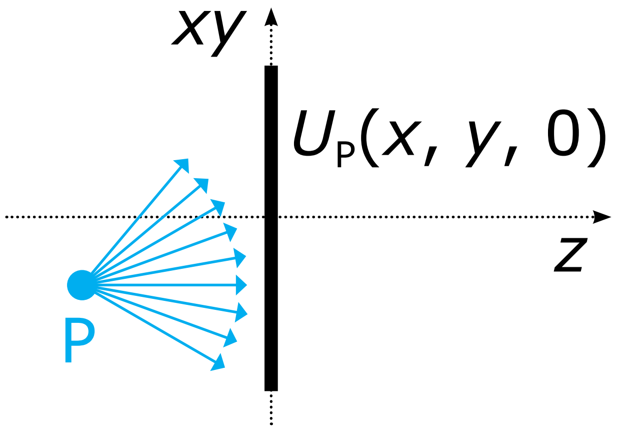
for every point (x, y) of the hologram:

- get the amplitude A_{obj} and the phase φ_{obj} of the object wave in (x, y)
- get the amplitude A_{ref} and the phase φ_{ref} of the reference wave in (x, y)
- calculate captured intensity in (x, y)
  $I(x, y) = |A_{obj} \exp(-\mathrm{j}\, \varphi_{obj}) + A_{ref} \exp(-\mathrm{j}\, \varphi_{ref})|^2$

Object wave

Complex amplitude of a point source

- point source P
  at (x_P, y_P, z_P), $z_P < 0$
- light amplitude A_P, phase φ_P,
  wavelength $\lambda \cong 630$ nm
  ($\Rightarrow k = 2\pi / \lambda \cong 10^7$)
- hologram plane $z = 0$



- $U_P(x, y, 0) = \dfrac{A_P}{r_P} \exp(-j[kr_P + \varphi_P])$

  $r_P = [(x - x_P)^2 + (y - y_P)^2 + z_P^2]^{1/2}$

Object wave

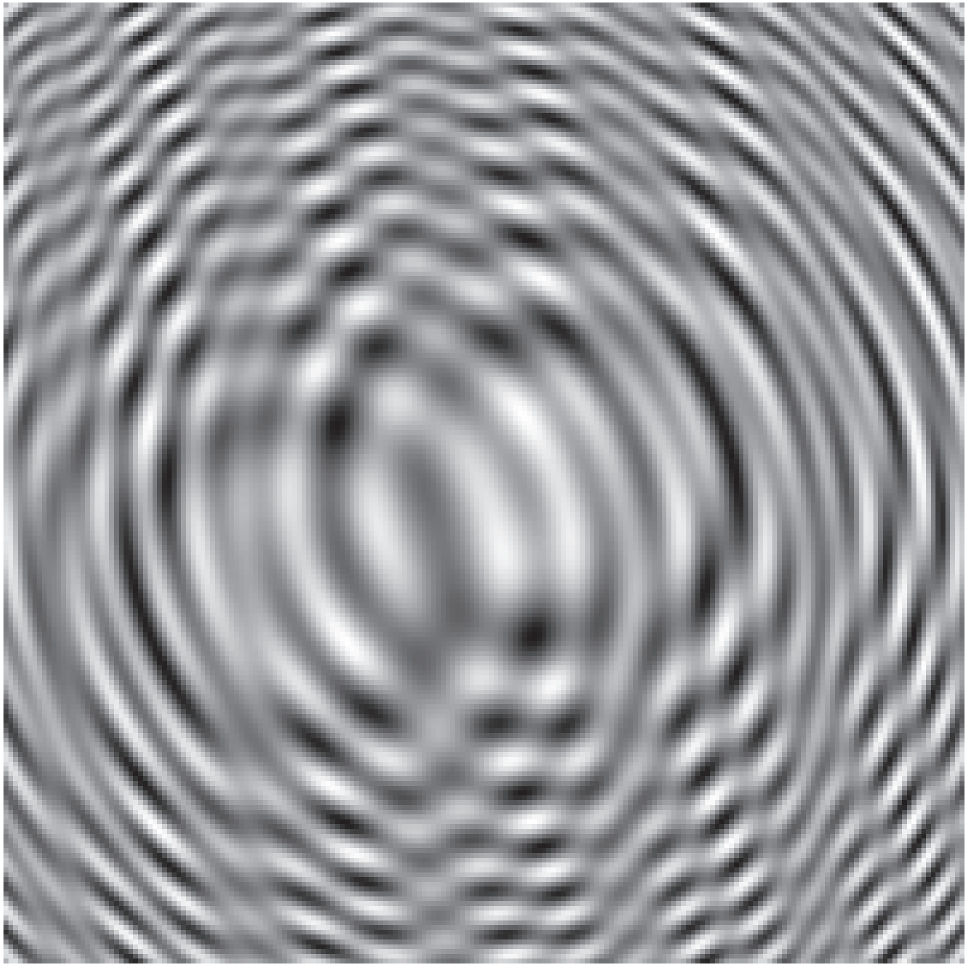
Really unoptimized Matlab code

```
lambda     = 630e-9;
k          = 2*pi/lambda;
res_x      = 200;
res_y      = 200;
hologram_z = 0;
sampling   = 20e-6;
corner_x   = -(res_x-1) * sampling / 2;
corner_y   = -(res_y-1) * sampling / 2;
sources    = [0, 0, -0.5; 20*sampling, 0, -0.5;
             -40*sampling, 20*sampling, -0.5];
```

Object wave

```
objectwave = zeros(res_y, res_x);
for source = 1:rows(sources)
  for column = 1:res_x
    for row = 1:res_y
      x = (column-1) * sampling + corner_x;
      y = (row-1) * sampling + corner_y;
      objectwave(row,column) +=
         exp(i*k*sqrt((x-sources(source, 1))**2
          + (y-sources(source, 2))**2
          + (hologram_z - sources(source, 3))**2));
    endfor
  endfor
endfor
```

Object wave

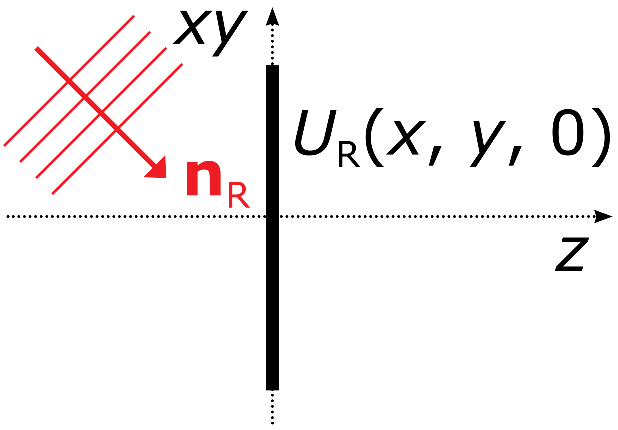


Real part of the object wave
(Just for information;
it has no physical meaning!)

Reference wave

Complex amplitude of a reference wave

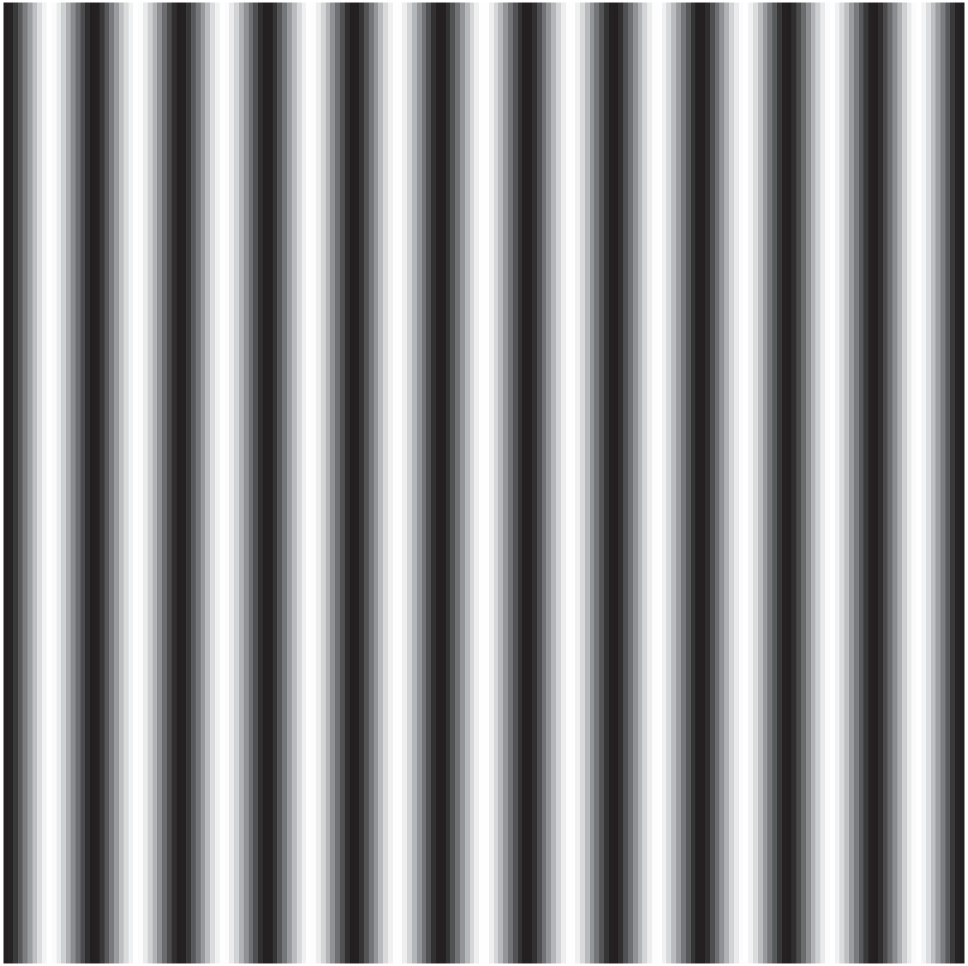
- plane wave with direction vector $\mathbf{n}_\mathrm{R} = (n_{\mathrm{R}x}, n_{\mathrm{R}y}, n_{\mathrm{R}z})$, $|\mathbf{n}_\mathrm{R}| = 1$ and amplitude A_R
- let us ignore constant phase ($\Rightarrow \varphi = 0$)
- $U_\mathrm{R}(x, y, 0) = A_\mathrm{R} \exp(-\mathrm{j}[k\,\mathbf{n}_\mathrm{R} \cdot \mathbf{x} + \varphi]) =$
  $= A_\mathrm{R} \exp(-\mathrm{j}k\,[x\,n_{\mathrm{R}x} + y\,n_{\mathrm{R}y}])$



Reference wave

```
refwave = zeros(res_y, res_x);
ref_x = cos(89.9 * pi/180) * k;
ref_y = cos(90 * pi/180) * k;

for column = 1:res_x
  for row = 1:res_y
    x = (column-1) * sampling + corner_x;
    y = (row-1) * sampling + corner_y;
    refwave(row,column) =
                    exp(i*(ref_x * x + ref_y * y));
  endfor
endfor
```

Reference wave



Real part of the reference wave
(Just for information;
it has no physical meaning!)

Hologram calculation

Intensity calculation

- $$\begin{aligned} I(x, y, 0) &= |U_R(x, y, 0) + U_P(x, y, 0)|^2 \\ &= [U_R(x, y, 0) + U_P(x, y, 0)] \times \\ &\quad \times [U_R(x, y, 0) + U_P(x, y, 0)]^* \\ &= \underbrace{U_R U_R^*}_{a} + \underbrace{U_P U_P^*}_{b} + \underbrace{U_R U_P^* + U_P U_R^*}_{c} \end{aligned}$$

a) reference wave intensity

b) object points interference (if U_P is a complex wave)

c) object points and reference wave interference (bipolar intensity)

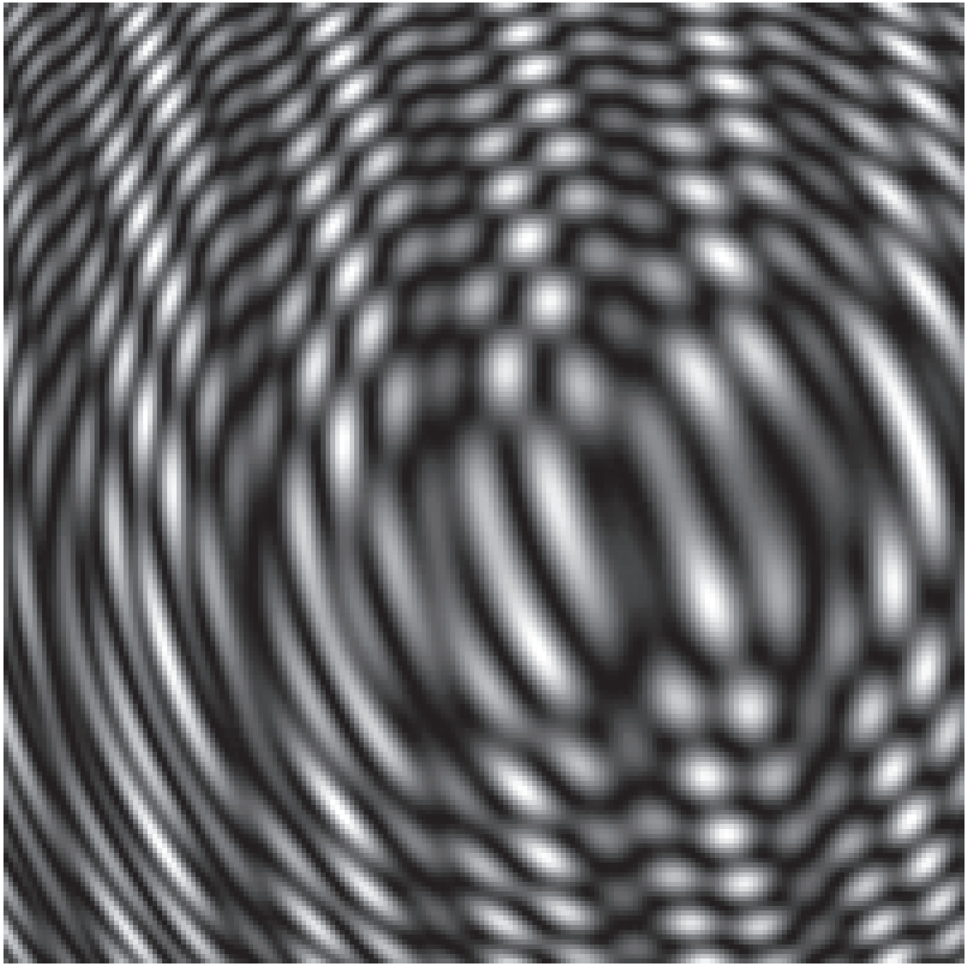
Hologram calculation

```
hologram = objectwave + refwave;
hologram = hologram .* conj(hologram);
```

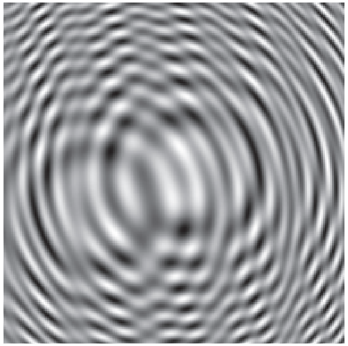
- alternative (bipolar intensity):

```
hologram = real(objectwave) .* real(refwave) +
           imag(objectwave) .* imag(refwave)
```

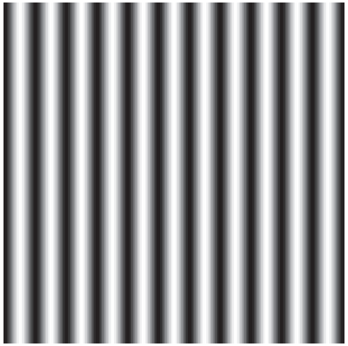
Hologram calculation



The hologram (intensity picture)

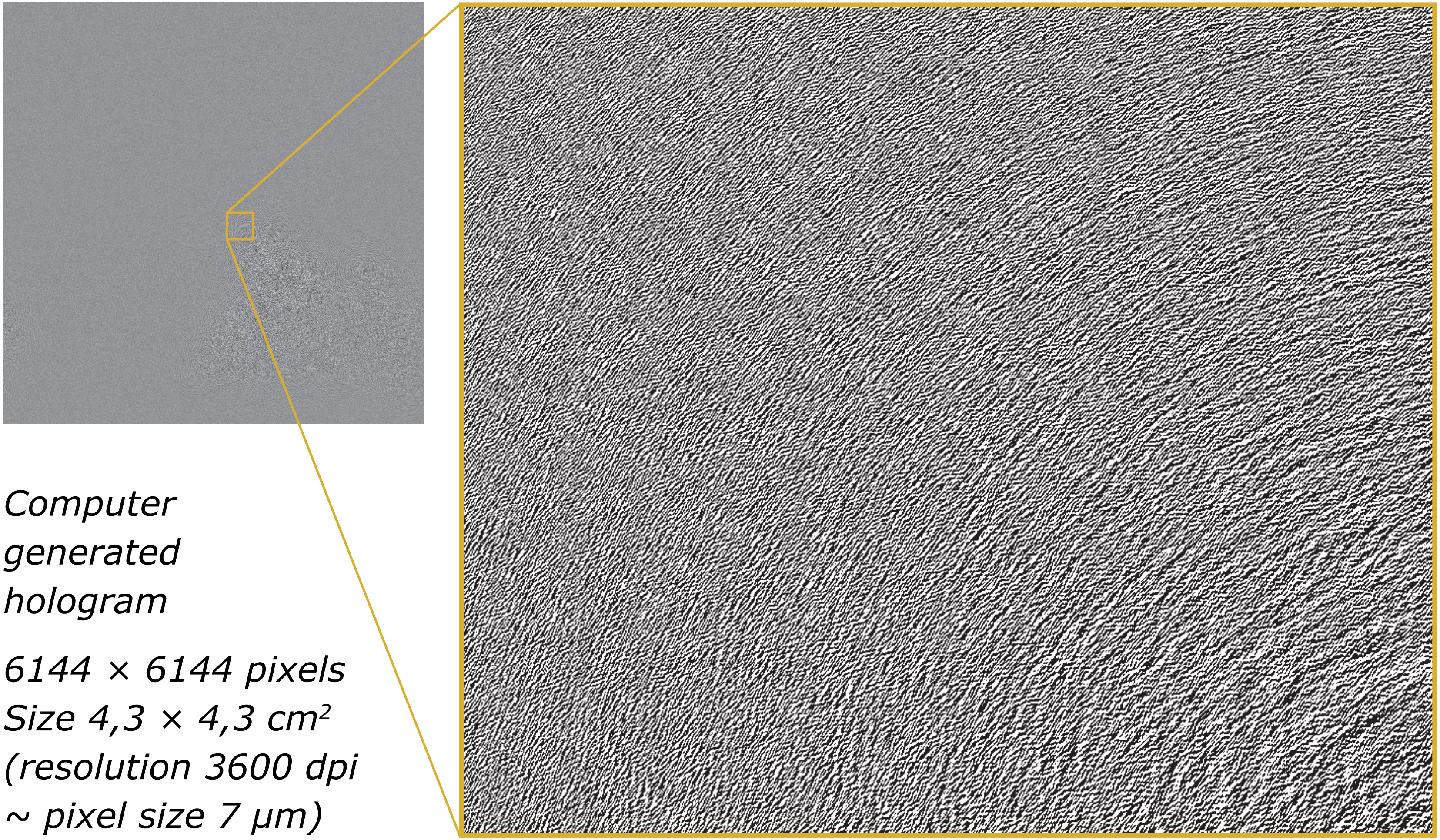


Object wave



Reference wave

Hologram calculation

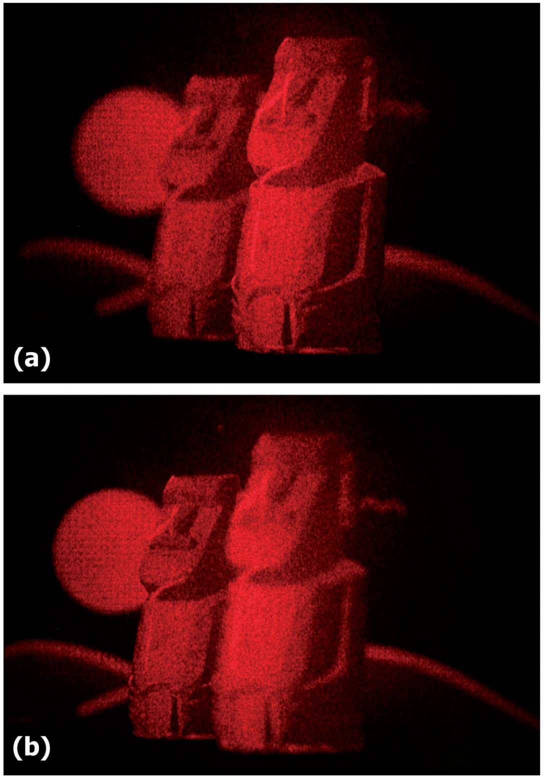


Computer generated hologram

6144 × 6144 pixels
Size 4,3 × 4,3 cm²
(resolution 3600 dpi ~ pixel size 7 µm)

Static high resolution holograms

- electron beam litography
  - 0.1 µm details
    ⇒ diffraction up to 90°
  - extremely expensive,
    recording 1 mm^2/min
- laser litography
  - 1 µm details
    ⇒ diffraction up to 20°
  - very expensive,
    recording 4 mm^2/min

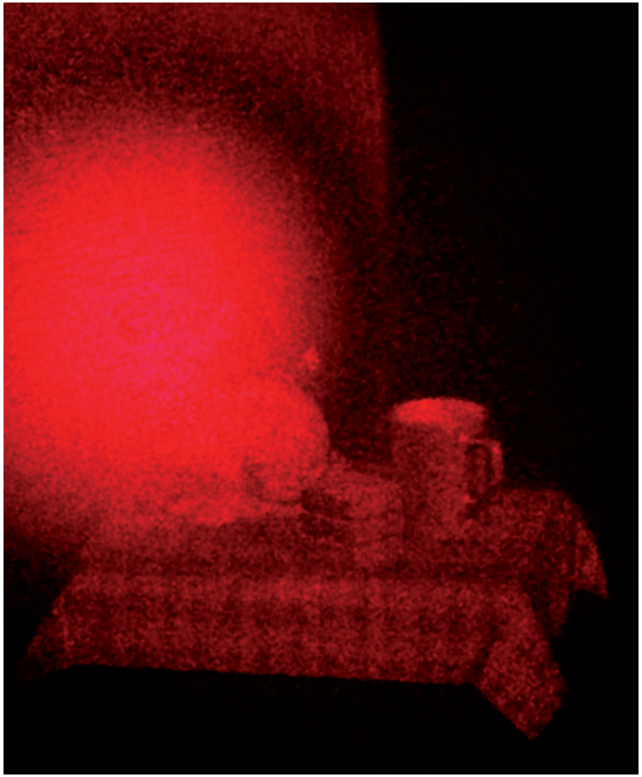


Hologram by K. Matsushima

Hologram portrayal

Home made static holograms

- imagesetter
  - 10 μm details
    - ⇒ diffraction up to 2°
  - price ~ 5 € per A4
- laser printer
  - 100 μm details
    - ⇒ diffraction up to 0.5°

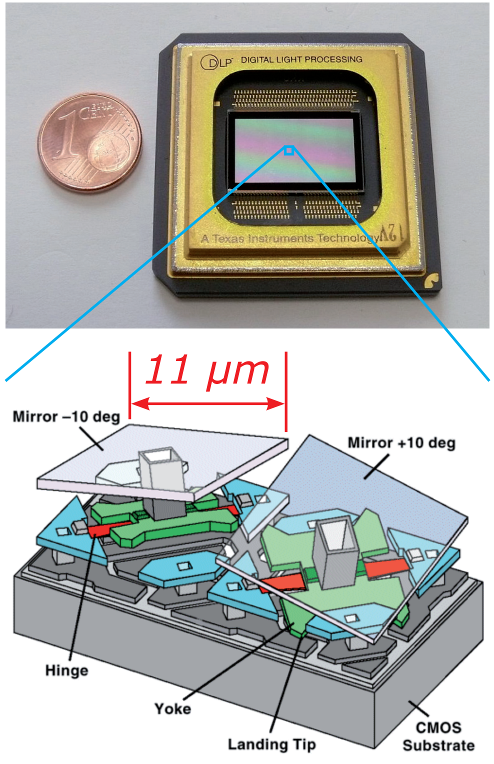


Hologram by I. Hanák, M. Janda

Hologram portrayal

Laboratory holographic displays

- based on DMD chips (DLP projectors), phase only spatial light modulators or acousto-optic modulators: (Bilkent University, MIT Media Lab, ...)
- based on intermediate optical photorefracive memory (University of Arizona)

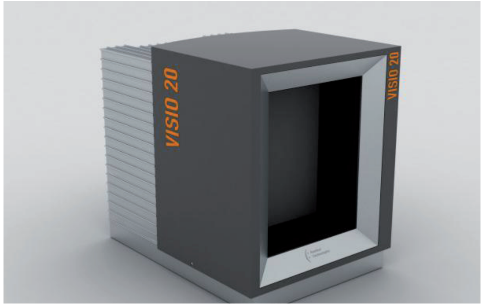


DMD chip by Texas Instruments

Hologram portrayal

Early stage commercial displays

- Zebra Imaging
- SeeReal Technologies spatial light modulators plus eye tracking
- QinetiQ spatial light modulator plus intermediate optical memory



SeeReal Visio 20″

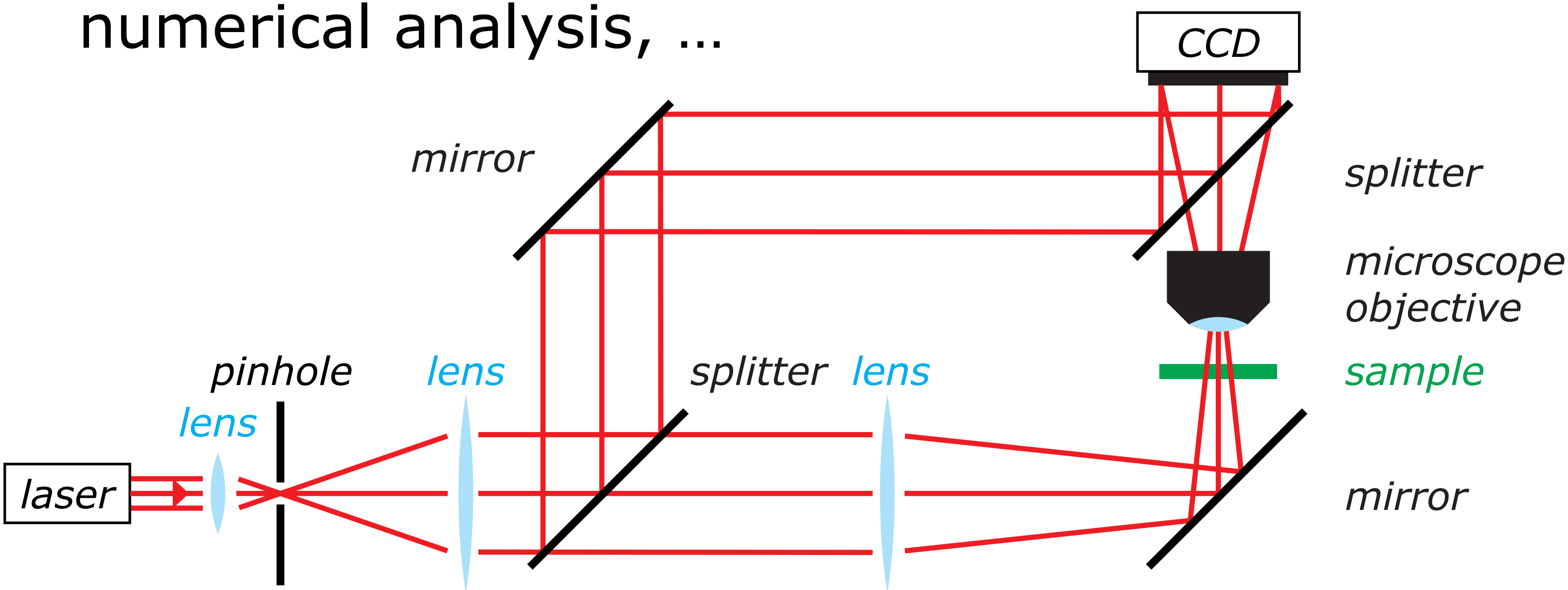
Zebra Imaging ZScape motion display

Digital holography applications

Digital holographic microscopy

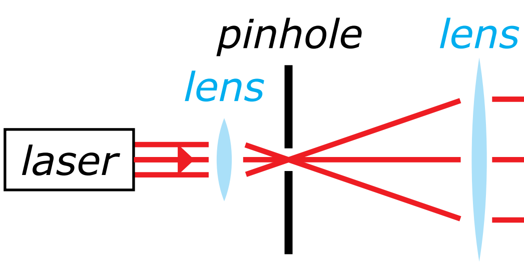
- acquisition of digital hologram
- numerical reconstruction

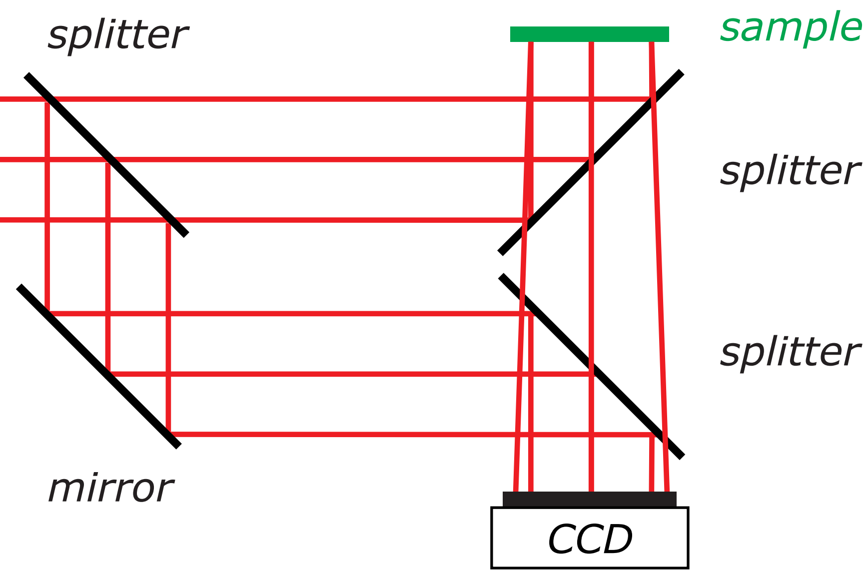
⇒ signal filtering, unwanted diffraction removal, numerical analysis, …

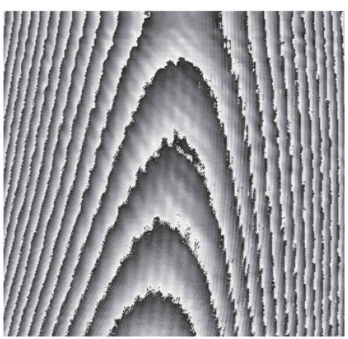


Surface metrology

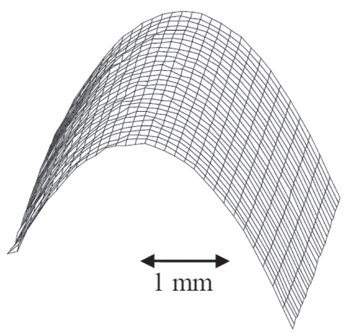
- real object numerical reconstruction
- reconstructed phase ~ surface bumpiness







captured phase

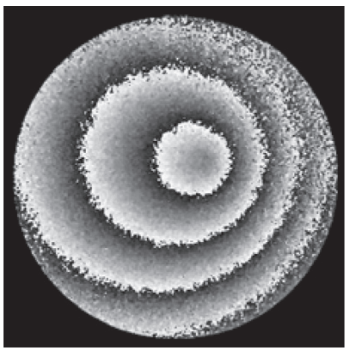


unwrapped phase (Jüptner, Schnars: Digital Holography)

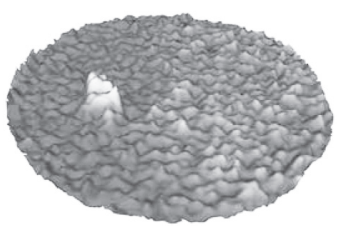
Digital holography applications

Comparative digital holography

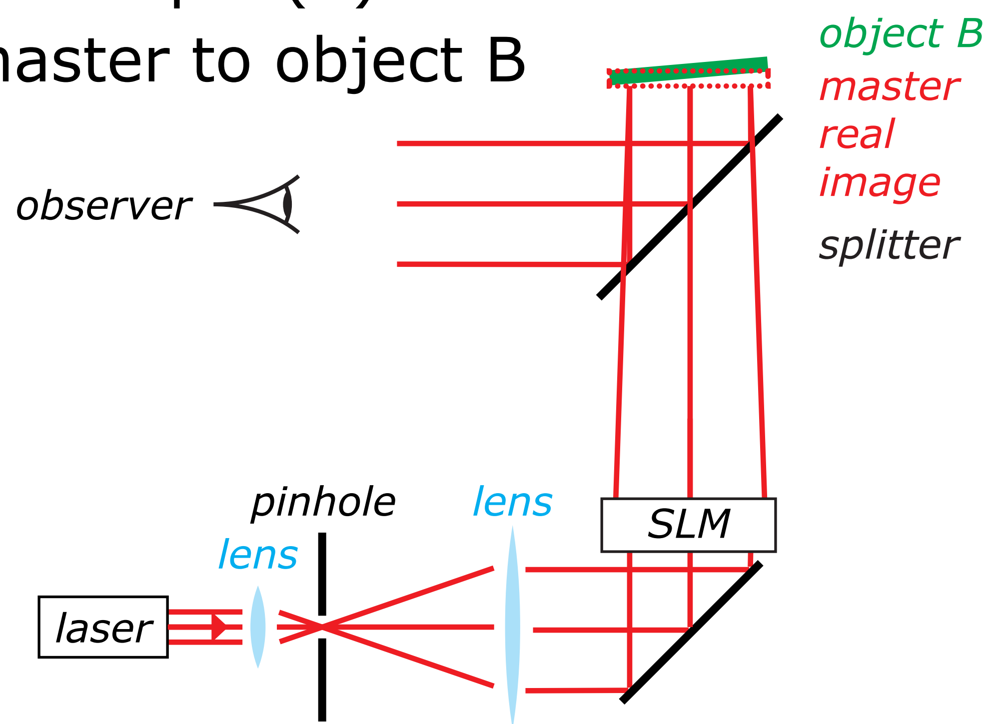
- hologram of master sample (A)
- reconstruction of master to object B



captured phase



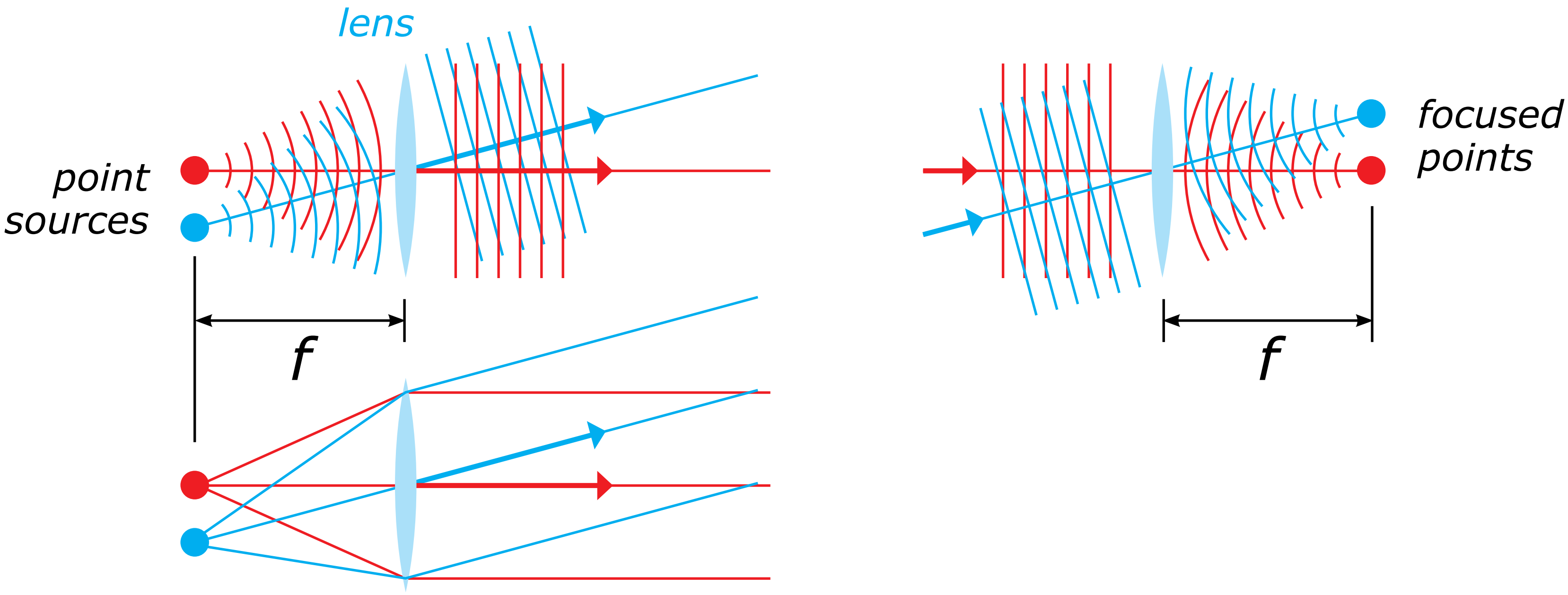
unwrapped phase



(Jüptner, Schnars: Digital Holography)

Signal processing

- conversion between plane and spherical wave: convex lens of focal length f



Digital holography applications

Signal processing

- complex amplitude of plane wave at plane $z = 0$:
  $U(x, y) = A_{ab} \exp(-\mathrm{j}[ax + by])$
- a, b depend on wave inclination
- illumination with many plane waves:
  $U(x, y) = \int_a \int_b A_{ab} \exp(-\mathrm{j}[ax + by]) \, \mathrm{d}a \, \mathrm{d}b$

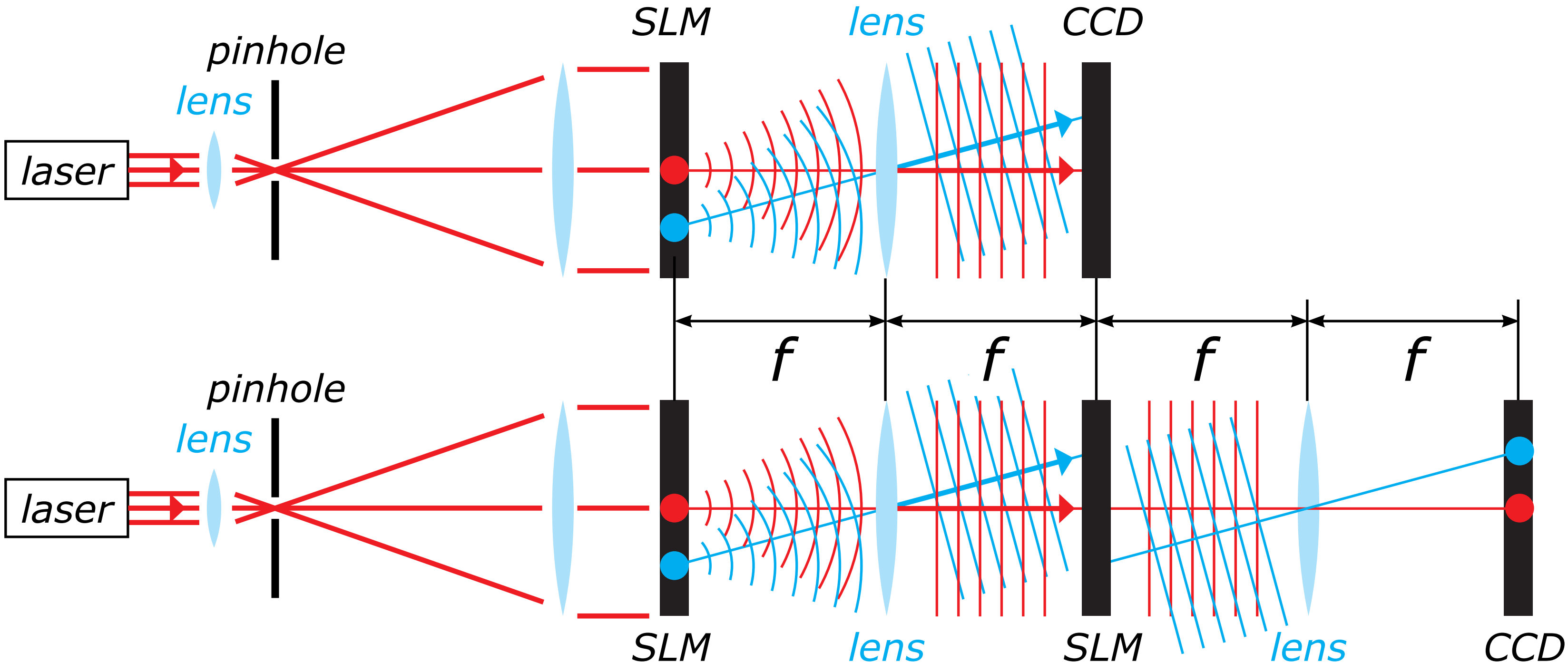
$\Rightarrow$ can be considered as Fourier transform of A_{ab}

- Fourier transform (**not a proper definition!**):
  $\mathrm{FT}\{A(a, b)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_{ab} \exp(-\mathrm{j}[ax + by]) \, \mathrm{d}a \, \mathrm{d}b$

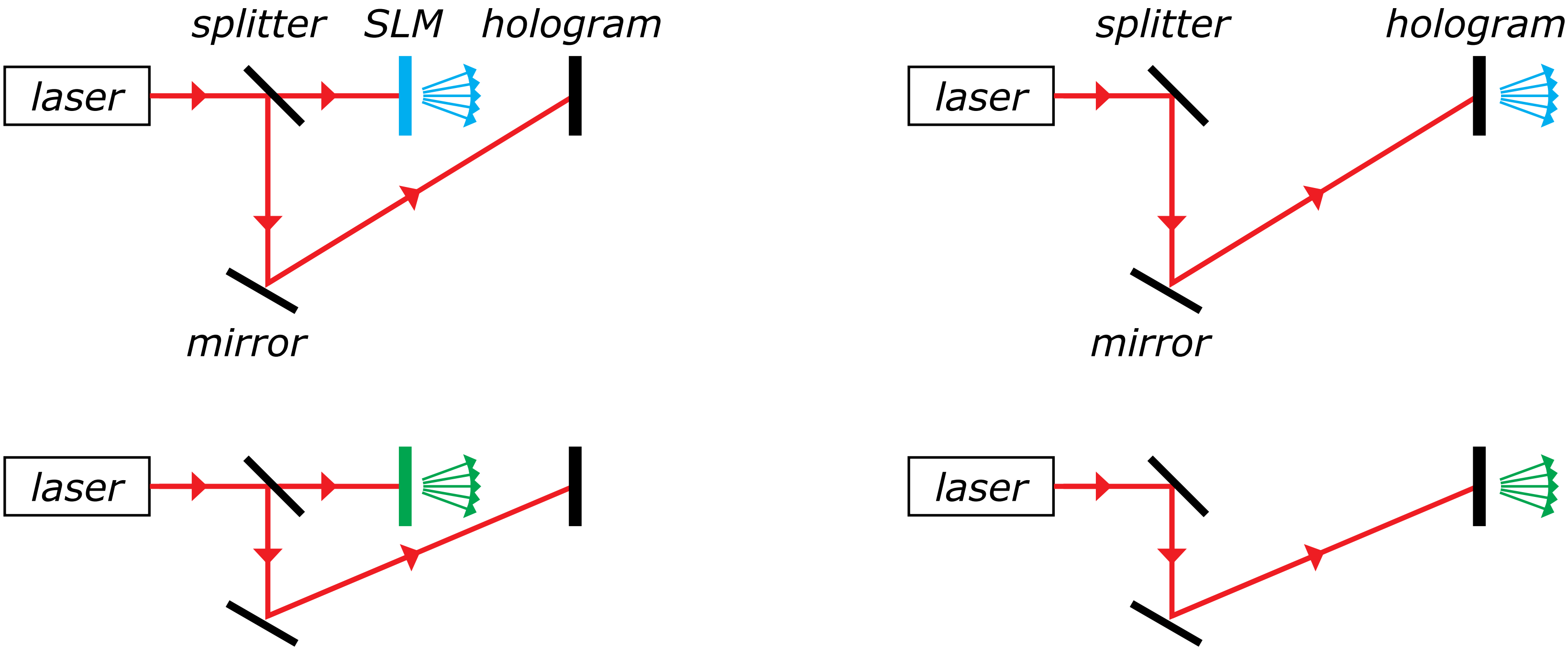
Digital holography applications

Signal processing

- 2*f* system – optical Fourier transform unit
- 4*f* system – optical filtering system



Holographic memory



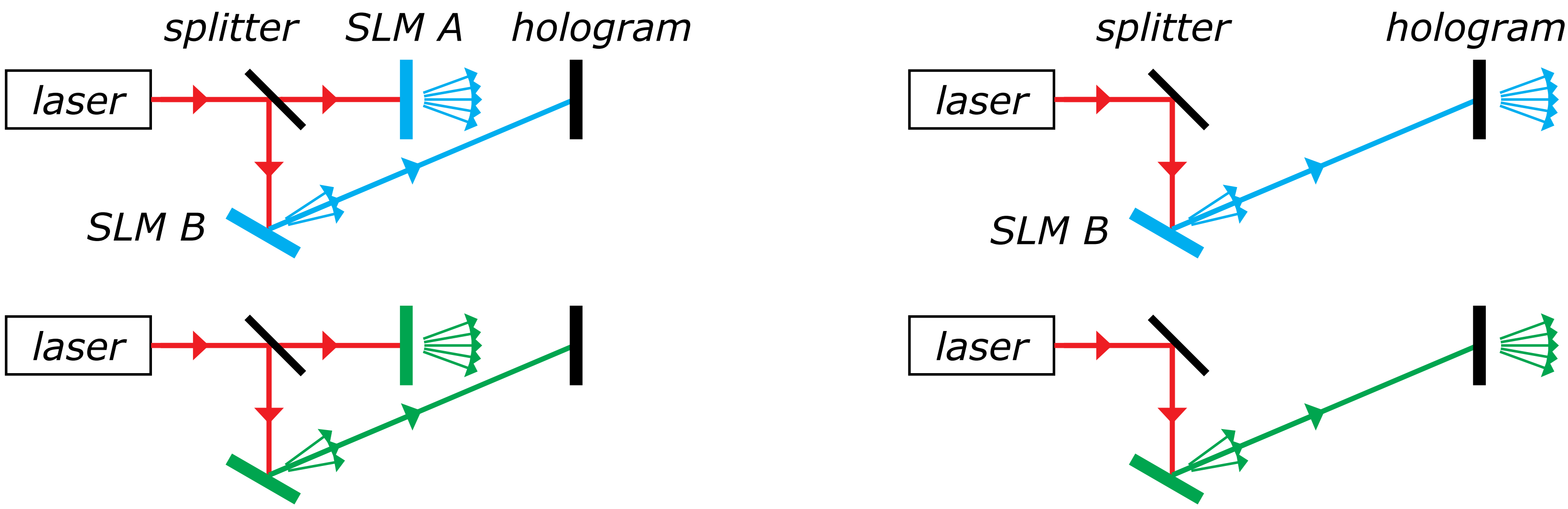
multiple exposure of single hologram

selective reconstruction by reconstruction wave change

Digital holography applications

Holographic memory

- spatial light modulator (SLM) A: data
- SLM B: address



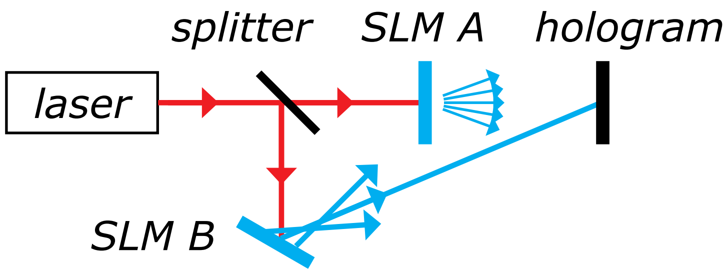
multiple exposure of single hologram

selective reconstruction by reconstruction wave change

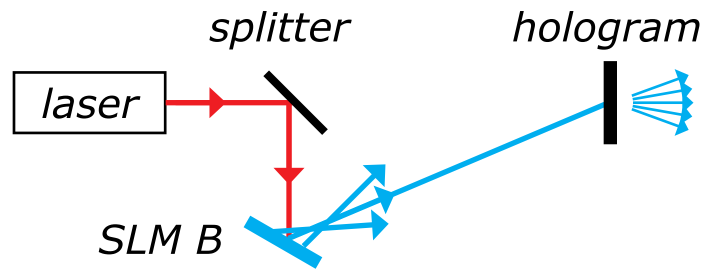
Digital holography applications

Holographic cryptography

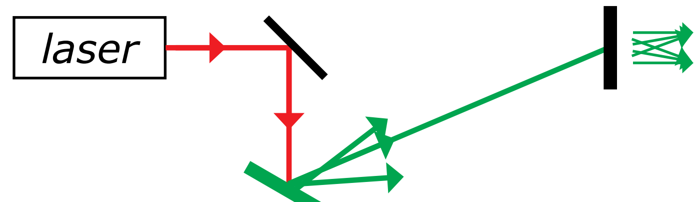
- SLM A: data, SLM B: key
- wrong key reconstruction: scrambled output



encryption



correct key reconstruction



wrong key reconstruction

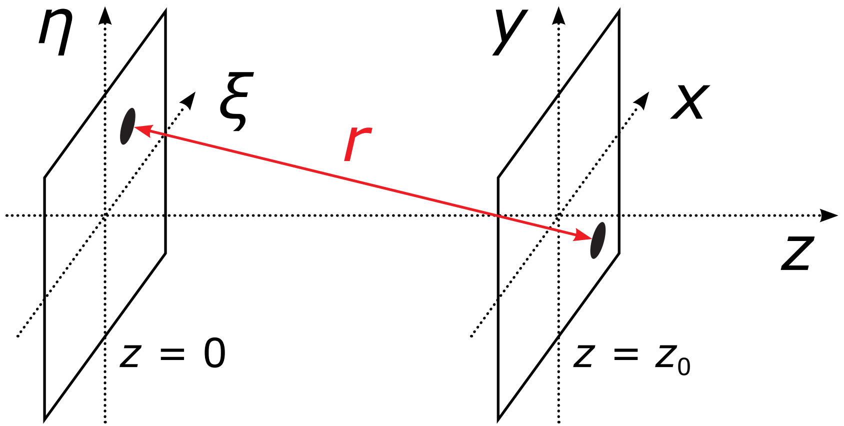
Numerical propagation

Rayleigh-Sommerfeld integral

$$U(x, y, z_0) = -\frac{1}{2\pi} \iint_{hologram} U(\xi, \eta, 0) \times$$

$$\times \left(-jk - \frac{1}{r}\right) \frac{\exp(-jkr)}{r} \frac{z_0}{r} \, d\xi \, d\eta$$

$$r = [(x - \xi)^2 + (y - \eta)^2 + z_0^2]^{1/2}$$



Numerical propagation

Discrete calculation

- discretization of areas to $M \times N$ samples
- samples distance Δ
- $x = (m - M/2)\Delta$, $y = (n - N/2)\Delta$
- $$U'[p, q] = -\frac{1}{2\pi}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} U[m, n]\, K[p-m, q-n]$$

$$K[\mathrm{p}, \mathrm{q}] = (-\mathrm{j}k - \frac{1}{r})\frac{\exp(-\mathrm{j}kr)}{r}\frac{z_0}{r}$$

$$r = [(p\Delta)^2 + (q\Delta)^2 + {z_0}^2]^{1/2}$$

Numerical propagation

Discrete calculation

- $U'[p, q] = -\frac{1}{2\pi}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} U[m, n]\, K[p - m, q - n]$
- $p = 0, m = M - 1; \quad q = 0, n = N - 1$
  $\Rightarrow$ minimal indices K: $-(M - 1), -(N - 1)$
- $p = M - 1, m = 0; \quad q = N - 1, n = 0$
  $\Rightarrow$ maximal indices K: $+(M - 1), +(N - 1)$

- K has to be known in $(2M - 1) \times (2N - 1)$ samples

Numerical propagation

Discrete cyclic convolution

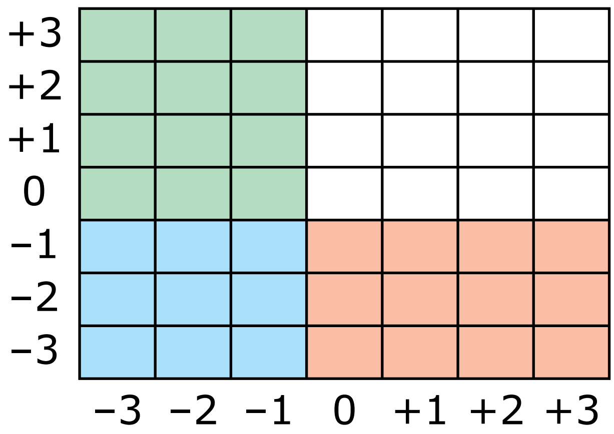
- padding $U[m, n]$ with zeros to $(2M - 1) \times (2N - 1)$
- $$U'[p, q] = -\frac{1}{2\pi} \sum_{m=0}^{2M-2} \sum_{n=0}^{2N-2} U[m, n] \times$$
  $$\times K_c[p - m \ (\mathrm{mod}\ 2M - 1), q - n \ (\mathrm{mod}\ 2N - 1)]$$
  $$= -\frac{1}{2\pi} \mathbf{IDFT}\{\mathbf{DFT}(\mathrm{U}) \odot \mathbf{DFT}(\mathrm{K})\}$$

DFT	discrete Fourier transform
IDFT	inverse discrete Fourier transform
$\odot$	element-by-element multiplication

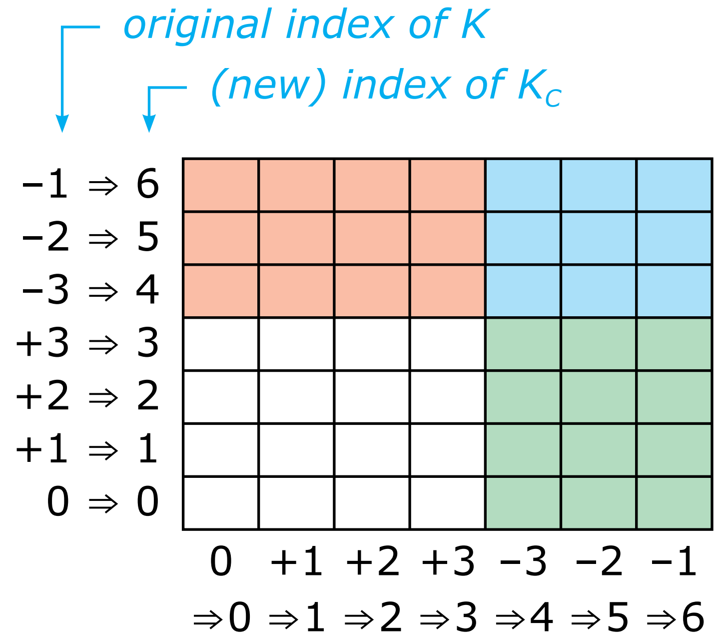
Numerical propagation

Discrete cyclic convolution

- example for $M = N = 4$



structure of K



structure of K_c

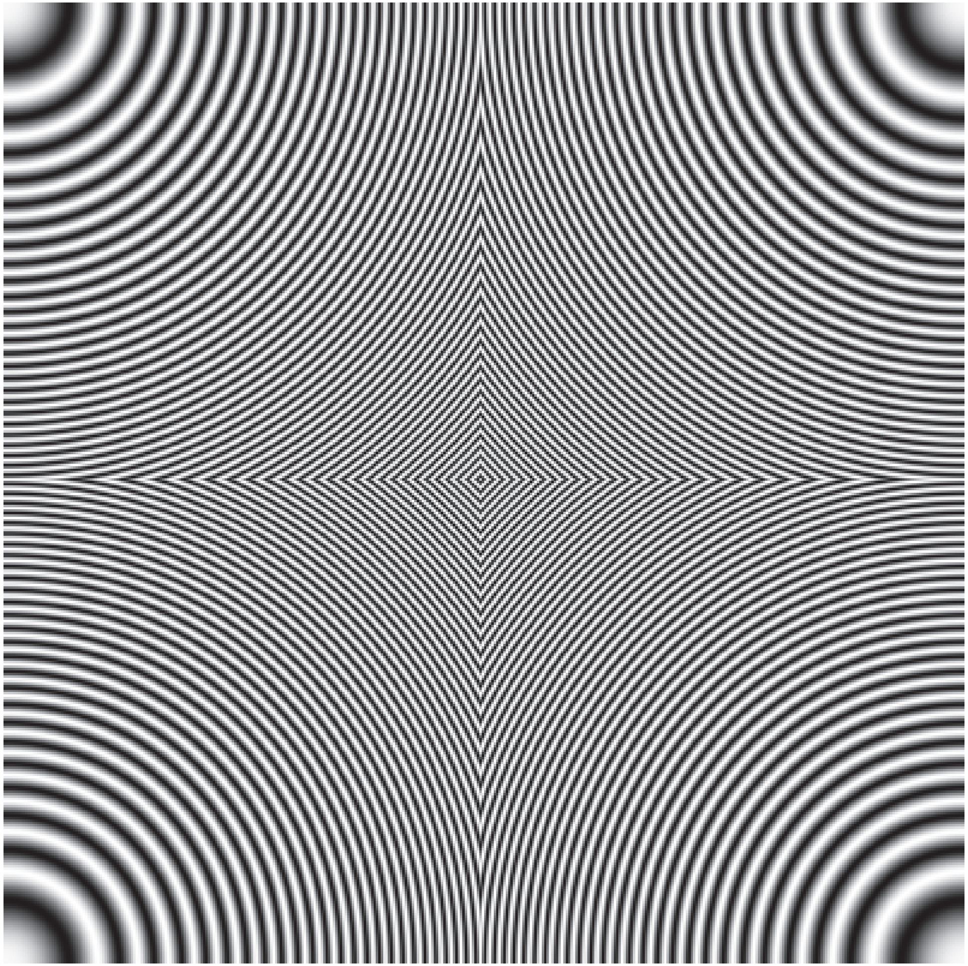
Numerical propagation

```
propag_z   = -0.5;

kernel = zeros(2*res_y, 2*res_x);
if (propag_z < 0) ii = -i; else ii = i; endif

for column = 1:2*res_x
  for row = 1:2*res_y

    if (column < res_x)
      x = (column-1) * sampling;
    else
      x = (column-2*res_x-1) * sampling;
    endif
```

Numerical propagation

```
    if (row < res_y)
      y = (row-1) * sampling;
    else
      y = (row-2*res_y-1) * sampling;
    endif

    r2 = x**2 + y**2 + propag_z**2;
    kernel(row,column) =
                  ii * k * exp(ii*k*sqrt(r2)) / r2;
  endfor
endfor
```

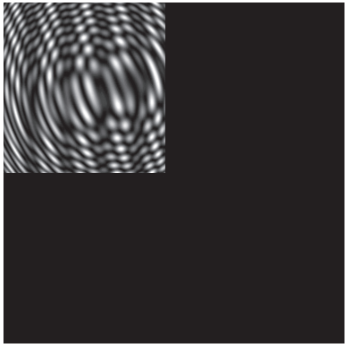
Numerical propagation



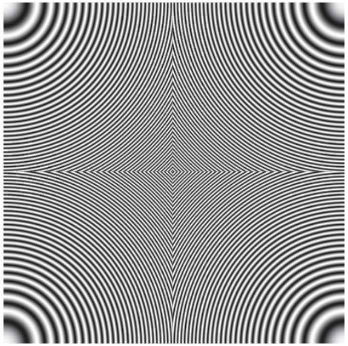
Real part of the Rayleigh-Sommerfeld cyclic convolution kernel (Just for information; it has no physical meaning!)

Numerical propagation

```
field = zeros(2*res_y, 2*res_x);
field(1:res_y, 1:res_x) = hologram;

FTfield = fft2(field);
FTkernel = fft2(kernel);

FTfield2 = FTfield .* FTkernel;

field2 = ifft2(FTfield2);
image = field2(1:res_y, 1:res_x);
```

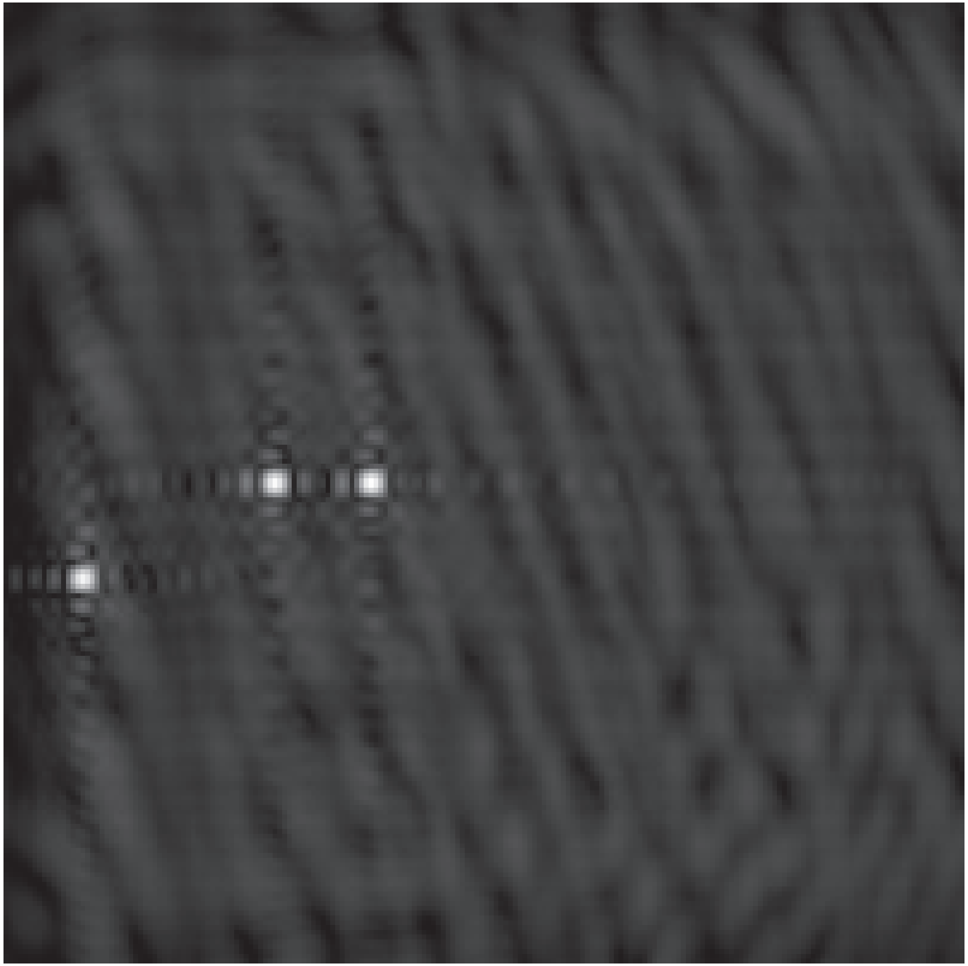


real(field)



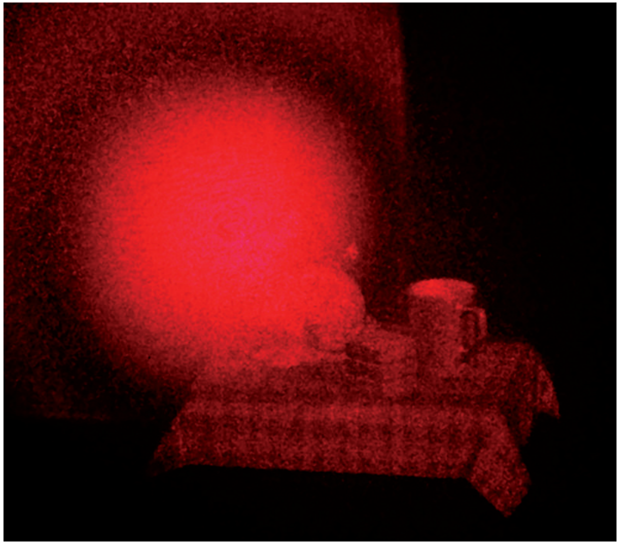
real(kernel)

Numerical propagation



Numerical simulation of hologram propagation (Intensity picture – this would be actually captured)

Numerical propagation



Optical reconstruction

Numerical reconstruction

Numerical propagation

- forward propagation
  - in the z+ axis direction
  - hologram propagation – in a distance $z_0 > 0$ real image appears – on-screen projection
  - original complex field propagation $U_P(x, y, 0)$ – no real image on z+ axis
- backward propagation
  - propagation to a distance $z_0 < 0$
  - convolution kernel K_C has to be complex conjugate

Numerical propagation

Lens simulation

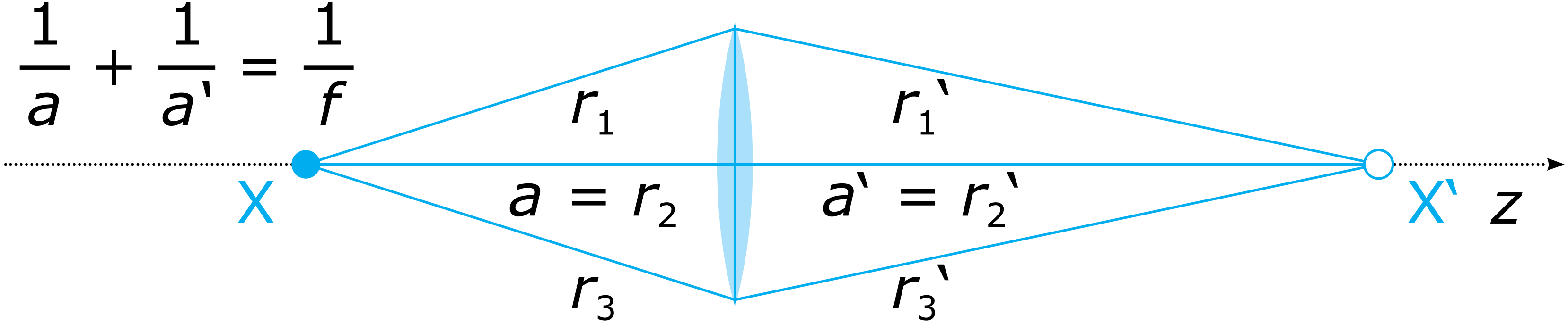
1. propagation to a distance r: phase shift kr
2. propagation in a lens: phase shift φ
3. propagation to a distance r‘: phase shift kr‘

- all contributions in phase in point X‘

⇒ phase function of a lens $\varphi = -(kr + kr')$

⇒ in (x, y, 0): $\varphi = -k[(x^2+y^2+a^2)^{1/2} + (x^2+y^2+a'^2)^{1/2}]$

$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}$$



Hologram of a 3D scene

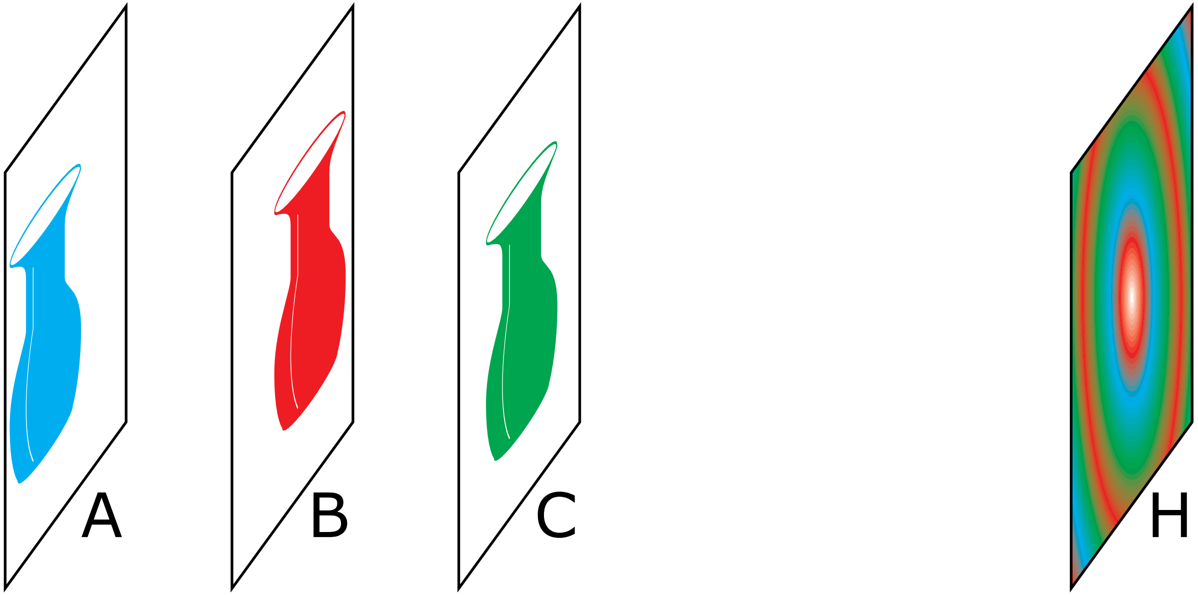
- object replacement with a point cloud
  - extraordinary number of lights needed ⇒ slow
  - does not count with visibility
  - easy parallelization ⇒ fast for thousands of points

Hologram of a 3D scene

- object replacement with a flat image
  - the same as hologram propagation – use of DFT
  - not for a 3D scene

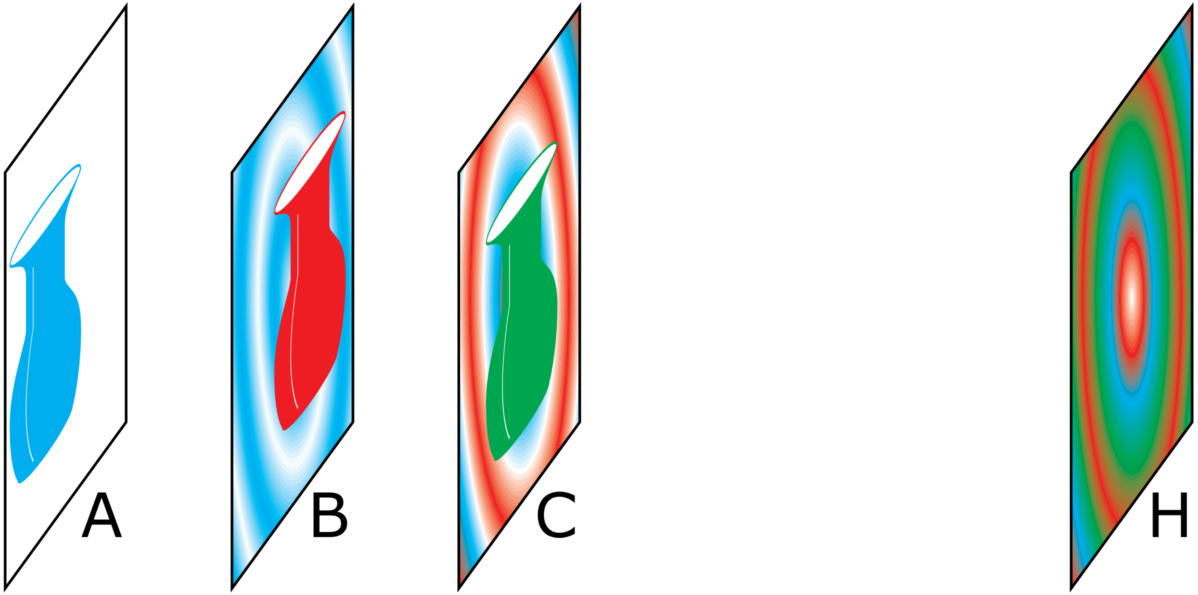
Hologram of a 3D scene

- object replacement with series of flat images
  - propagation A→H, B→H, C→H, sum
  - simulation of 3D scene, use of DFT
  - does not count with visibility



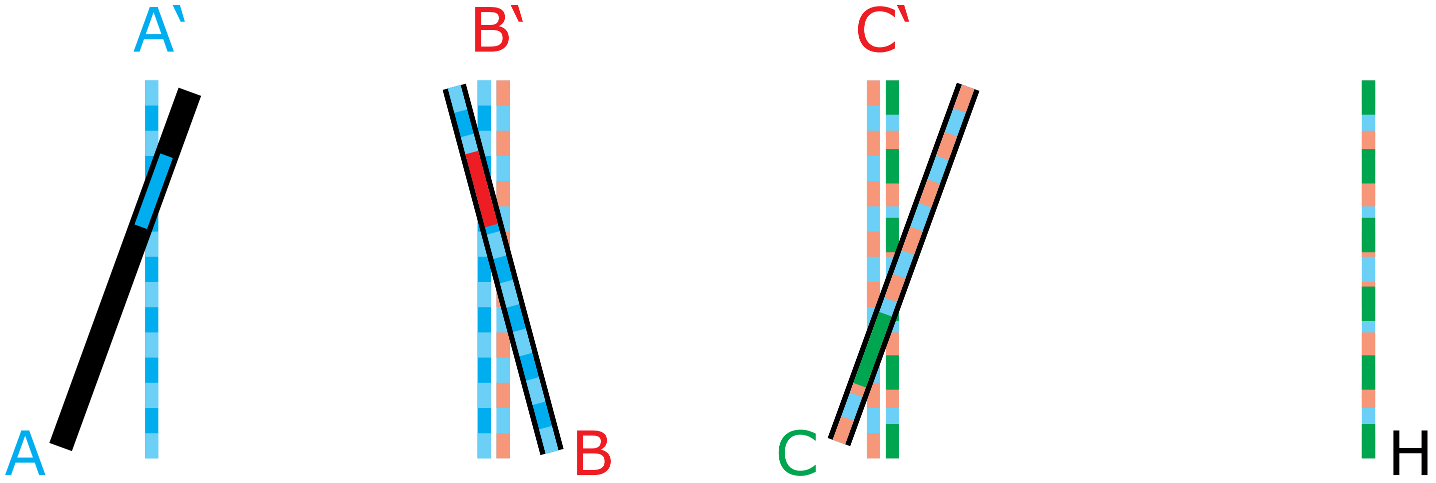
Hologram of a 3D scene

- step-by-step propagation
  - propagatiom A→B, masking, B→C, masking, C→H
  - enables to replace 3D scene with several slices



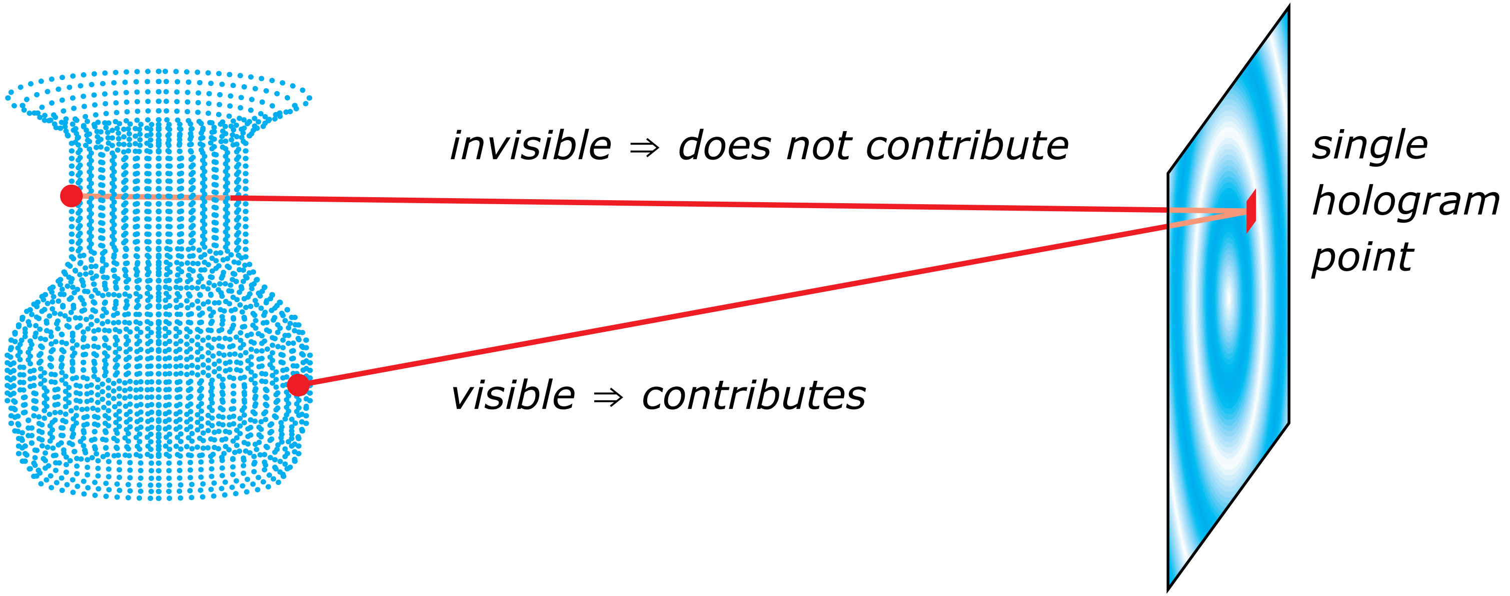
Hologram of a 3D scene

- general step-by-step propagation
  - rotation A→A‵, propagation A‵→B‵, rotation B‵→B, masking, rotation B→B‵, propagation B‵→C‵, …
  - enables to render a scene with textured polygons



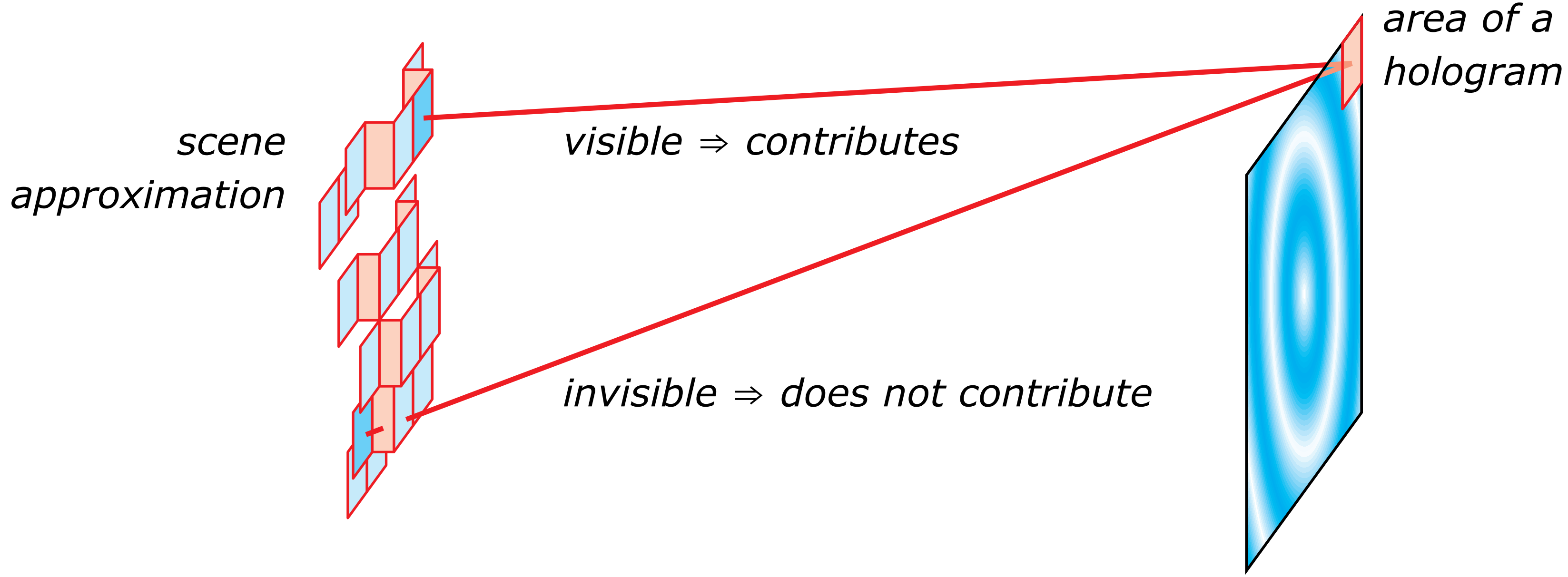
Hologram of a 3D scene

- point cloud rendering enhanced with ray casting for visibility testing
  - extremly slow



Hologram of a 3D scene

- scene breakup to rectangular patches
  - common visibility solution
    for a number of point sources
    and a number of hologram points



Hologram of a 3D scene

- analytic triangle patch propagation formula
  - visibility solution in one view only (mostly)
  - problem with diffuse surface reflection
- analytic line propagation formula
  - for wireframe models

Hologram of a 3D scene

- precalculated table of point sources fields, their fast summation on GPU
- approximation of light propagation
  - Rayleigh-Sommerfeld convolution 3× DFT
  - angular spectrum decomposition 2× DFT, direct calculation of DFT(kernel)
  - Fresnel approximation 1× DFT, paraxial
  - Fraunhofer approximation 1× DFT, paraxial, big distances

Angular spectrum decomposition

- a plane wave hitting plane z = 0:
  $U(x, y, 0) = A \exp\{-jk(ax + by)\}$
  propagation vector $\mathbf{n} = (a, b, [1 - a^2 - b^2]^{1/2})$
  $\left.\begin{array}{l} a = \mathbf{n} \cdot (1, 0, 0) = \cos\theta_x \\ b = \mathbf{n} \cdot (0, 1, 0) = \cos\theta_y \end{array}\right\}$ direction cosines
- many plane waves hitting plane z = 0:
  $U(x, y, 0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A(a/\lambda, b/\lambda) \exp\{-jk(ax + by)\}\, da\, db$
  with $A(a/\lambda, b/\lambda) = 0$ for $|a| > 1$, $|b| > 1$
  - definition of $A(a/\lambda, b/\lambda)$ instead of clearer $A(a, b)$ will be advantageous in a while

Angular spectrum decomposition

- more often: $f_x = a/\lambda$, $f_y = b/\lambda$
  i.e.
  $$U(x,y,0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A(f_x,f_y) \exp\{-2\pi j(f_x x + f_y y)\}\, df_x df_y$$
  $$= \mathrm{FT}\{ A(f_x,f_y) \}$$
  i.e.
  $$A(f_x,f_y) = \mathrm{FT}^{-1}\{ U(x,y,0) \}$$
  $$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} U(x,y,0) \exp\{2\pi j(f_x x + f_y y)\}\, dx dy$$

Angular spectrum decomposition

- a plane wave hitting plane $z = z_0$:

$$U(x,y,z_0) = A \exp\{-jk(ax + by + cz_0)\}$$
$$= A \exp\{-jk(ax + by)\} \exp\{-jkz_0c\}$$
$$= A \exp\{-jk(ax + by)\} \exp\{-jkz_0[1 - a^2 - b^2]^{1/2})\}$$

- many planes hitting plane $z = z_0$:

$$U(x,y,z_0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A(f_x,f_y) \exp\{-j2\pi z_0[1/\lambda^2 - f_x{}^2 + f_y{}^2]^{1/2})\} \exp\{-j2\pi(f_x x + f_y y)\}\, df_x df_y$$
$$= FT\{ A(f_x,f_y) \exp\{-j2\pi z_0[1/\lambda^2 - f_x{}^2 + f_y{}^2]^{1/2})\}\}$$

Angular spectrum decomposition

Angular spectrum propagation

input: $U(x, y, 0)$

output: $U(x, y, z_0)$

calculation:

$$A(f_x,f_y) = \mathrm{FT}^{-1}\{\, \mathrm{U}(x,y,0) \,\}$$

$$\mathrm{U}(x,y,z_0) = \mathrm{FT}\{\, A(f_x,f_y) \exp\{-j2\pi z_0[1/\lambda^2 - f_x^{\,2} + f_y^{\,2}]^{1/2})\}\}$$

- mathematically equivalent to the R-S convolution
- just two Fourier transforms
- numerically easier for small z_0
  (R-S is better for bigger z_0 – see kernel sampling)

Fresnel approximation

Rayleigh-Sommerfeld solution

$$U(x, y, z_0) = -\frac{1}{2\pi} \iint_{hologram} U(\xi, \eta, 0) \times$$

$$\times \left(-jk - \frac{1}{r}\right) \frac{\exp(-jkr)}{r} \frac{z_0}{r} \, d\xi \, d\eta$$

$$\begin{aligned}
r &= [(x - \xi)^2 + (y - \eta)^2 + z_0^2]^{1/2} \\
&= z_0 \, [1 + (x - \xi)^2/z_0^2 + (y - \eta)^2/z_0^2]^{1/2} \\
&\doteq z_0 \, [1 + (x - \xi)^2/2z_0^2 + (y - \eta)^2/2z_0^2] \\
&= z_0 + (x - \xi)^2/2z_0 + (y - \eta)^2/2z_0 \\
&= z_0 + (x^2 + y^2)/2z_0 + (\xi^2 + \eta^2)/2z_0 - (x\xi + y\eta)/z_0
\end{aligned}$$

Fresnel approximation

For $z_0 \gg$ x, y:

$$U(\mathrm{x}, \mathrm{y}, z_0) = -\frac{1}{2\pi} \iint_{hologram} \mathrm{U}(\xi, \eta, 0) \times$$

$$\times \left(-\mathrm{j}k - \frac{1}{r}\right) \frac{\exp(-\mathrm{j}kr)}{r} \frac{z_0}{r} \, \mathrm{d}\xi \, \mathrm{d}\eta$$

$$\doteq \frac{\mathrm{j}kz_0}{2\pi} \iint_{hologram} \mathrm{U}(\xi, \eta, 0) \frac{\exp(-\mathrm{j}kr)}{r^2} \, \mathrm{d}\xi \, \mathrm{d}\eta$$

$$\doteq \frac{\mathrm{j}k}{2\pi z_0} \iint_{hologram} \mathrm{U}(\xi, \eta, 0) \exp(-\mathrm{j}kr) \, \mathrm{d}\xi \, \mathrm{d}\eta$$

Fresnel approximation

$$\doteq \frac{\mathrm{j}k}{2\pi z_0} \iint_{hologram} \mathrm{U}(\xi, \eta, 0) \times$$

$$\exp(-\mathrm{j}k[z_0 + (x^2 + y^2)/2z_0 +$$

$$(\xi^2 + \eta^2)/2z_0 - (x\xi + y\eta)/z_0]) \,\mathrm{d}\xi \,\mathrm{d}\eta$$

$$= \frac{\mathrm{j}k}{2\pi z_0} \exp(-\mathrm{j}kz_0) \exp(-\mathrm{j}k(x^2 + y^2)/2z_0) \times$$

$$\iint_{hologram} \mathrm{U}(\xi, \eta, 0) \exp(-\mathrm{j}k(\xi^2 + \eta^2)/2z_0) \times$$

$$\exp(-\mathrm{j}2\pi(x\xi + y\eta)/\lambda z_0) \,\mathrm{d}\xi \,\mathrm{d}\eta$$

Fresnel approximation

$$= \frac{\mathrm{j}k}{2\pi z_0} \exp(-\mathrm{j}kz_0) \exp(-\mathrm{j}k(x^2 + y^2)/2z_0) \times$$

$$\mathrm{FT}\{ U(\xi, \eta, 0) \exp(-\mathrm{j}k(\xi^2 + \eta^2)/2z_0) \}$$

where after FT calculation substitute

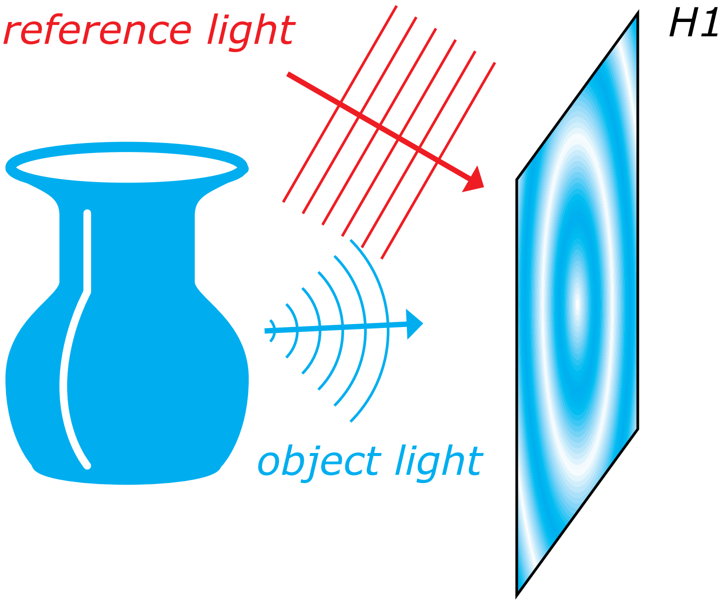
$$f_x = x/\lambda z_0$$

$$f_y = y/\lambda z_0$$

- approximation valid for on-axis case, big z_0
  $z_0{}^3 \gg \pi/4\lambda \max\{[(x - \xi)^2 + (y - \eta)^2]^2\}$
- just one Fourier transform

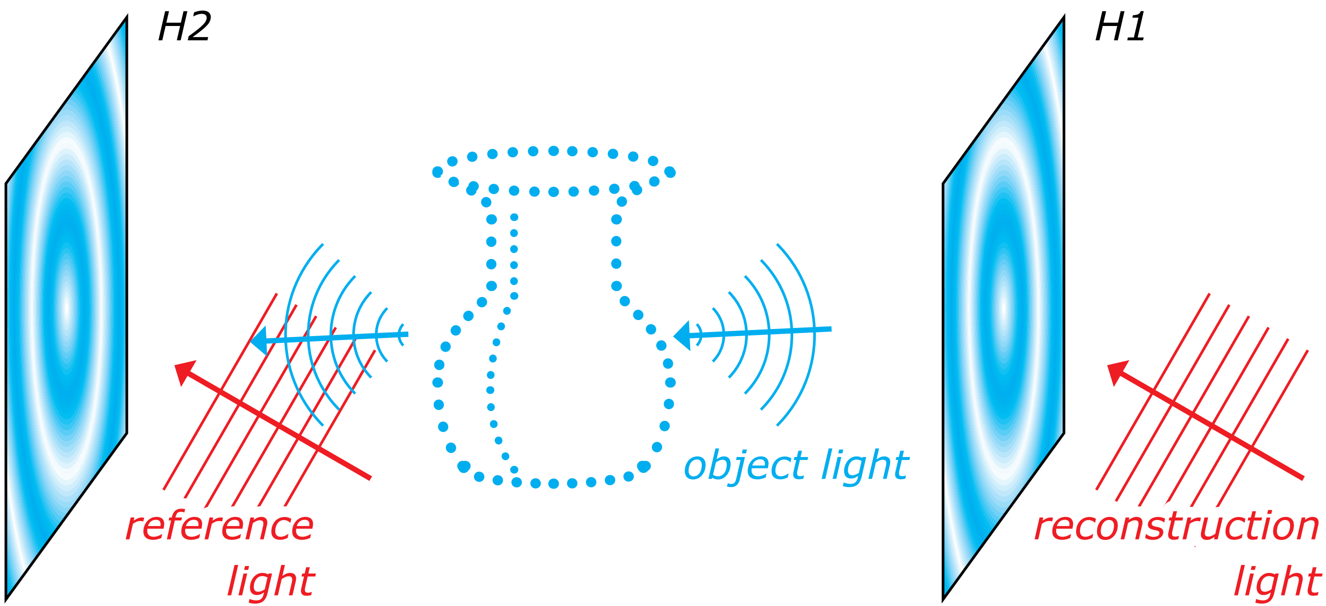
Hologram of a 3D scene

- classical H1 – H2 process
  1. make a classical hologram (H1)



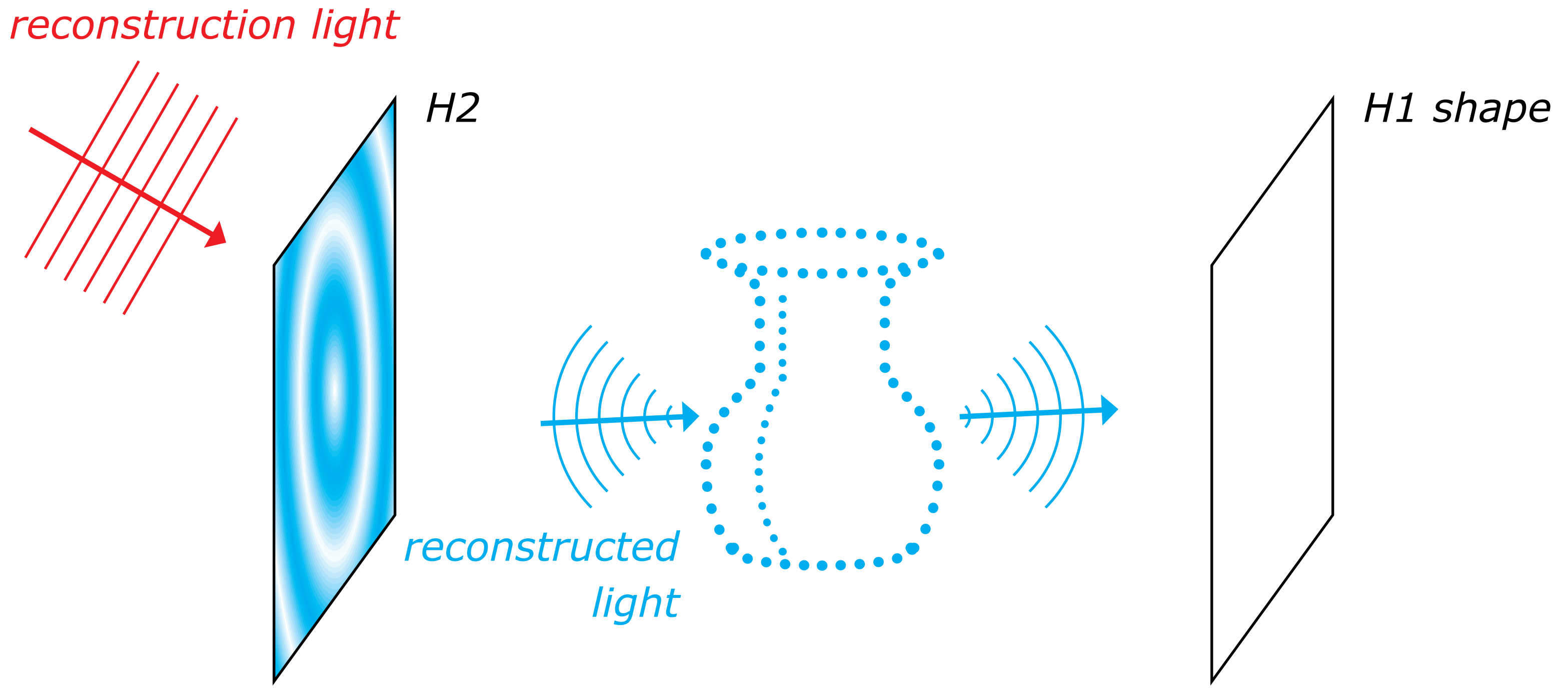
Hologram of a 3D scene

- classical H1 – H2 process
  2. illuminate H1 with a conjugate wave
  3. make a hologram of a hologram (H2)



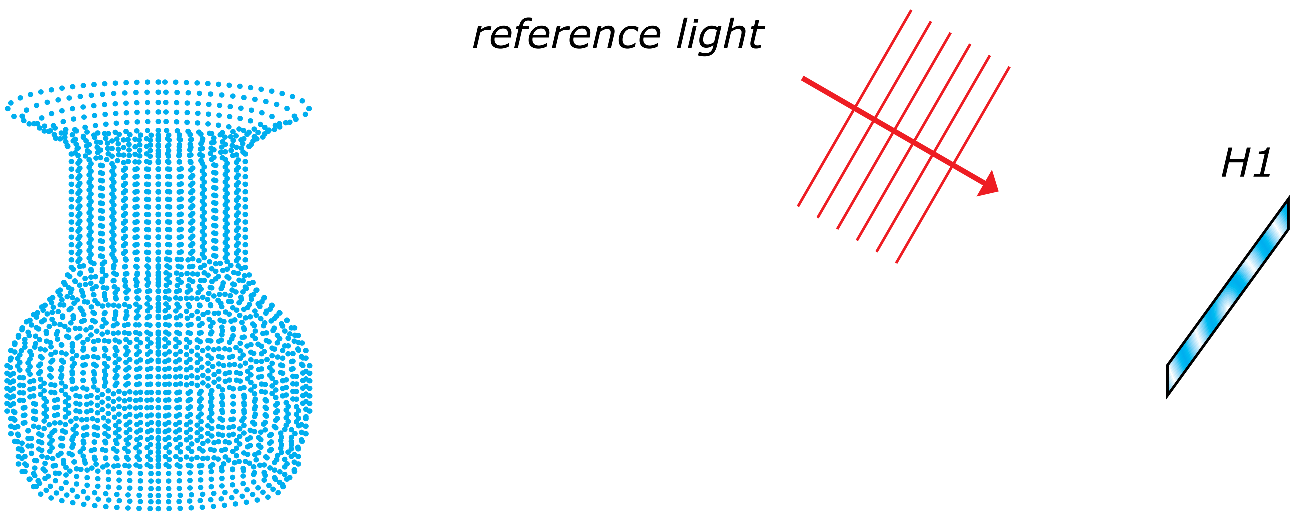
Hologram of a 3D scene

- classical H1 – H2 process
  4. illuminate H2 with a conjugate wave
  – an orthoscopic image, viewing aperture H1



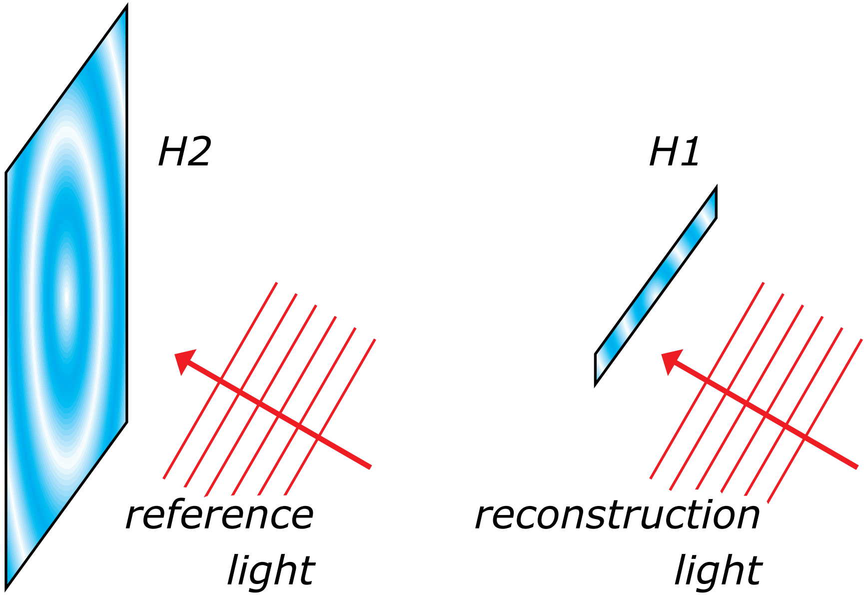
Hologram of a 3D scene

- classical white light hologram
  - H1 – hologram of a scene
    "viewed through a narrow window"
- digitally: slow calculation, small H1 surface



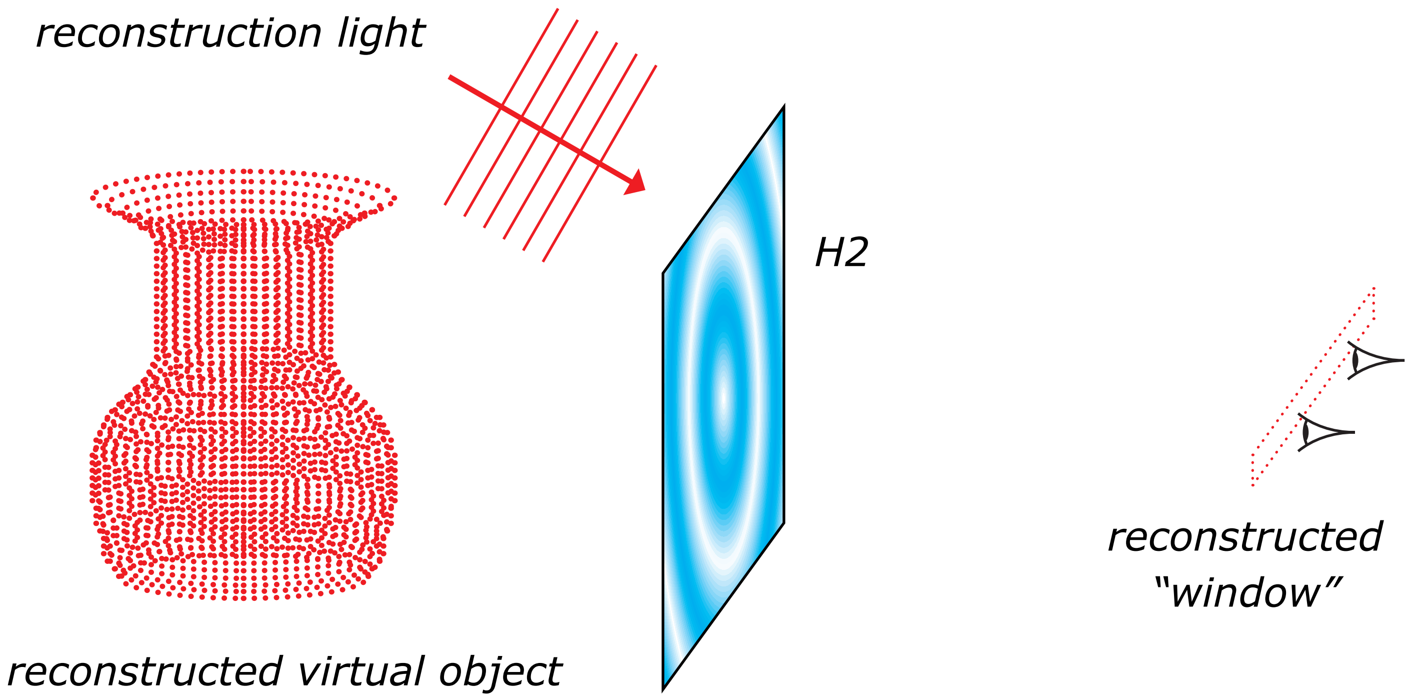
Hologram of a 3D scene

- classical white light hologram
  - H2 – hologram of the H1 hologram
- digitally: no visibility checks ⇒ fast calculation



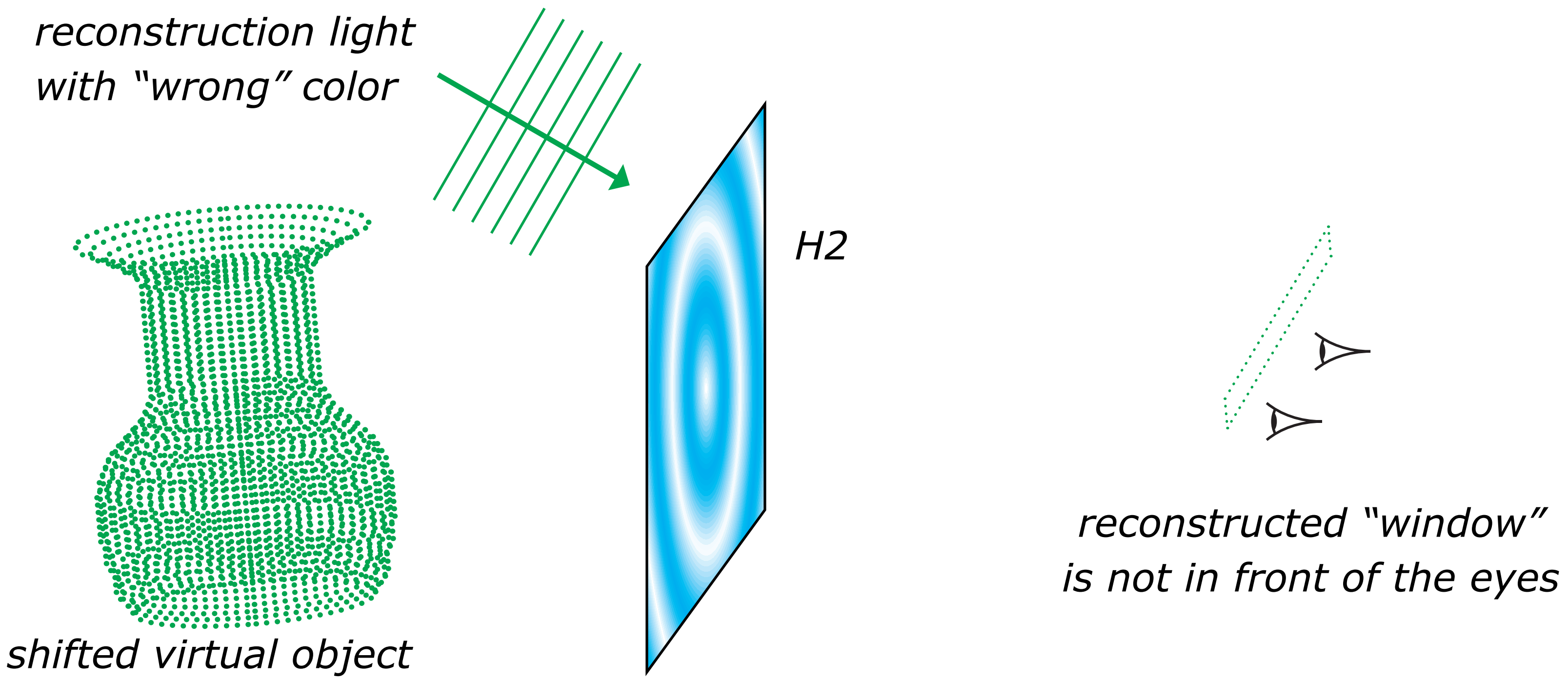
Hologram of a 3D scene

- classical white light hologram reconstruction
  - resembles view through a narrow window
  - horizontal parallax only image



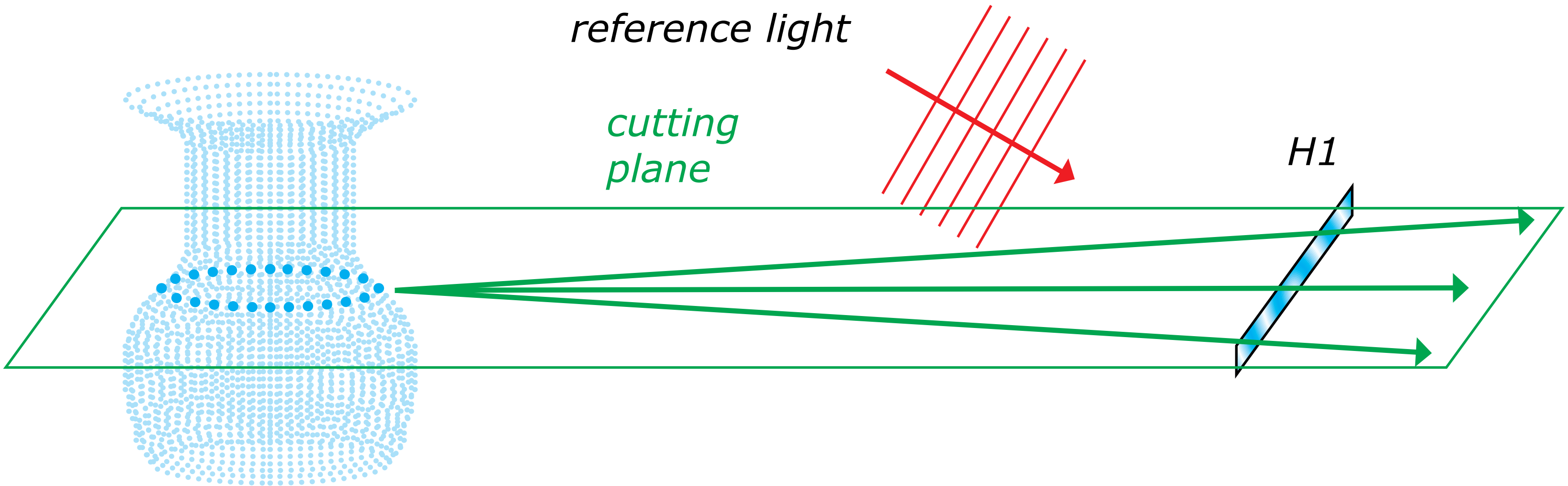
Hologram of a 3D scene

- classical white light hologram reconstruction
  - "wrong" reconstruction color shifts reconstruction
  - H2 extracts "the right" color from white light



Hologram of a 3D scene

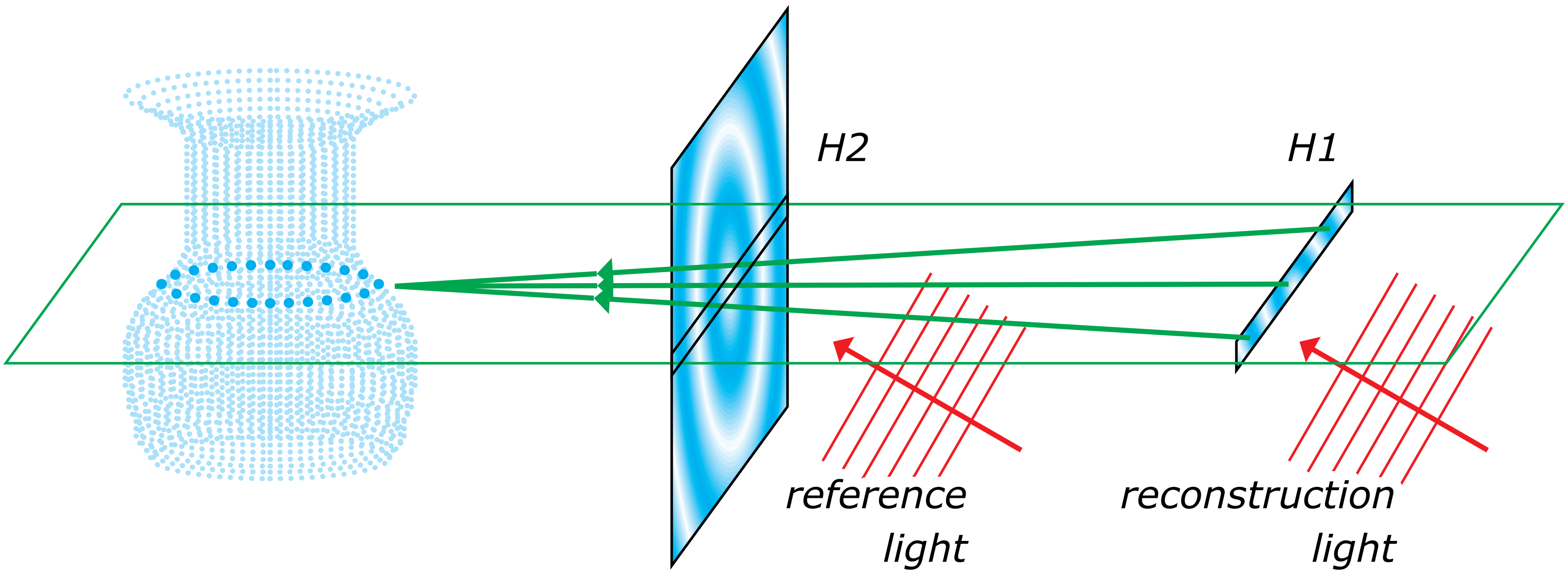
- white light hologram structure
  - just “bold” points will be visible due to rays in the cutting plane



Hologram of a 3D scene

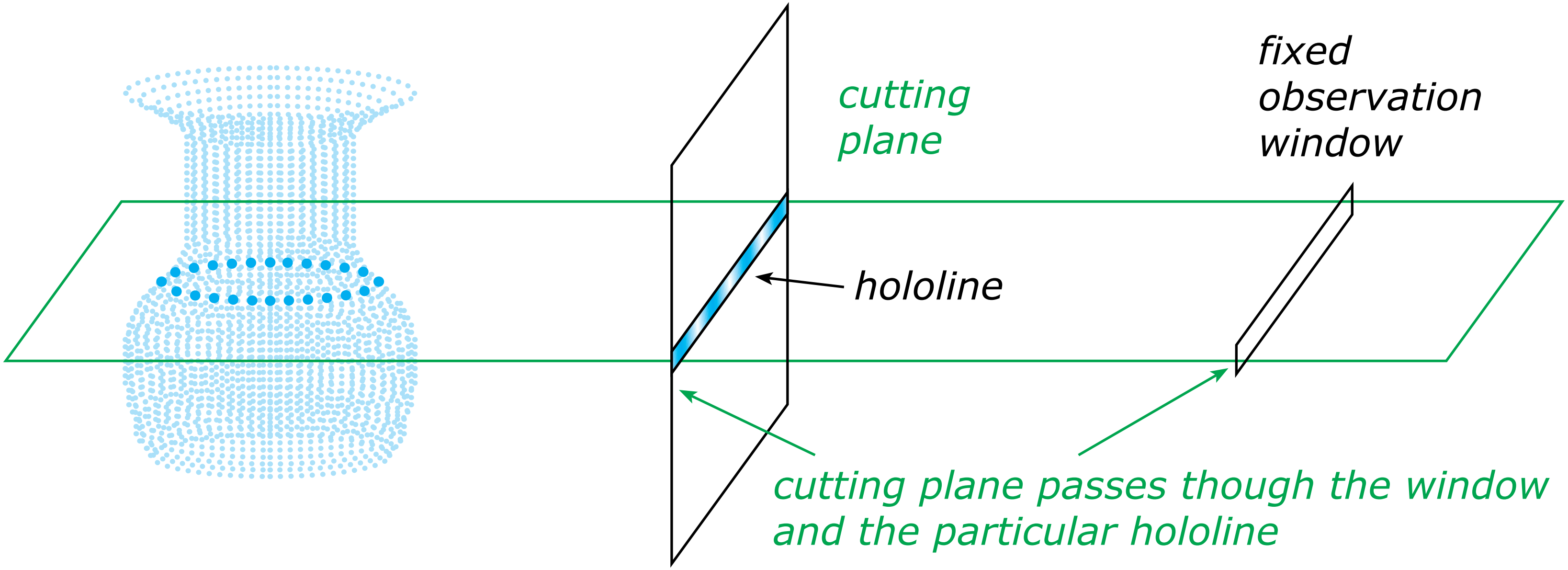
- white light hologram structure
  - in H2 recording, those “bold” point will affect only a part of the H2

  ⇒ “bold” points affect a part of H2 only



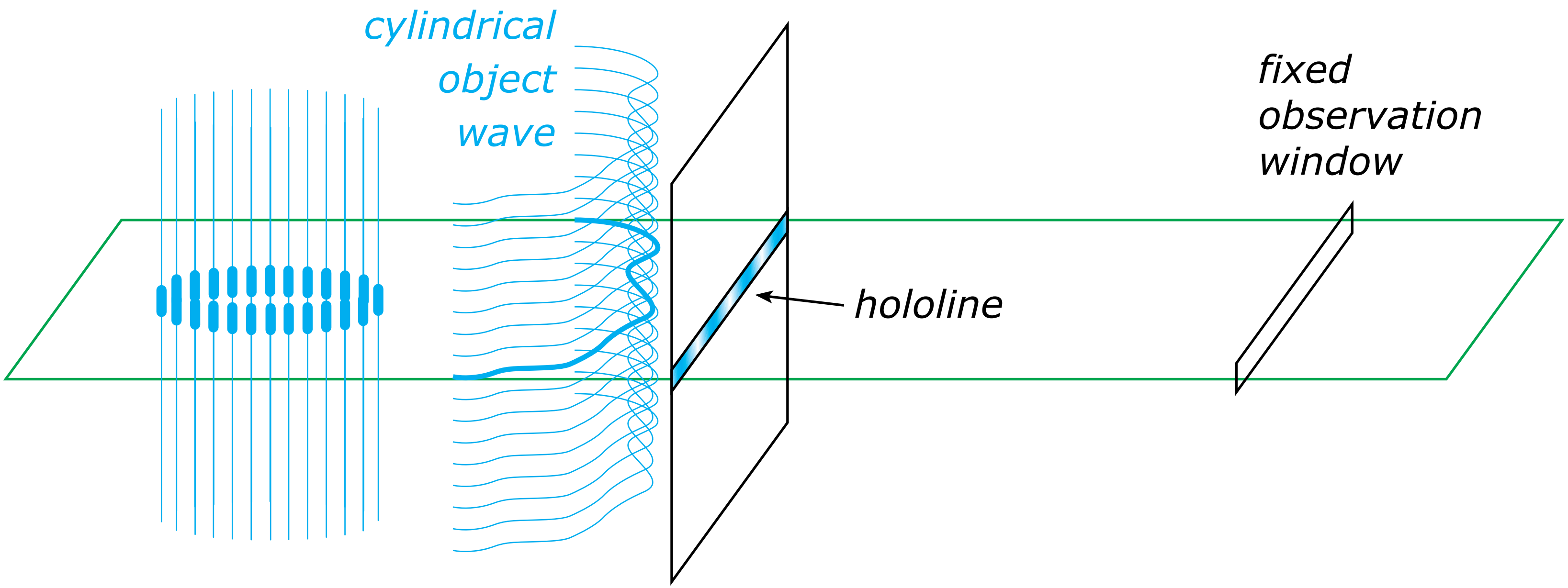
Hologram of a 3D scene

- digital white light HPO hologram (1)
  - split the H2 into parts – hololines
  - just one line of the hololine is considered
  - calculate the hololine using “bold” points only



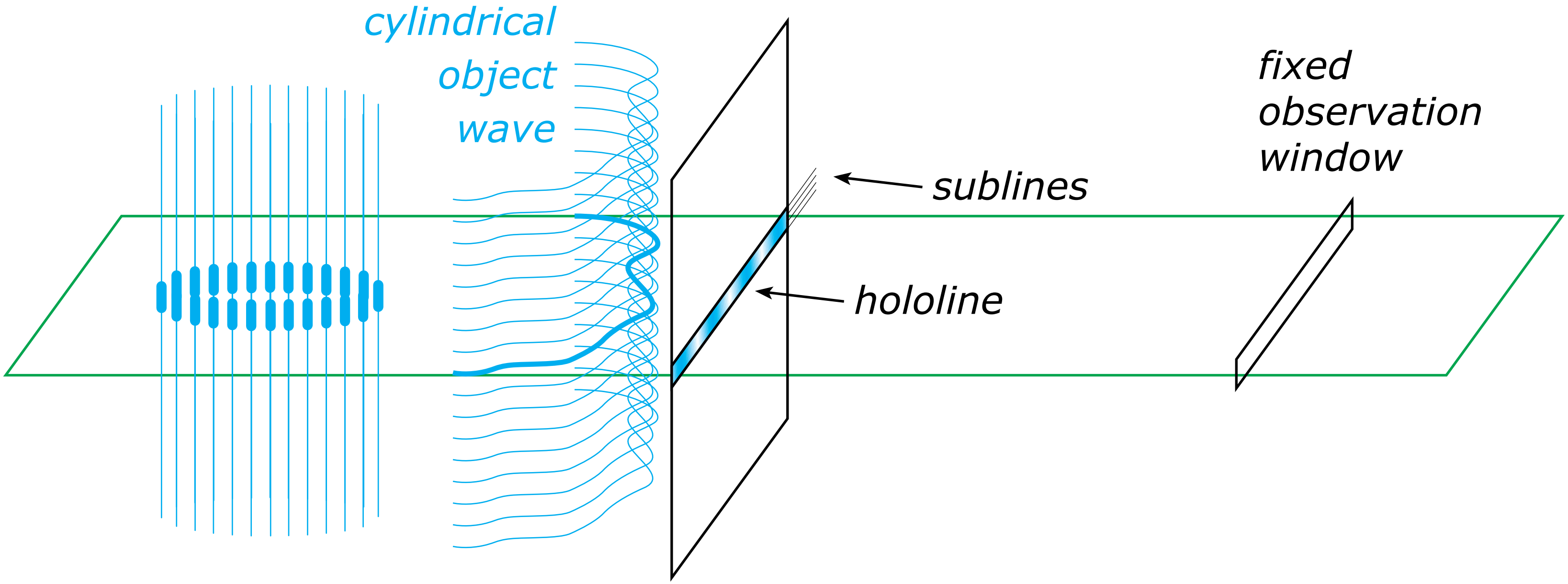
Hologram of a 3D scene

- digital white light HPO hologram (2)
  - – assume the “bold” points to be lines
  - ⇒ they emit cylindrical wave
  - ⇒ object wave constant in vertical direction



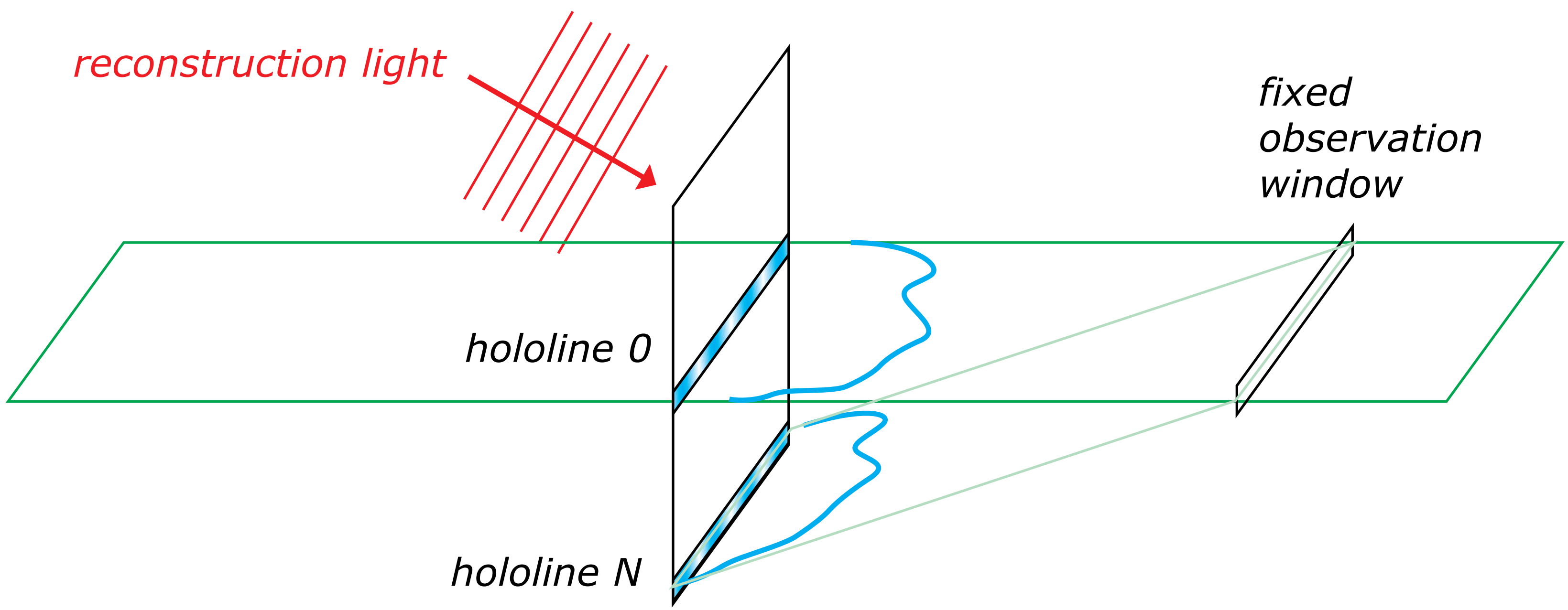
Hologram of a 3D scene

- digital white light HPO hologram (3)
  - hololine has the area *width* × *height*
  - object wave in every horizontal line (subline) is the same ⇒ calculate once & copy



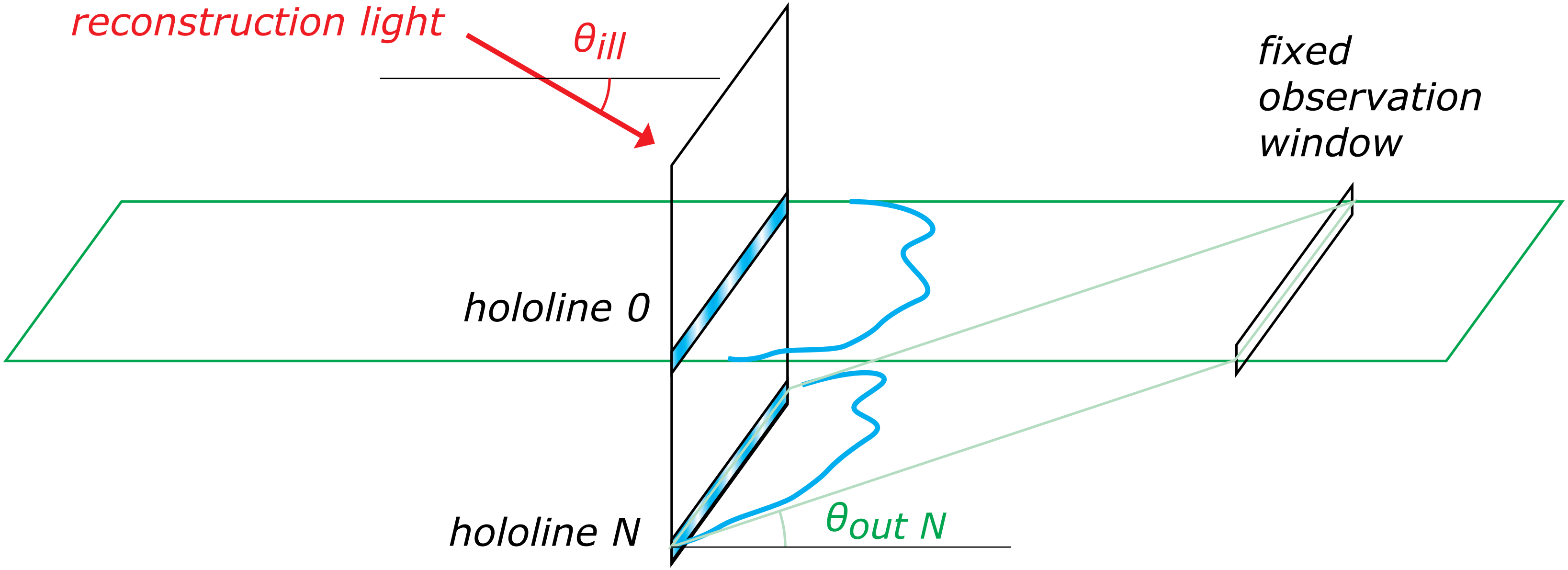
Hologram of a 3D scene

- digital white light HPO hologram (4)
  - in reconstruction, the wave from a hololine should hit the observation window



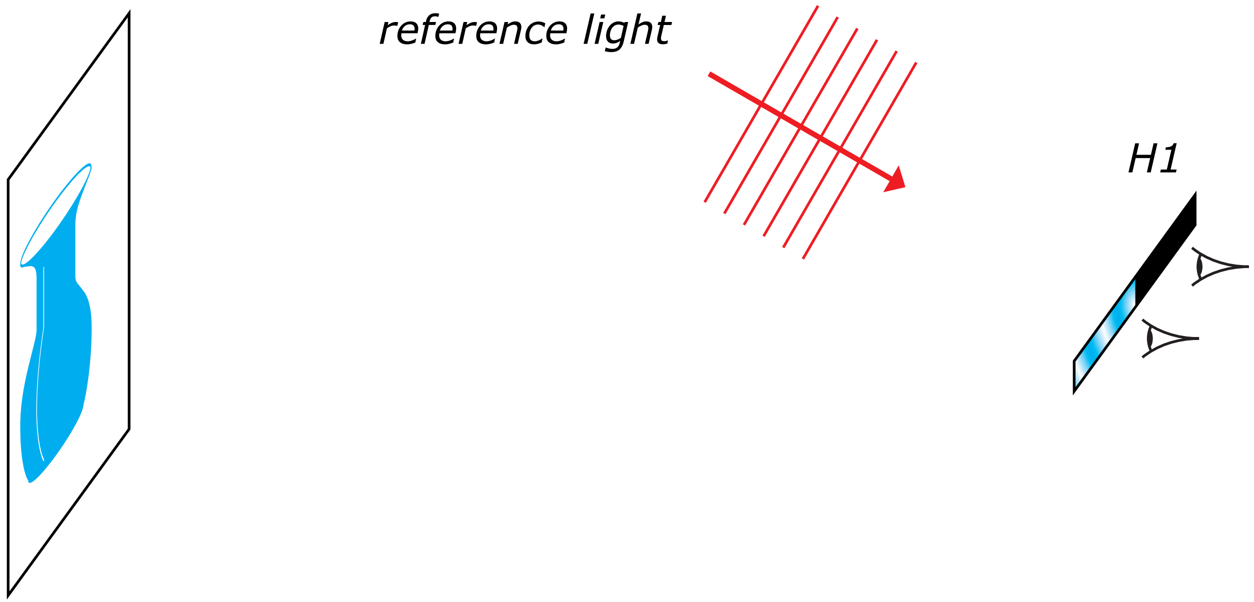
Hologram of a 3D scene

- digital white light HPO hologram (4)
  - the light has to change its angle from θ_{ill} to $\theta_{out\ N}$
  - in hololine N, add reference wave with angle $\sin\theta_{ref} = \sin\theta_{ill} - \sin\theta_{out\ N}$



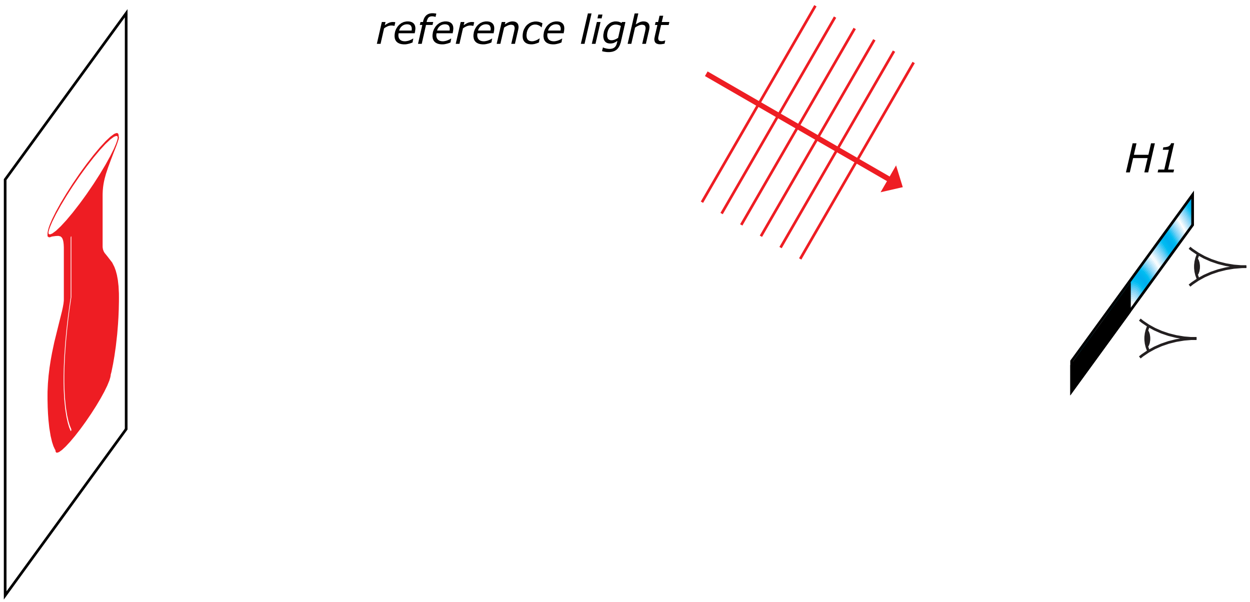
Hologram of a 3D scene

- classical holographic stereogram (1)
  - record left image to the left part of H1 only



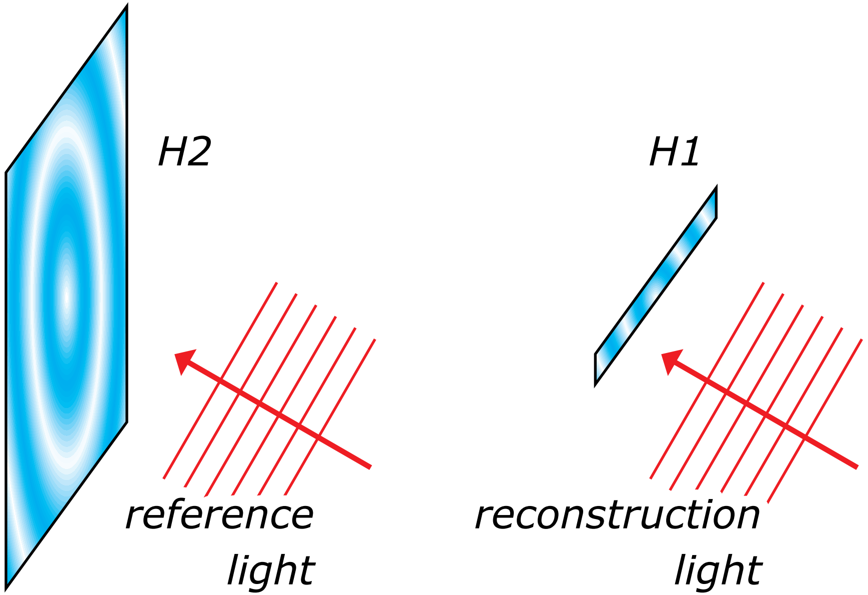
Hologram of a 3D scene

- classical holographic stereogram (2)
  - record right image to the right part of H1 only



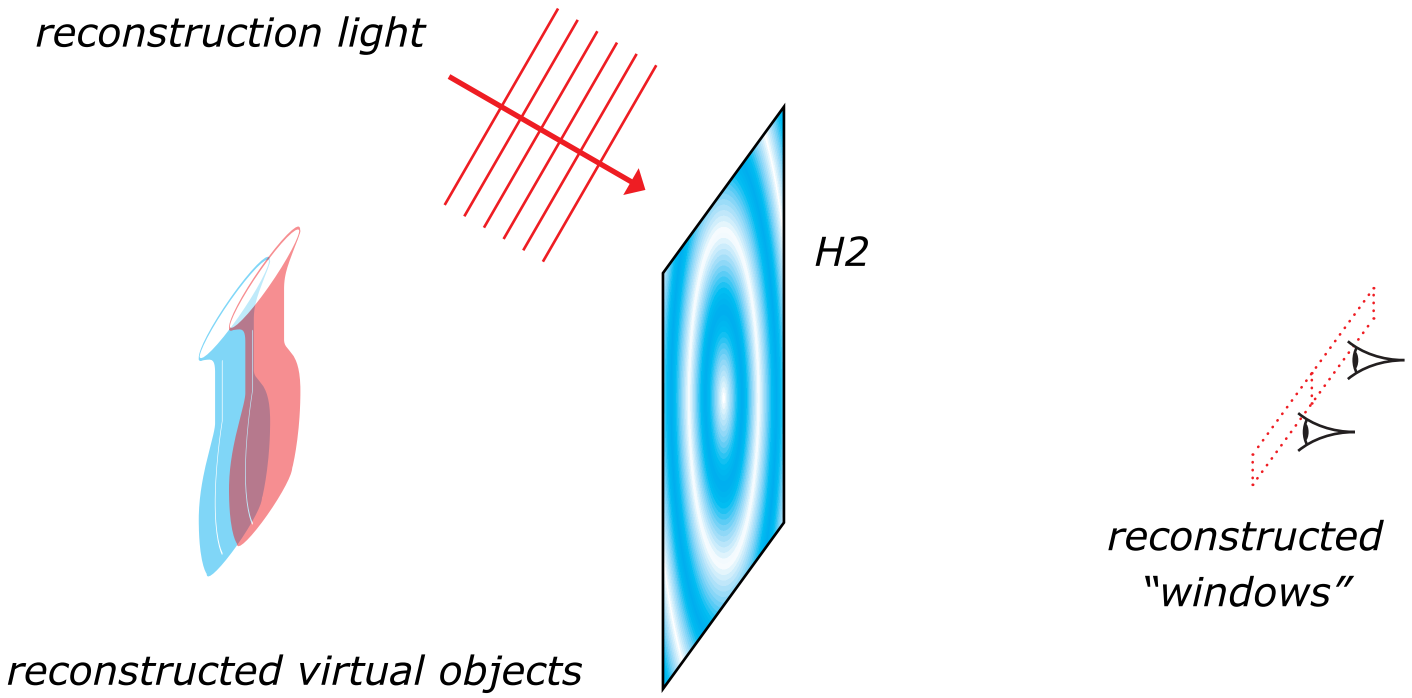
Hologram of a 3D scene

- classical holographic stereogram (3)
  - record H2 in a common way



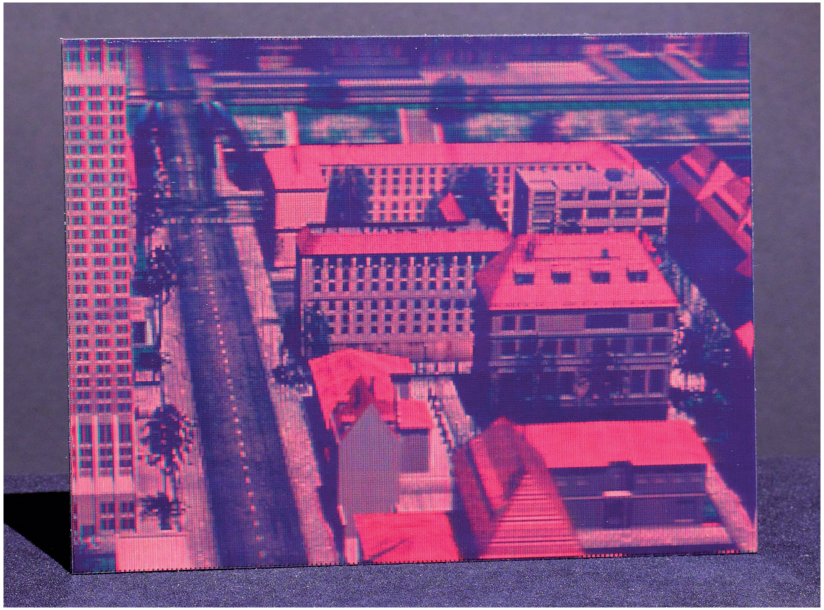
Hologram of a 3D scene

- classical holographic stereogram (4)
  - after illumination, the left eye watches the left image, the right eye watches the right image



Hologram of a 3D scene

- digital holographic stereogram
  - visibility solving in particular directions using computer graphics (ray optics)
  - hologram has to display right image in the right direction
  - compatible with common imaging cameras



Holographic stereogram by Geola Digital

FACULTY
OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

DEPARTMENT OF
COMPUTER SCIENCE
AND ENGINEERING

CENTRE
OF COMPUTER GRAPHICS
AND VISUALIZATION

PLZEŇ
CZECH REPUBLIC

COMPUTER GENERATED
HOLOGRAPHY

3D VISION AND BEYOND

Questions?

- graphics.zcu.cz
- holo.zcu.cz
- www.kiv.zcu.cz